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Vibration Analysis of the Moving Plates Subjected to the Force of Gravity

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ABSTRACT

The use of frequency-dependent dynamic stiffness matrix (or spectral element matrix) in structural dynamics may provide very accurate solutions, while it reduces the number of degrees-of-freedom to improve the computational efficiency and cost problems. Thus, this paper develops a spectral element model for the thin plates moving with constant speed under uniform in-plane tension and gravity. The concept of Kantorovich method and the principle of virtual displacement is used in the frequency-domain to formulate the dynamic stiffness matrix. The present spectral element model is evaluated by comparing its solutions with the exact analytical solutions. The effects of moving speed, in-plane tension and gravity on the natural frequencies of the plate are numerically investigated.

1. INTRODUCTION

Axially moving plates may experience severe vibrations above a critical speed to result in structural failures. Examples of such systems are the high speed band saw, magnetic and paper tapes, paper webs, films, wide moving bands and belts, cooling tower strips and the like. An extensive literature review on axially moving materials is given in reference(1). Though there have been so many publications on axially moving one-dimensional structures such as strings and beams, there is relatively a small number of publications on the axially moving plates which are subjected to tensile forces in the transport direction.

In 1968, Soler(2) used a simple bending-torsion plate model for the band moving with constant speed. Ulsoy and Mote(3) is the first who have studied the vibration of a moving plate as a model of a wide bandsaw blade by using the classical Ritz method and finite element-Ritz method. Lengoc and Mccallion(4) applied the extended Galerkin method to the plate model by Ulsoy and Mote(3) and investigate effects of in-plane stresses on the natural frequencies. Lin and Mote(5) used the von Karman nonlinear plate theory to investigate the large equilibrium displacement and stress distribution of a web under transverse loading. Later on, Lin(6) investigated the stability of a moving plate with two simply supported and two free edges by using the canonical form of the equations of motion. Wang(7) developed a mixed finite element formulation for a moving orthotropic plate based on the Mindlin-Reissner plate model. Damaren and Langoc(8) applied the Rayleigh-Ritz method to formulate the discrete-parameter motion equations for active control applications.

The vibration of plates has been a subject of considerable research in the structural dynamics community. Because the analytical solution is not available in a closed form for the plates that do not have at least two parallel edges simply supported(9-11), thus the approximate methods have been widely used: Rayleigh-Ritz method (RRM)(12-18), Kantorovich method(19-24), finite element method (FEM)(25), finite strip method (FSM)(26-29), and the spectral element method (SEM)(30-33).

The RRM is one of the most commonly used methods in free vibration analysis of plates. The accuracy of RRM strongly depends on the shape functions used in the analysis. In the literature, the various shape functions have been proposed in the literature. However, Bhat *et al.*(14)(16)(23) showed that a Rayleigh-type assumption of shape functions, even with an optimizing exponent, will not give very good results for a plate with some edges free.

In the Kantorovich method, the displacements of a plate are described by the functions given as the products of assumed shape functions and unknown functions, which reduces the partial differential equations of motion to the ordinary differential equations for unknown functions. The assumed shape functions have to satisfy the boundary conditions at two parallel edges in one direction. In contrast to FSM, the unknown functions are determined to satisfy the boundary conditions at two parallel edges in the other direction by solving the reduced ordinary differential equations in an analytical way. Accordingly, the conventional Kantorovich method is not a member of element methods such as FEM and FSM.

The FEM is a representative element method, in which the displacements of a finite element are described by using local, piecewise continuous polynomial functions. The FEM is certainly an extremely versatile and powerful technique widely used for solving diverse boundary and initial value problems. However, it requires very fine meshes to improve the solutions and accordingly a very large memory. It is well known that the FEM solutions become increasingly inaccurate as the frequency increases.

