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수정된 Lanczos 벡터의 중첩을 통한 구조물의 동적해석

Dynamic Analysis of Structures by Superposition of Modified Lanczos Vectors

김 병 완* 정 형 조** 김 운 학*** 이 인 원****
Kim, Byoung-Wan Jung, Hyung-Jo Kim, Woon-Hak Lee, In-Won

ABSTRACT

This paper proposes modified Lanczos vector superposition method for efficient dynamic analysis of structures. Proposed method is based on the modified Lanczos algorithm that generates stiffness-orthonormal Lanczos vectors. Proposed method has better computing efficiency than the conventional Lanczos vector superposition method in the analysis of multi-input-loaded structures. The efficiency of proposed method is verified through numerical examples. Comparison with other vector superposition methods is also presented through numerical examples.

1. Introduction

The eigenvector superposition method is widely used for solving the following dynamic equaiton of motion of structures

$$\mathbf{M}\ddot{\mathbf{u}} + (\alpha\mathbf{M} + \beta\mathbf{K})\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ and are the n by n mass and stiffness matrices, respectively, $\mathbf{f}$ is force vector and $\mathbf{u}$ is displacement vector. α and β are the Rayleigh damping coefficients. The mode acceleration method(1) is also frequently used to correct errors of the eigenvector superposition method. Those two methods perform eigenvalue analysis of structures to obtain eigenvectors.

* 한국과학기술원 건설및환경공학과 박사과정
** 정회원 · 세종대학교 토목환경공학과 조교수
*** 정회원 · 국립한경대학교 토목공학과 교수
**** 정회원 · 한국과학기술원 건설및환경공학과 교수

Wilson et al.[5] proposed the Ritz vector superposition method that does not perform eigenvalue analysis of structures. They showed that the method yields better computing efficiency than the eigenvector superposition method. Nour-Omid and Clough[2] proposed the Lanczos vector superposition method that does not also perform eigenvalue analysis of structures. They showed that the method has better computing efficiency than the eigenvector superposition method. Although the Lanczos vector superposition method is very efficient, it has some drawback. If multi-input loads such as moving loads on bridges, winds acting on high-rise buildings and wave forces applying to large offshore structures are applied to structures, the calculation of transformed force vector is somewhat costly in the method. Therefore, improvement of the Lanczos vector superposition method is required to overcome the above shortcoming.

2. Improved Lanczos vector superposition method

In this paper, the modified Lanczos algorithm is proposed to improve the conventional Lanczos vector superposition method for structures under multi-input loads. Proposed algorithm uses modified Lanczos vectors, $\mathbf{y}_i$'s that satisfy the following stiffness-orthonormality condition instead of mass-orthonormality condition.

$$\mathbf{y}_i^T \mathbf{K} \mathbf{y}_j = \delta_{ij} \tag{2}$$

Then, modified Lanczos recursion can be written as

$$\tilde{\mathbf{y}}_i = \mathbf{K}^{-1} \mathbf{M} \mathbf{y}_i - \xi_i \mathbf{y}_i - \eta_{i-1} \mathbf{y}_{i-1} \tag{3}$$

Since $\tilde{\mathbf{y}}_i$ is the next Lanczos vector before normalization, $\tilde{\mathbf{y}}_i$ can be expressed by

$$\tilde{\mathbf{y}}_i = \eta_i \mathbf{y}_{i+1} \tag{4}$$

Then, (3) can be rewritten as

$$\eta_i \mathbf{y}_{i+1} = \mathbf{K}^{-1} \mathbf{M} \mathbf{y}_i - \xi_i \mathbf{y}_i - \eta_{i-1} \mathbf{y}_{i-1} \tag{5}$$

Premultiplying (5) by $\mathbf{y}_i^T \mathbf{K}$ and using (2), we get

$$\xi_i = \mathbf{y}_i^T \mathbf{M} \mathbf{y}_i \tag{6}$$

From (4)

$$\tilde{\mathbf{y}}_i^T \mathbf{K} \tilde{\mathbf{y}}_i = \eta_i^2 \mathbf{y}_{i+1} \mathbf{K} \mathbf{y}_{i+1} = \eta_i^2 \tag{7}$$

and therefore

$$\eta_i = (\tilde{\mathbf{y}}_i^T \mathbf{K} \tilde{\mathbf{y}}_i)^{1/2} \tag{8}$$

