

## Reliability Analysis of the Expected Overtopping Probability of Rubble Mound Breakwater

마루높이 설정을 위한 월파확률의 신뢰성 해석

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### Abstract

The reliability analysis of overtopping probability is proposed. In order to estimate the expected overtopping probability of the rubble mound breakwater, the experimental results of individual wave runup height is applied for the analysis of irregular wave system. The joint distribution of wave heights and periods is used for the input data of runup calculation because the runup height depends on the wave height and period.

The runup heights during the one event that the design wave attacks the rubble mound breakwater extend to the one life cycle of 60 years. Utilizing the Monte-Carlo method, the one life cycle is tried more about 60 times for obtaining the expected value of overtopping probability.

It is found that the inclusion of the variability of wave tidal and wave steepness has great influence on the computation of the expected overtopping probability of rubble mound breakwater. The previous design disregarding the tidal fluctuation largely overestimates or underestimates the expected overtopping probability depending on tidal range and wave steepness.

### 1. Introduction

The reliability design method has been developed for breakwater designs since the mid-1980s, especially in Europe and Japan. In Europe, van deer Meer (1988) proposed a probability approach for the design of breakwater armor layers, and Burcharth(1991) introduced the partial safety factors in the reliability design of rubble mound breakwaters. Recently Burcharth and Sorensen (1999) established the partial safety factor systems for rubble mound and vertical breakwater by summarizing the results of the PIANC Working Groups.

The European reliability design methods belong to the Level 2 method. On the other hand, in Japan the Level 3 reliability design methods have been developed, in which the expected sliding distance of a caisson of a vertical breakwater (Shimosako and Takahashi, 1999; Goda and Takagi, 2000) or the expected damage of the armor blocks of a horizontally composite breakwater (Hanzawa et al., 1996) during its lifetime is estimated through the Monte Carlo simulations by taking the uncertainties of various design factors into computation. The Level 2 method with partial safety factors is easier for engineers to use, but the Level 3 method gives in general more useful design information.

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However, the method has mainly focused on the stability investigation of the breakwater. Among the design procedures of rubble mound breakwater, determination of the crest elevation is one of the important factors. The crest elevation of rubble mound has been designed by the allowable overtopping probability or volume of it. In this study, for the design of crest elevation, the estimation method of overtopping probability by reliability analysis is proposed. We employ the Level 3 reliability design method so as to take into account the variability of wave height, period and tidal level.

## 2. Computational Procedure

The basis of the reliability design method is a due consideration of the stochastic behaviors of loads and resistances. As described in the introduction, the present study employs the Level 3 design method, which introduces the probability density functions of design factors. Use of random variation of design factors is explained below in conjunction with the computational N-S chart sketched in Fig. 2.

### 2.1 Overtopping probability during one storm event

The analysis of wave overtopping requires incorporation of the probability distribution of individual wave heights and periods. If regular wave data for the overtopping are directly applied to the design of a rubble mound breakwater, the error introduced in the estimate of the overtopping probability may be small. Because generally the rubble mound breakwater has been constructed at the deeper water where the surf beat could be induced little by wave breaking.

In this study, the estimation method of the overtopping probability based on the reliability analysis is proposed. The joint distribution of wave height and period is introduced and the Rayleigh distribution for the wave height is applied. The Rayleigh distribution is shown eq.(1).

$$p(x) = \frac{\pi}{2} x \exp\left[-\frac{\pi}{4} x^2\right] \quad (1)$$

where,  $p(x)$  is the probability density,  $H$  is the wave height,  $\bar{H}$  is the means of wave heights,  $H$  is the wave

height,  $\bar{H}$  is the means of wave heights,  $x = H/\bar{H}$ .

The distribution of wave periods in a certain range of wave height can be derived by the formulation for the conditional probability density function as

$$p(\tau/x) = \frac{p(x, \tau)}{p(x)} = \frac{ax}{\sqrt{\pi\nu}} \exp\left[-\frac{a^2 x^2}{\nu^2} (\tau-1)^2\right] \quad (2)$$

$$\text{where, } a = \sqrt{\pi}/2, \quad x = H/\bar{H}, \quad \tau = T/\bar{T} \quad (3)$$

Bretschneider-Mitsuyasu spectrum yields ( $\approx 0.425$ , whereas values of ) in observed wave spectra typically lies in the range of about 0.3 to 0.8.