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The FSM is very similar to the Kantorovich method in philosophy, but can be considered as a special form of FEM. In FSM, the prob lem domain is divided into strip elements and the displacements of a strip element are described by the functions, which are given as products of trigonometrical/hyperbolic series and polynomials. The series have to satisfy *a priori* boundary conditions at the end of the strips. Therefore, FSM is between FEM and RM. The memory required for FSM is usually much smaller than for FEM.

The SEM is a member of element methods. Contrary to the conventional FEM and FSM, the frequency-dependent dynamic stiffness matrix (often called 'spectral element matrix' in the literature) is used in SEM[30-33]. Thus, it is known to provide very accurate solutions with using only a small number of degrees-of-freedom (DOF). It is quite straightforward to apply the SEM to one-dimensional structures. However, for the plate structures, the application has been limited to Levy-type plates because it is not easy to derive the exact dynamic stiffness matrix for non-Levy-type plates.

The purpose of this study is to formulate the spectral element model for the thin plates moving with constant speed under gravity and uniform in-plane tension in the transport direction. The concept of Kantorovich method will be used in part to formulate the approximate dynamic stiffness matrix for the present model and the principle of virtual displacement will be used to formulate the dynamic stiffness matrices for constant speed, in-plane tension and gravity.

2. EQUATION OF MOTION

Consider a rectangular thin plate moving with constant speed c in the opposite direction of gravitation, as shown in Fig. 1.

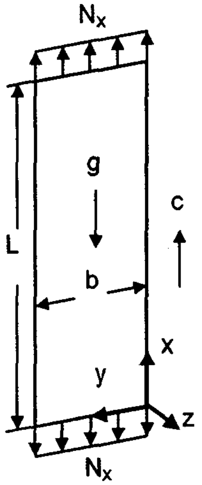

Fig.1 An axially moving plate under in-plane tension and gravity.

The plate has the length L, width b, and thickness h in the x, y, z-directions, respectively. The material properties of the plate are the mass density ρ, Young's modulus E, and Poisson's ration ν. The plate has the free boundary conditions on $y = 0$ and $y = b$.

The equation of motion is

$$D\,\nabla^4 w + \rho h\left(\frac{\partial^2 w}{\partial t^2} + 2c\frac{\partial^2 w}{\partial t \partial x} + c^2\frac{\partial^2 w}{\partial x^2}\right) - (N_x + \rho h g x)\frac{\partial^2 w}{\partial x^2} - \rho h g\frac{\partial w}{\partial x} = p(x,y,t) \qquad (1)$$

where $w(x,y,t)$ is the transverse displacement, $D = Eh^3/(12(1-\nu^2))$ is the plate rigidity, ρ is the mass density per unit volume, N_x is the in-plane tension in the x-direction measured per unit width, g is the acceleration of gravity and $p(x,y,t)$ is the external force distribution normal to the plate. $\overline{M}_x$ and $\overline{V}_x$ denote the bending moment and shear force specified on the boundaries at $x = 0$ and $x = L$.

and the boundary conditions on $x = 0$ and $x = L$,

$$V_x = \overline{V}_x \quad \text{or} \quad w = \overline{w}, \qquad M_x = \overline{M}_x \quad \text{or} \quad \frac{\partial w}{\partial x} = \overline{\theta} \qquad (2)$$

on $y = 0$ and $y = b$,

$$V_y = 0 \quad \text{and} \quad M_y = 0 \qquad (3)$$

In the above equations, ∇^4 is the bi-harmonic operator in rectangular coordinates. V_x and V_y are the resultant Kirchhoff effective shear forces per unit length in the y- and x-direction, and M_x and M_y are the resultant bending moments per unit length in the y- and x-direction, respectively. They are related to the transverse displacement as

$$
\begin{aligned}
V_x &= -D\left[\frac{\partial^3 w}{\partial x^3}+(2-\nu)\frac{\partial^3 w}{\partial x \partial y^2}\right]-\rho h c\frac{\partial w}{\partial t}-\rho h c^2\frac{\partial w}{\partial x}+(N_x+\rho h g x)\frac{\partial w}{\partial x}\\
V_y &= -D\left[\frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right]\\
M_x &= -D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right), \qquad M_y = -D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)
\end{aligned} \tag{4}
$$