After m (<< n) steps, we have a set of Lanczos vectors, $\mathbf{Y}=[\mathbf{y}_1\,\mathbf{y}_2\cdots\mathbf{y}_m]$, and $\mathbf{Y}$ satisfies following relations from (2)

$$\mathbf{Y}^T\mathbf{K}\mathbf{Y}=\mathbf{I} \tag{9}$$

where $\mathbf{I}$ is identity matrix of order m. Lanczos recursion, (5), can be rearranged in matrix form

$$\mathbf{K}^{-1}\mathbf{M}\mathbf{Y}-\mathbf{Y}\mathbf{S}=\eta_m\mathbf{y}_{m+1}\mathbf{e}_m^T \tag{10}$$

where $\mathbf{e}_m$ is the last column of $\mathbf{I}$ and $\mathbf{S}$ is a tridiagonal matrix of the form

$$\mathbf{S}=\begin{bmatrix}\xi_1 & \eta_1 & & & \\ \eta_1 & \xi_2 & \eta_2 & & \\ & & \ddots & & \\ & & \eta_{m-2} & \xi_{m-1} & \eta_{m-1} \\ & & & \eta_{m-1} & \xi_m\end{bmatrix} \tag{11}$$

Premultiplying (10) by $\mathbf{Y}^T\mathbf{K}$ and applying the stiffness-orthonormality condition, we get

$$\mathbf{Y}^T\mathbf{M}\mathbf{Y}=\mathbf{S} \tag{12}$$

Now, performing transformation in (1) with modified Lanczos coordinates, $\mathbf{u}=\mathbf{Y}\mathbf{r}$, and premultiplying both sides by $\mathbf{Y}^T$, we get

$$\mathbf{Y}^T\mathbf{M}\mathbf{Y}\ddot{\mathbf{r}}+(\alpha\mathbf{Y}^T\mathbf{M}\mathbf{Y}+\beta\mathbf{Y}^T\mathbf{K}\mathbf{Y})\dot{\mathbf{r}}+\mathbf{Y}^T\mathbf{K}\mathbf{Y}\mathbf{r}=\mathbf{Y}^T\mathbf{f} \tag{13}$$

From Equations (9) and (12), this is rewritten in tridiagonal form

$$\mathbf{S}\ddot{\mathbf{r}}+(\alpha\mathbf{S}+\beta\mathbf{I})\dot{\mathbf{r}}+\mathbf{r}=\mathbf{Y}^T\mathbf{f} \tag{14}$$

If single input loads are applied to structures, $\mathbf{f}=\mathbf{a}p$. $\mathbf{a}$ is the spatial load distribution vector and p is the time variation function. By taking $\mathbf{K}^{-1}\mathbf{a}$ as a starting vector in the modified Lanczos algorithm, the right side of (13) can be reduced as follows:

$$\mathbf{Y}^T\mathbf{f}=\mathbf{Y}^T\mathbf{K}\mathbf{K}^{-1}\mathbf{a}p=\mathbf{Y}^T\mathbf{K}\eta_0\mathbf{y}_1p=\mathbf{Y}^T\mathbf{K}\mathbf{Y}\mathbf{e}_1\eta_0p=\eta_0\mathbf{e}_1p \tag{15}$$

where $\mathbf{e}_1$ is the first column of $\mathbf{I}$. Finally, (14) will be

$$\mathbf{S}\ddot{\mathbf{r}}+(\alpha\mathbf{S}+\beta\mathbf{I})\dot{\mathbf{r}}+\mathbf{r}=\eta_0\mathbf{e}_1p \tag{16}$$

When multi-input loads are applied to structures, $\mathbf{f}=\mathbf{A}\mathbf{p}$. $\mathbf{A}$ is the n by k spatial load distribution matrix, $\mathbf{p}$ is the k by 1 time variation function vector and k is the number of input loads. Then (14) becomes

$$\mathbf{S}\ddot{\mathbf{r}}+(\alpha\mathbf{S}+\beta\mathbf{I})\dot{\mathbf{r}}+\mathbf{r}=\mathbf{Y}^T\mathbf{A}\mathbf{p} \tag{17}$$

On the other hand, the conventional Lanczos vector superposition method has the following equation of motion for multi-input-loaded structures

$$\mathbf{T}\ddot{\mathbf{q}} + (\alpha\mathbf{T} + \beta\mathbf{I})\dot{\mathbf{q}} + \mathbf{q} = \mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{A}\mathbf{p} \tag{18}$$

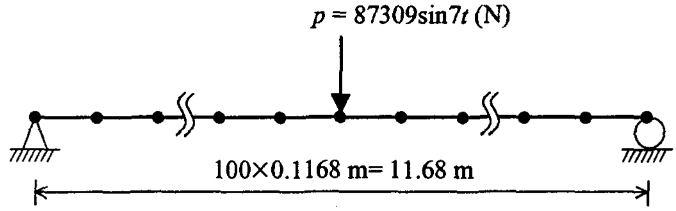
The number of operations for computing $\mathbf{Y}^T\mathbf{A}$ in (17) is nmk. In (18), total number of operations for computing $\mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{A}$ is $2nh(m + k) + nm(k + 1)$. h is the half-bandwidth of system matrices. It is clear that nmk is smaller than $2nh(m + k) + nm(k + 1)$. Therefore, (17) requires less computing time than (18).