For the calculation of the eq. (1), the means of wave heights of assigns to the relationship of eq.(4) by Rayleigh distribution.

$$H_{1/3} = 1.60 \bar{H} \quad (4)$$

where,  $H_{1/3}$  is the significant wave height.

Utilizing the general relationship of the filed data, the wave period in the eq.(2) has the range of eq. (5)

$$T_{1/3} = (0.9 \sim 1.4) \bar{T} \quad (5)$$

Utilizing the experimental data of Day and Kamel(Per Brunn, 1985), runup height of individual wave is estimated. The experimental data assign to the curve fitting of eq. (6)

$$R_u/H = A[1 - \exp(-B \cdot I_r)] \quad (6)$$

where,  $R_u$  is the runup height.  $H$  the individual wave height.  $A$  and  $B$  are the values of 1.3698 and -0.5964 respectively for rubble mound.  $I_r$  denotes the Iribarren Number. The Iribarren Number is shown in eq. (7).

$$I_r = \sqrt{g/2\pi} \cdot T \left( \frac{\tan \alpha}{\sqrt{H}} \right) \quad (7)$$

where,  $\tan \alpha$  is the seaward slope of breakwater.

The crest elevation of the previous design is determined by the design wave height and tidal level. However, the tidal level fluctuates in a range. Fig. 1 shows the definition of crest elevation.

For the count of wave overtopping, the comparison

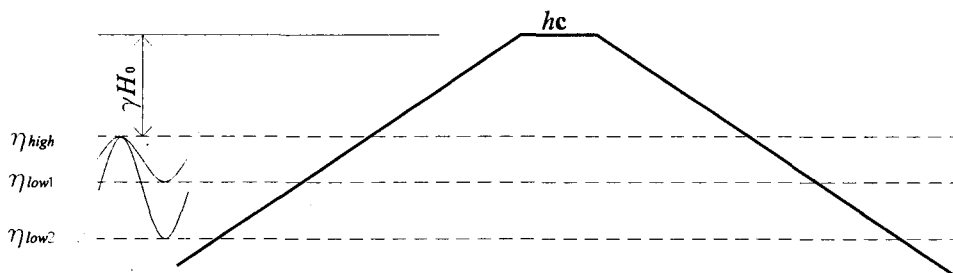


Fig. 1. Definition of crest elevation.

between the run-up and crest elevation is made. In the calculation of run-up height, the individual wave is applied to the run-up relationship based on regular wave data one by one. Accumulated count number is divided by total number of wave for the overtopping probability.

## 2.2 Reliability Analysis

The expected overtopping probability during its lifetime is estimated through the Monte Carlo simulations by taking the uncertainties of various design factors into computation.

In the present study, the following Weibull distribution function is employed for the annual maximum wave heights:

$$F(x) = 1 - \exp\left\{-\left[\frac{x-B}{A}\right]^k\right\} \quad (8)$$

where  $x$  stands for the annual maximum significant wave height,  $A$  and  $B$  are the scale and location parameters, respectively, and  $k$  is the shape parameter.

The Weibull distribution function with  $k=2.0$ ,  $A=2.23$ ,  $B=4.78$  and was used as the extreme distribution of the offshore wave height, which gave the design deepwater wave height with the return period of 50 years to be 9.2 m. An annual maximum significant wave height is randomly sampled from the distribution function and is denoted as  $H_{0e}$ . This wave height is further given a stochastic variation with the normal distribution with the mean  $H_0$  and the standard deviation  $\sigma_{H_0}$ . This variation represents the uncertainty in the estimate of extremal distribution function owing to the limited sample size of extreme wave data or the inaccuracy in wave hindcasts.

The mean wave height and the standard deviation are assumed to have following relations with  $H_{0e}$  (Takayama and Ikeda, 1994):

$$H_0 = (1 + \alpha_{H_0}) H_{0e}, \quad \sigma_{H_0} = \gamma_{H_0} H_{0e} \quad (9)$$

where  $\alpha_{H_0}$  and  $\gamma_{H_0}$  denote the bias and deviation coefficient, respectively. The sample offshore wave height  $H_{0c}$  to be used in the calculation is then determined by a normalized random number based on Eq. (9). The parameters expressing the uncertainties for the offshore wave height were  $\alpha_{H_0}=0.0$  and  $\gamma_{H_0}=0.1$ . The corresponding significant wave period is determined so as to yield wave steepness (0.02 ~ 0.08) at the design site:

$$T_s = \left(\frac{2\pi H_{0c}}{g (H_{0c}/L_0)}\right)^{1/2} \quad (10)$$

where  $g$  is the gravitational acceleration.