3. SPECTRAL ELEMENT FORMULATION

The free vibration response of the plate can be written in the spectral form

$$
w(x,y,t)=\sum_{n=1}^{N} W_n(x,y)e^{i\omega_n t} \tag{5}
$$

where W_n are the spectral components corresponding to the discrete frequencies $\omega_n = 2\pi n / T$. N denotes the number of spectral components to be taken into account in the analysis. Substituting Eq. (5) into Eq. (1) gives

$$
\nabla^4 W_n+\left(c_p^2-\frac{N_x+\rho h g x}{D}\right)\frac{\partial^2 W_n}{\partial x^2}+\left(2ic_p\Omega_n^2-\frac{\rho h g}{D}\right)\frac{\partial W_n}{\partial x}-\Omega_n^4 W_n=0 \tag{6}
$$

where

$$
c_p = c\sqrt{\frac{\rho h}{D}}, \qquad \Omega_n^2=\omega_n\sqrt{\frac{\rho h}{D}} \tag{7}
$$

By substituting Eq. (5) into Eq. (4), the resultant shear forces and moments can be written as

$$
\begin{aligned}
V_x(x,y,t)&=\sum_{n=1}^{N} V_{xn}(x,y)e^{i\omega_n t}, \quad V_y(x,y,t)=\sum_{n=1}^{N} V_{yn}(x,y)e^{i\omega_n t}\\
M_x(x,y,t)&=\sum_{n=1}^{N} M_{xn}(x,y)e^{i\omega_n t}, \quad M_y(x,y,t)=\sum_{n=1}^{N} M_{yn}(x,y)e^{i\omega_n t}
\end{aligned} \tag{8}
$$

The general solution of Eq. (6) is assumed in the form

$$
W_n(x,y)=\sum_{r=1}^{M} X_{nr}(x)Y_r(y) \tag{9}
$$

where $Y_r(y)$ ($r = 1, 2, \ldots, M$) are the known functions chosen a priori, and X_{nr} are the unknown functions to be analytically determined. In the present study, the eigenfunctions of the free-free Bernoulli-Euler beam (simply, beam functions) are chosen for $Y_r(x)$, which satisfy the following orthogonality conditions[27]:

$$
\int_0^b Y_r(y)Y_m(y)dy=0, \quad \int_0^b Y_r''''(y)Y_m(y)dy=0, \quad \int_0^b Y_r''(y)Y_m(y)dy=0 \qquad (\text{for } r\neq m) \tag{10}
$$

The beam functions Y_m ($m = 1, 2, \ldots, M$) cannot exactly satisfy the free-free plate boundary conditions (*i.e.*, Eq. 12) by themselves,. Thus, for chosen Y_m, the functions X_{nm} should be determined to satisfy

$$
X_{nm}\frac{d^3Y_m}{dy^3}=-(2-\nu)\frac{d^2X_{nm}}{dx^2}\frac{dY_m}{dy}, \quad X_{nm}\frac{d^2Y_m}{dy^2}=-\nu\frac{d^2X_{nm}}{dx^2}Y_m \tag{11}
$$

on $y = 0$ and $y = b$.

Substituting Eq. (9) into Eq. (6) and multiplying by $Y_m(x)$, and integrating from $y = 0$ to $y = b$ by use of the integral by parts yield

$$\frac{d^4 X_{nm}}{dx^4} + \left(-2\xi_{m1} + c_p^2 - \frac{N_x}{D}\right)\frac{d^2 X_{nm}}{dx^2} + \left(2ic_p\Omega_n^2 + \frac{X}{D}\right)\frac{dX_{nm}}{dx} + \left(\xi_{m2} - \Omega_n^4\right)X_{nm}$$
$$+\frac{1}{\xi_{m0}}\left(2\frac{d^2 X_{nm}}{dx^2}\frac{dY_m}{dy}Y_m + X_{nm}\frac{d^3 Y_m}{dy^3}Y_m - X_{nm}\frac{d^2 Y_m}{dy^2}\frac{dY_m}{dy}\right)\Bigg|_0^b = 0 \tag{12}$$

where

$$\xi_{m0} = \int_0^b Y_m^{\,2} dy, \quad \xi_{mj} = \int_0^b \left(d^j Y_m / dy^j\right)^2 dy \Big/ \xi_{m0} \tag{13}$$