3. Numerical examples

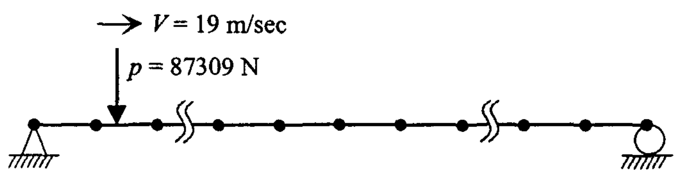
A simple span beam[3] and a multi-span continuous bridge[4] are analyzed to verify the effectiveness of the proposed Lanczos vector superposition method. The results are compared with those of the eigenvector superposition method, the mode acceleration method, the Ritz vector superposition method and the conventional Lanczos vector superposition method. The accuracy and computing time are examined to compare each method. The following normalized RMS (root mean squre) error is used in the examination of accuracy.

$$\varepsilon = \frac{\sqrt{\frac{1}{T_d}\int_0^{T_d}(\mathbf{u}_{exact} - \mathbf{u})^T(\mathbf{u}_{exact} - \mathbf{u})\,dt}}{\sqrt{\frac{1}{T_d}\int_0^{T_d}\mathbf{u}_{exact}^T\mathbf{u}_{exact}\,dt}} \tag{19}$$

where T_d is time duration. The results obtained by the direct integration method are taken as the exact solutions. The geometric and loading configurations of example structures are shown in Figs. 1 ~ 2. Since arrival times of moving load are different at all nodal points, moving load is a multi-input load.

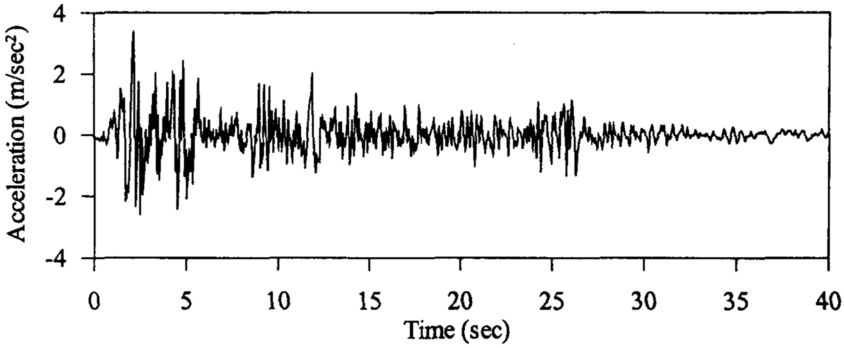


(a) Single input load (sinusoidal load)

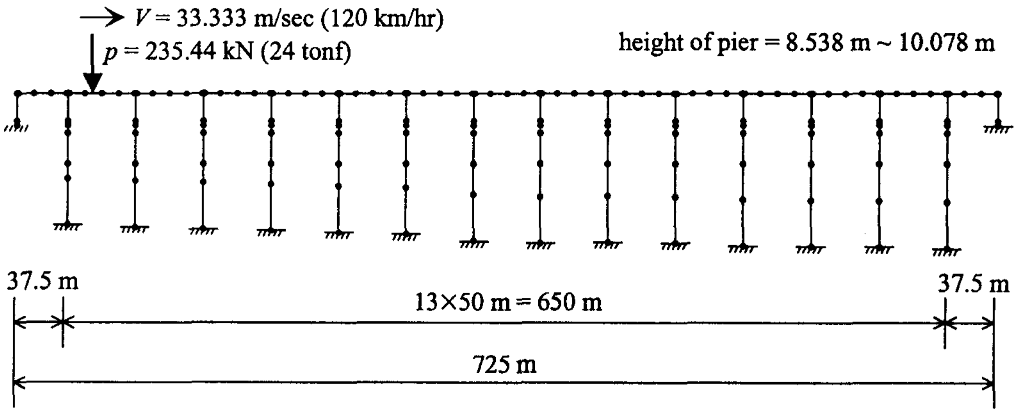


(b) Multi-input load (moving load)