The variation of water level by tide is assumed to have a sinusoidal variation between the low water level (e.g. LWL) and the depth equivalent to the tidal range (e.g. HWL). A sample of tidal elevation  $\eta$  with respect to the low water level is determined based on this assumption using a random number uniformly distributed between 0 and  $2\pi$  as a phase of the sinusoidal curve. Fig. 2 shows the total procedure of computation of expected overtopping probability by reliability analysis.

The method described above is the procedure for expected overtopping probability up to a certain year, and overtopping the total damage accumulated within one lifetime is calculated by repeating this process for the

Number of Life Cycle, ( N=60 times )
Life Cycle ( 50 years )
Annual Maximum Wave Height $H_{oc}$ utilizing by the Weibull distribution at a position of breakwater with the return period of 50 years to be 9.2 m ( $a=2.23, b=4.78, k=2.0$ )
Life Cycle ( 50 years )
1) Uncertainty of wave height : Sampling of $H_{oc}$ has average value of $H_0$ and S.D. of $N \geq H_0$
2) Uncertainty of variation of water level by tide
① Sinusoidal variation between the design water level(e.g. LWL) and the depth equivalent to the tidal range (e.g. HWL)
② A random number uniformly distributed between 0 and as a phase of the sinusoidal curve.
4) One Storm Event (probability of overtopping)
a. Wave height, $H_{oc}$ - period relationship
b. Set the Crest Elevation(C.E.)
C.E. = (Range of Tide) + $H_0$
c. Comparison of Runup height and Crest elevation
Runup = Runup + Tide
d. if (RunUp.GT.C.E.) then Sum the probability density of the wave
Overtopping probability during one storm event
1. Calculation of averaged overtopping probability during one life cycle
2. Calculation of 1/3 averaged overtopping probability during one life cycle Calculation of expected overtopping probability

Fig. 2. N-S chart for the calculation of expected overtopping probability.

corresponding years of the lifetime of the breakwater. The process of one lifetime cycle is shown in Fig. 2.

This process is repeated 60 times, and the overtopping probability during one life cycle obtained are added together to yield the expected value of expected overtopping probability. The number of 60 times is employed here shows that a stable statistical result can be obtained by doing so.

### 3. Illustrative Examples

In this section, we present the effect of tidal fluctuation and wave steepness on the expected overtopping probability. The crest elevation is set to the depth of high tide added to the design wave height. Fig. 3 shows the computational results of expected overtopping probability according to the tidal range.

As shown in Fig. 3, the expected overtopping probability becomes smaller by the increasing of tidal range.

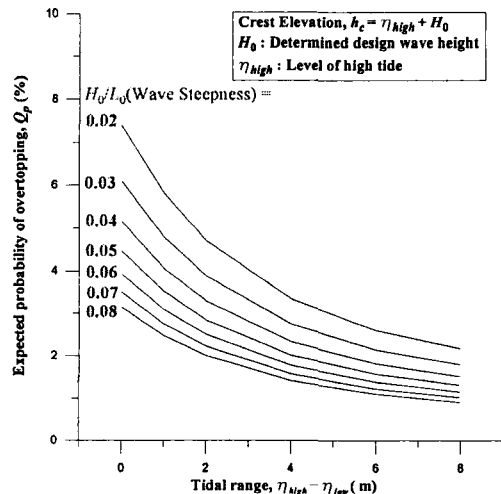


Fig. 3. Expected overtopping probability according to the tidal range.

Especially, the value of tidal range equals to 0 same as the previous design condition shows almost 7 % higher

than that of tidal range of 8 m. The effect of wave steepness is acceptable because the smaller value of it gives higher runup height. Others for the different crest elevation is shown in the appendix.