Substituting Eq. (11) into Eq. (12) to obtain

$$\frac{d^4 X_{nm}}{dx^4} + \alpha_m^4 \frac{d^2 X_{nm}}{dx^2} + \beta_{nm}\frac{dX_{nm}}{dx} + \gamma_{nm} X_{nm} = 0 \tag{14}$$

where

$$\alpha_m = -2\xi_{m1} + c_p^2 - \frac{N_x + \rho hgx_i}{D} + 2\nu\varsigma_m, \quad \beta_{nm} = 2ic_p\Omega_n^2 - \frac{\rho hg}{D},$$
$$\gamma_{nm} = \xi_{m2} - \Omega_n^4, \quad \varsigma_m = \frac{1}{\xi_{m0}}\frac{dY_m}{dy}Y_m\Bigg|_0^b \tag{15}$$

Assuming the x in α_m to be x_i, the center of element, the solution of Eq. (14) can be obtained in the form

$$X_{nm}(x) = a_{nm1}e^{ik_{nm1}x} + a_{nm2}e^{ik_{nm2}x} + a_{nm3}e^{ik_{nm3}x} + a_{nm4}e^{ik_{nm4}x} = \left[E_{nm}(x)\right]\{a_{nm}\} \tag{16}$$

where

$$\left[E_{nm}(x)\right] = \left[e^{ik_{nm1}x} \quad e^{ik_{nm2}x} \quad e^{ik_{nm3}x} \quad e^{ik_{nm4}x}\right]$$
$$\{a_{nm}\} = \{a_{nm1} \quad a_{nm2} \quad a_{nm3} \quad a_{nm4}\}^T \tag{17}$$

In Eq. (16), k_{nmi} $(i = 1, 2, 3, 4)$ are the wavenumbers determined from the following dispersion relation

$$k_{nm}^4 + \alpha_m k_{nm}^2 + \beta_{nm} k_{nm} + \gamma_{nm} = 0 \tag{18}$$

The constants a_{nmi} $(i = 1, 2, 3, 4)$ are determined to satisfy the boundary conditions on $x = 0$ and $x = L$. Substituting Eq. (16) into Eq. (9) gives the general solutions W_n.

$$W_n(x,y) = \sum_{r=1}^{M} X_{nr}(x)Y_r(y) = \sum_{r=1}^{M}\left[E_{nr}(x)\right]Y_r(y)\{a_{nr}\} = \sum_{r=1}^{M}\left[P_{nr}(x,y)\right]\{a_{nr}\} = \left[P_n(x,y)\right]\{a_n\}$$
$$\Theta_n(x,y) = \left[P_n'(x,y)\right]\{a_n\} \qquad \text{where, } ' = \frac{\partial}{\partial x} \tag{19}$$

The spectral generalized displacements d_{nr} $(r = 1,2,\ldots, M)$ are defined by

$$\{d_n\}^T = \{W_n(0,y) \quad \Theta_n(0,y) \quad W_n(L,y) \quad \Theta_n(L,y)\} \tag{20}$$