Fig. 1. Simple span beam



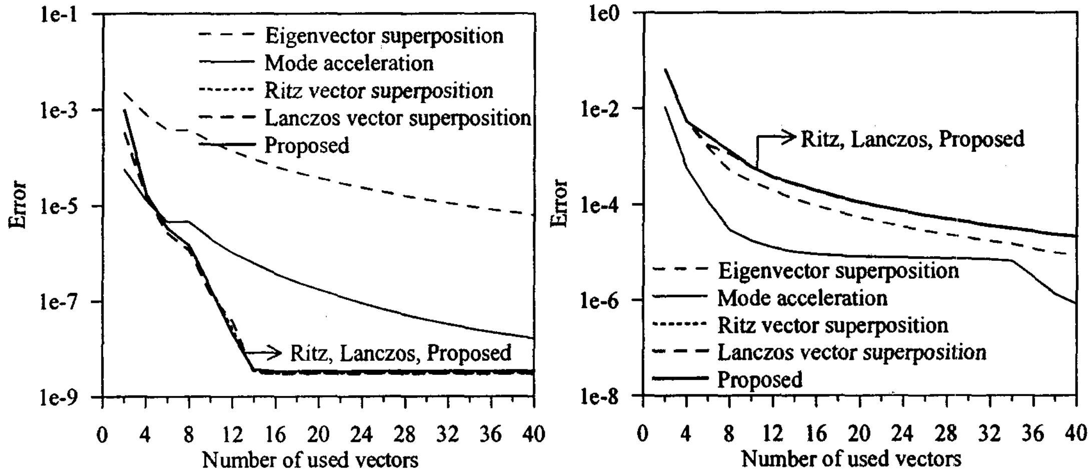
(a) Single input load (El Centro earthquake)



(b) Multi-input load (moving load)

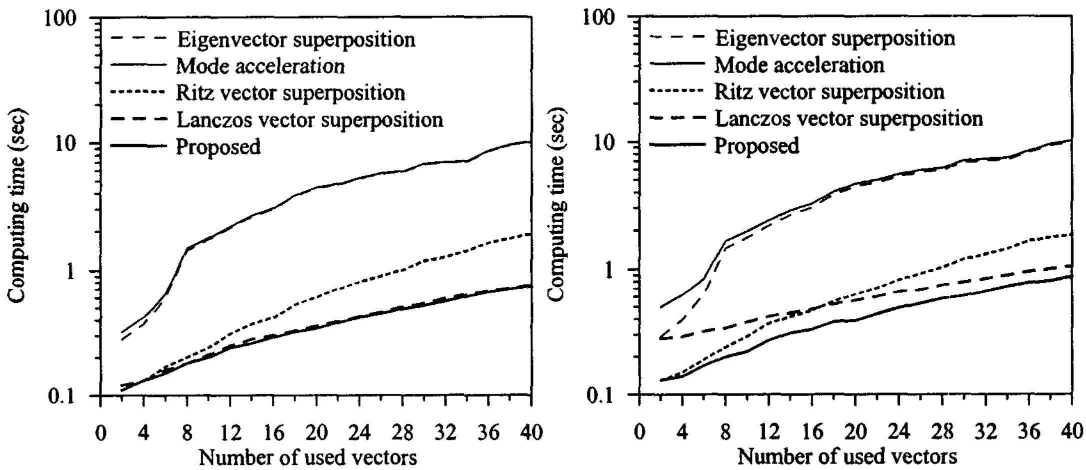
Fig. 2. Multi-span continuous bridge

Some results are shown Figs. 3 ~ 6. Proposed and conventional Lanczos vector superposition methods and the Ritz vector superposition method has almost the same accuracy. The three methods have better accuracy than the eigenvector superposition method and the mode acceleration method in the case of single input load. For the multi-input loading case, the eigenvector superposition method and the mode acceleration method have better accuracy than the others. The accuracy of the mode acceleration method is the best. Since proposed and conventional Lanczos vector superposition methods and the Ritz vector superposition method do not perform eigenvalue analysis of structures, the three methods have less computing time than the eigenvector superposition method and the mode acceleration method. When the number of used vectors is large, the Ritz vector super position method is slightly costly than proposed and conventional Lanczos vector superposition methods because it requires additional operation for computing eigensolution of reduced system. In the case of single input load, the computing efficiency of proposed and conventional Lanczos vector superposition methods is nearly the same. In the case of multi-input load, the conventional Lanczos vector superposition method is a little more costly than the Ritz vector superposition method when the number of used vectors is small. This is due to the calculation of transformed force vector. It can bee seen that proposed method that reduces the operation for calculating transformed force vector has the best computing efficiency for multi-input loading case.