#### 4. Conclusion

It is found that the inclusion of the variability of wave tidal and wave steepness has great influence on the computation of the expected overtopping probability of rubble mound breakwater. The previous design disregarding the tidal fluctuation largely overestimates or underestimates the expected overtopping probability depending on tidal range and wave steepness.

Reliability analysis for the proper crest elevation could be the reasonable methodology.

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## Appendix

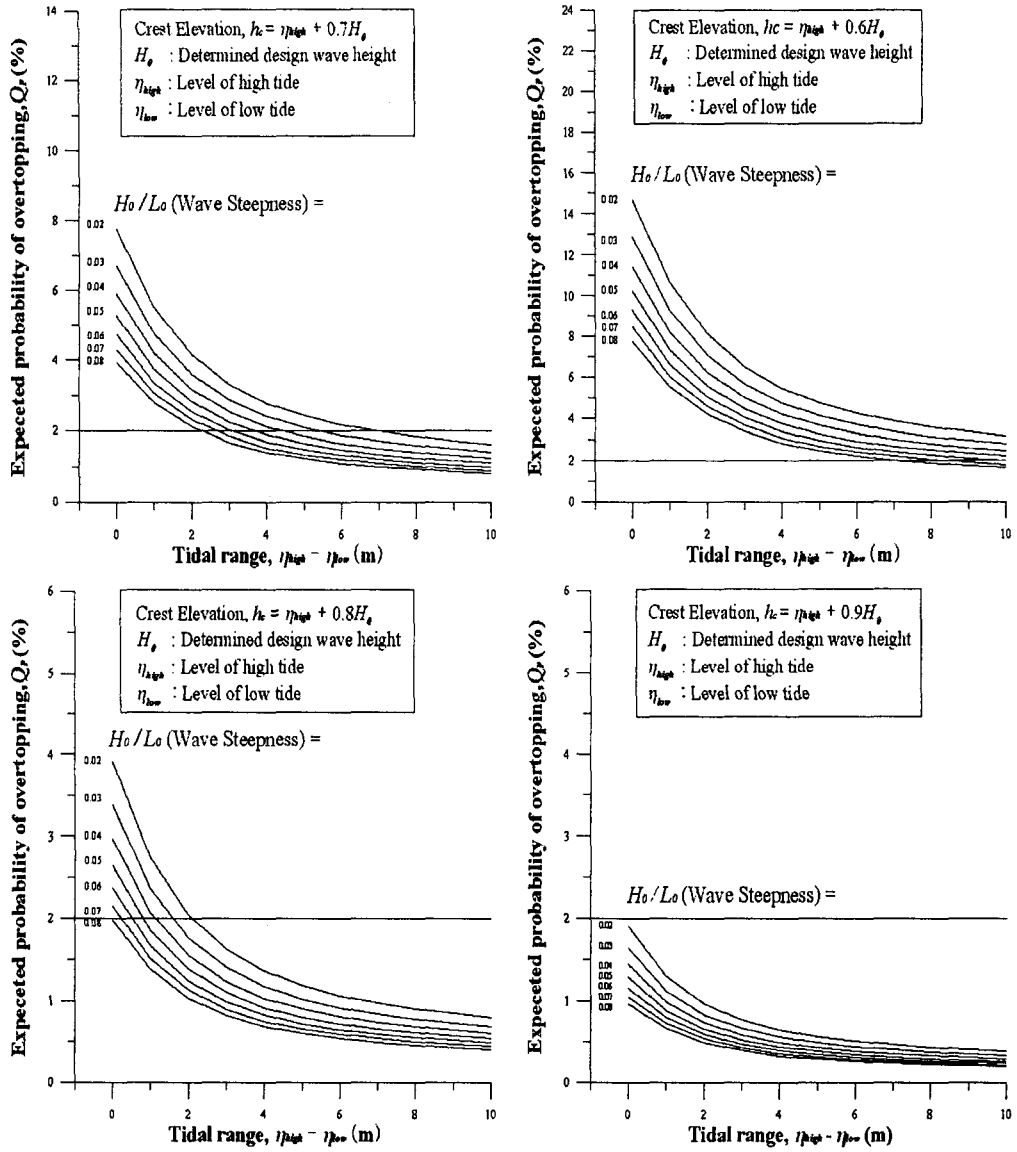


Fig. A-1. Overtopping Probability Covered With T-T-P.