By multiplying Eq. (20) by $Y_m(x)$, integrating from $y = 0$ to $y = b$, and using Eqs. (10), the spectral generalized displacements can be obtained

$$\{d_{nm}\} \equiv \int_0^b \{d_n\}Y_m dy = \left[A_{nm}\right]\{a_{nm}\} \qquad \text{where,} \quad \left[A_{nm}\right] = \int_0^b \begin{bmatrix} P_{nm}(0,y) \\ P_{nm}'(0,y) \\ P_{nm}(L,y) \\ P_{nm}'(L,y) \end{bmatrix} Y_m dy \tag{21}$$

thus,

$$\{a_{nm}\} = [A_{nm}]^{-1}\{d_{nm}\} \tag{22}$$

Substituting Eq. (22) into Eq. (19) gives

$$W_n(x,y) = \sum_{r=1}^{M}[P_{nr}(x,y)][A_{nr}]^{-1}\{d_{nr}\} = \sum_{r=1}^{M}[N_{nr}(x,y)]\{d_{nr}\} = [N_n(x,y)]\{d_n\} \tag{23}$$

where, $[N_n(x,y)]$ is the frequency-dependent dynamic shape function.

The weak form of Eq. (6) can be obtained from

$$\int_0^b\int_0^L\left\{\nabla^4 W_n + \left(c_p^2 - \frac{N_x + \rho hgx}{D}\right)\frac{\partial^2 W_n}{\partial x^2} + \left(2ic_p\Omega_n^2 - \frac{\rho hg}{D}\right)\frac{\partial W_n}{\partial x} - \Omega_n^4 W_n = 0\right\}\delta W_n\, dxdy = 0 \tag{24}$$

By taking the integral by parts and then applying the boundary conditions on $y=0$ and $y=b$, one may obtain

$$\begin{aligned}
&\int_0^b\int_0^L\left\{\left(\frac{\partial^2 W_n}{\partial x^2} + \nu\frac{\partial^2 W_n}{\partial y^2}\right)\frac{\partial^2\delta W_n}{\partial x^2} + \left(\frac{\partial^2 W_n}{\partial y^2} + \nu\frac{\partial^2 W_n}{\partial x^2}\right)\frac{\partial^2\delta W_n}{\partial y^2} + 2(1-\nu)\frac{\partial^2 W_n}{\partial x\partial y}\frac{\partial^2\delta W_n}{\partial x\partial y}\right.\\
&\left.-\left(c_p^2 - \frac{N_x + \rho hgx}{D}\right)\frac{\partial W_n}{\partial x}\frac{\partial\delta W_n}{\partial x} + ic_p\Omega_n^2\left(\frac{\partial W_n}{\partial x}\delta W_n - W_n\frac{\partial\delta W_n}{\partial x}\right) - \Omega_n^4 W_n\,\delta W_n\right\}dx\,dy\\
&+\int_0^b M_{xn}(x,y)\frac{\partial\delta W_n}{\partial x}\bigg|_0^L dy - \int_0^b V_{xn}(x,y)\delta W_n\bigg|_0^L dy = 0
\end{aligned} \tag{25}$$

where the following definitions are used.

$$\begin{aligned}
V_{xn}(x,y) &= -D\left[\frac{\partial^3 W_n}{\partial x^3} + (2-\nu)\frac{\partial^3 W_n}{\partial x\partial y^2}\right] - i\rho hc_p\omega_n\frac{\partial W_n}{\partial t} - \rho hc_p{}^2\frac{\partial W_n}{\partial x} + (N_x + \rho hgx)\frac{\partial W_n}{\partial x}\\
M_{xn}(x,y) &= -D\left(\frac{\partial^2 W_n}{\partial x^2} + \nu\frac{\partial^2 W_n}{\partial y^2}\right)
\end{aligned} \tag{26}$$