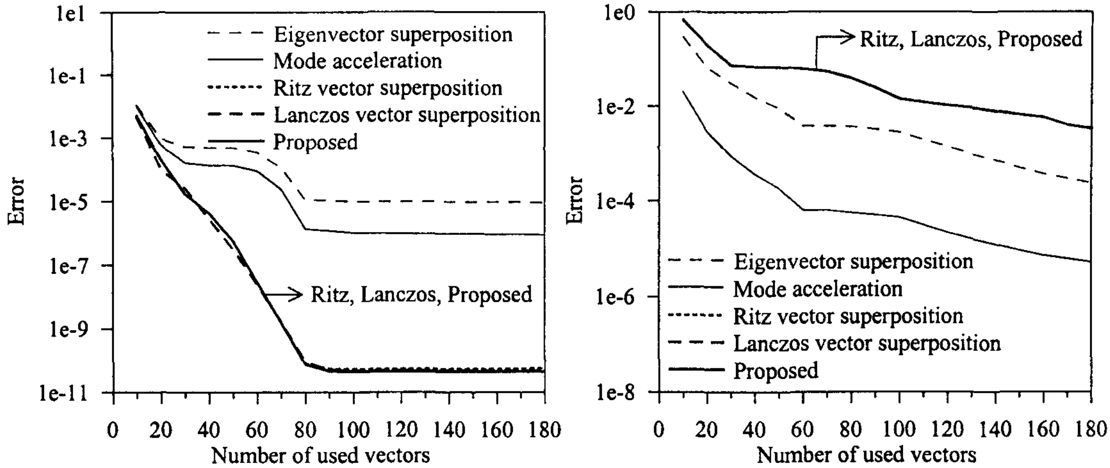
(a) Single input load **(b) Multi-input load**

Fig. 3. Accuracy of simple span beam



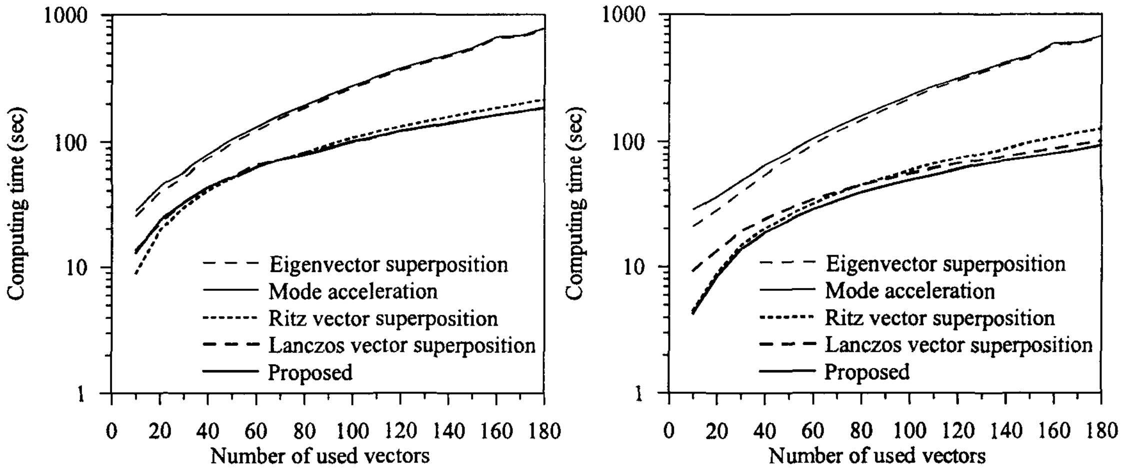
(a) Single input load **(b) Multi-input load**

Fig. 4. Computing time of simple span beam



(a) Single input load **(b) Multi-input load**

Fig. 5. Accuracy of multi-span continuous bridge



(a) Single input load **(b) Multi-input load**

Fig. 6. Computing time of multi-span continuous bridge

4. Conclusions

Improved Lanczos vector superposition method based on the modified Lanczos algorithm that generates stiffness-orthonormal Lanczos vectors is proposed for efficient dynamic analysis of structures. From numerical analysis, the characteristics of proposed Lanczos vector superposition method can be summarized as follows:

(1) Proposed method has better computing efficiency than the conventional Lanczos vector superposition method in the analysis of structures under multi−input loads. For the single input loading case, two methods have almost the same computing efficiency.

(2) Proposed and conventional Lanczos vector superposition methods and the Ritz vector superposition method has almost the same accuracy. The three methods have better accuracy than the eigenvector superposition method and the mode acceleration method in the case of single input load. The eigenvector superposition method and the mode acceleration method have better accuracy than the others in the case of multi−input load.

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