Substituting Eq. (23) into Eq. (25) yields

$$\begin{aligned}
&\{\delta d_{nr}\}^{\mathrm{T}}\left(\left[\int_0^b\int_0^L\left\{\left[\frac{\partial^2 N_{nr}}{\partial x^2}\right]^{\mathrm{T}}\left[\frac{\partial^2 N_{nr}}{\partial x^2}\right] + \left[\frac{\partial^2 N_{nr}}{\partial x^2}\right]^{\mathrm{T}}\left[\frac{\partial^2 N_{nr}}{\partial y^2}\right] + \left[\frac{\partial^2 N_{nr}}{\partial y^2}\right]^{\mathrm{T}}\left[\frac{\partial^2 N_{nr}}{\partial y^2}\right] + \nu\left[\frac{\partial^2 N_{nr}}{\partial y^2}\right]^{\mathrm{T}}\left[\frac{\partial^2 N_{nr}}{\partial x^2}\right]\right.\right.\right.\\
&+2(1-\nu)\left[\frac{\partial^2 N_{nr}}{\partial x\partial y}\right]^{\mathrm{T}}\left[\frac{\partial^2 N_{nr}}{\partial x\partial y}\right] + ic_p\Omega_n^2\left([N_{nr}]^{\mathrm{T}}\left[\frac{\partial N_{nr}}{\partial x}\right] - \left[\frac{\partial N_{nr}}{\partial x}\right]^{\mathrm{T}}[N_{nr}]\right) - c_p^2\left[\frac{\partial N_{nr}}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N_{nr}}{\partial x}\right]\\
&\left.\left.+\frac{\rho hgx}{D}\left[\frac{\partial N_{nr}}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N_{nr}}{\partial x}\right] + \frac{N_x}{D}\left[\frac{\partial N_{nr}}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N_{nr}}{\partial x}\right] - \Omega_n^4[N_{nr}]^{\mathrm{T}}[N_{nr}]\right\}dx\,dy\right]\{d_{nr}\}\\
&\left.-\int_0^b\left[-V_{xnr}(0,y)\quad M_{xnr}(0,y)\quad V_{xnr}(L,y)\quad -M_{xnr}(L,y)\right]^{\mathrm{T}}dy\right) = 0
\end{aligned} \tag{27}$$

From Eq. (27), thus one may readily obtain

$$[S_{nr}]\{d_{nr}\} = \{f_{nr}\} \tag{28}$$

where, $[S_{nr}]$ is the dynamic stiffness matrix for the mode $Y_r(y)$ given by

$$[S_{nr}] = [S_{nr}]_{classical} + [S_{nr}]_c + [S_{nr}]_g + [S_{nr}]_N \tag{29}$$

$$\begin{aligned}[S_{nr}]_{classical} = \int_0^b \int_0^L \Bigg(& \left[\frac{\partial^2 N_{nr}}{\partial x^2}\right]^{\mathrm{T}} \left[\frac{\partial^2 N_{nr}}{\partial x^2}\right] + \left[\frac{\partial^2 N_{nr}}{\partial x^2}\right]^{\mathrm{T}} \left[\frac{\partial^2 N_{nr}}{\partial y^2}\right] + \left[\frac{\partial^2 N_{nr}}{\partial y^2}\right]^{\mathrm{T}} \left[\frac{\partial^2 N_{nr}}{\partial y^2}\right] \\ & + \nu \left[\frac{\partial^2 N_{nr}}{\partial y^2}\right]^{\mathrm{T}} \left[\frac{\partial^2 N_{nr}}{\partial x^2}\right] + 2(1-\nu) \left[\frac{\partial^2 N_{nr}}{\partial x \partial y}\right]^{\mathrm{T}} \left[\frac{\partial^2 N_{nr}}{\partial x \partial y}\right] - \Omega_n^4 [N_{nr}]^{\mathrm{T}} [N_{nr}] \Bigg) dx\,dy \end{aligned} \tag{30}$$

$$[S_{nr}]_c = \int_0^b \int_0^L \left\{ i c_p \Omega_n^2 \left([N_{nr}]^{\mathrm{T}} \left[\frac{\partial N_{nr}}{\partial x}\right] - \left[\frac{\partial N_{nr}}{\partial x}\right]^{\mathrm{T}} [N_{nr}] \right) - c_p^2 \left[\frac{\partial N_{nr}}{\partial x}\right]^{\mathrm{T}} \left[\frac{\partial N_{nr}}{\partial x}\right] \right\} dx\,dy \tag{31}$$

$$[S_{nr}]_g = \int_0^b \int_0^L \frac{\rho h g x}{D} \left[\frac{\partial N_{nr}}{\partial x}\right]^{\mathrm{T}} \left[\frac{\partial N_{nr}}{\partial x}\right] dx\,dy \tag{32}$$

$$[S_{nr}]_N = \int_0^b \int_0^L \frac{N_x}{D} \left[\frac{\partial N_{nr}}{\partial x}\right]^{\mathrm{T}} \left[\frac{\partial N_{nr}}{\partial x}\right] dx\,dy \tag{33}$$

and $\{f_{nr}\}$ is the nodal force vector defined by

$$\{f_{nr}\} = \int_0^b \left[-V_{xnr}(0,y) \quad M_{xnr}(0,y) \quad V_{xnr}(L,y) \quad -M_{xnr}(L,y) \right]^{\mathrm{T}} dy \tag{34}$$

4. NUMERICAL EXAMPLES AND DISCUSSION

For numerical illustrations, the rectangular thin plate shown in Fig. 1 is considered. The material properties of the plate are the Young's modulus $E = 200\ GPa$, Poisson's ratio $\nu = 0.333$, and the mass density $\rho = 7800\ kg/m^3$. The plate thickness, width, and length are $h = 0.003\ m$, $b = 1\ m$, and $L = 10\ m$, respectively. The two parallel edges on $x = 0\ m$, $10\ m$ are simply supported and the other two parallel edges on $y = 0\ m$, $1\ m$ are free.

To evaluate the present models, it is assumed that the plate is not axially moving for the time being, but subjected to uniform tension $N_x = 6\times10^4\ N/m$. In Table 1, the representative ten natural frequencies obtained by the present SEM models are compared with the exact analytical results from Blevins [10] as well as with the FEM results obtained by using the ACM plate element model [34].

Table 1. Natural frequencies for the stationary classical plate subjected to in-plane tension without gravity

Mode (x, y)	FEM					Analytical (Blevins[10])	SEM
	1 × 15	2 × 30	3 × 45	4 × 60	5 × 75		
1 (1, 1)	1.442	1.673	1.685	1.687	1.688	1.688	1.688
2 (1, 2)	1.779	1.769	1.791	1.796	1.797	1.799	1.799
3 (2, 1)	2.906	3.348	3.371	3.376	3.377	3.378	3.378
4 (2, 2)	3.559	3.539	3.584	3.594	3.597	3.599	3.599
5 (3, 1)	4.404	5.026	5.061	5.068	5.070	5.071	5.072
10 (5, 2)	8.928	8.876	8.991	9.014	9.021	9.027	9.028
12 (6, 2)	10.728	10.669	10.808	10.835	10.844	10.850	10.853
16 (8, 2)	14.031	14.286	14.474	14.511	14.522	14.530	14.535

Table 2. Natural frequencies for the moving plate subjected to in-plane tension and gravity ($c = 3$ m/s)

Mode (x, y)	FEM				SEM	
	3 × 45		4 × 60			
	Without Gravity	With Gravity	Without Gravity	With Gravity	Without Gravity	With Gravity
1 (1, 1)	1.674	1.675	1.677	1.678	1.682	1.707
2 (1, 2)	1.781	1.783	1.786	1.818	1.793	1.816
3 (2, 1)	3.362	3.362	3.366	3.367	3.366	3.415
4 (2, 2)	3.575	3.576	3.585	3.588	3.588	3.633
5 (3, 1)	5.050	5.051	5.057	5.059	5.055	5.127
6 (3, 2)	5.370	5.371	5.384	5.384	5.386	5.454
7 (4, 1)	6.742	6.744	6.751	6.752	6.748	6.845
8 (4, 2)	7.170	7.172	7.188	7.189	7.190	7.280
9 (5, 1)	8.442	8.444	8.453	8.454	8.450	8.569
10 (5, 2)	8.976	8.979	8.999	9.000	9.001	9.113

Compared with the FEM results, Table 1 also shows that the SEM model provides very competitive results in spite of using only one finite element. The FEM model is found to require more than about 375 finite elements to achieve the accuracy of the natural frequencies by the present SEM model.

Table 2 shows the natural frequency dependence of the moving speed of plate c and the force of gravity g. For a fixed moving speed and in-plane tension of plate, in general, the natural frequencies of gravity effect are increased compared with zero gravity condition.

5. CONCLUSIONS

This paper introduces a spectral element model for the thin plates, which are moving with constant speed under gravity and uniform in-plane tension. The concept of Kantorovich method is used to formulate the dynamic stiffness matrix and the principle of virtual displacement is used to formulate the dynamic shape function and dynamic stiffness matrices. The present SEM is found to provide very satisfactory natural frequencies, when compared with FEM.

The effect of moving speed, in-plane tension and gravity on the natural frequencies is numerically investigated.

REFERENCES

1. Wickert, J. A. and Mote, C, D., Jr., "Current Research on the Vibration and Stability of Axially-Moving Materials," *The Shock and Vibration Digest*, Vol. 20, No. 5, 1988, pp. 3-13.
2. Soler, A. I., "Vibrations and Stability of a Moving Band," *Journal of Franklin Institute*, Vol. 286, No. 4, 1968, pp. 295-307.
3. Ulsoy, A. G and Mote, C. D., Jr., "Vibration of Wide Band Saw Blades," *Journal of Engineering for Industry*, Vol. 104, 1982, pp. 71-78.
4. Lengoc, L. and McCallion, H., "Wide Bandsaw Blade Under Cutting Conditions, Part I: Vibration of a Plate Moving in Its Plane while Subjected to Tangential Edge Loading," *Journal of Sound and Vibration*, Vol. 186, No. 1, 1995, pp. 125-142.
5. Lin, C. C. and Mote, C. D., Jr., "Equilibrium Displacement and Stress Distribution in a Two-Dimensional, axially Moving Web Under Transverse Loading," *Journal of Applied Mechanics*, Vol. 62, 1995, pp. 772-779.
6. Lin, C. C., "Stability and Vibration Characteristics of Axially Moving Plates," *International Journal of Solids and Structures*, Vol. 34, No. 24, 1997, pp. 3179-3190.
7. Wang, X., "Numerical Analysis of Moving Orthotropic Thin Plates," *Computers & Structures*, Vol. 70, 1999, pp. 467-486.
8. Damaren, C. J. and Lengoc, L., "Robust Active Vibration Control of a Bandsaw Blade," *Journal of Vibration and Acoustics*, Vol. 122, 2000, pp. 69-76.

9. Leissa, A. W., *Vibration of Plates*, NASA SP-160, NASA, Washington, D. C., 1969.
10. Blevins, R. D., *Formulas for Natural Frequency and Mode Shape*, Van Nostrand Reinhold Company, New York, 1979.
11. Gorman, D. J., *Free Vibration Analysis of Rectangular Plates*, Elsevier North Holland, New York, 1982.
12. Young, D., "Vibration of Rectangular Plates by the Ritz Method," *Journal of Applied Mechanics*, Vol. 17, No. 4, 1950, pp. 448-453.
13. Bassily, S. F. and Dickinson, S. M., "On the Use of Beam Functions for Problems of Plates Involving Free Edges," *Journal of Applied Mechanics*, Vol. 42, 1975, pp. 858-864.
14. Bhat, R. B., "Natural Frequencies of Rectangular Plates Using Characteristic Orthogonal Polynomials in Rayleigh-Ritz Method," *Journal of Sound and Vibration*, Vol. 102, No. 4, 1985, pp. 493-499.
15. Dickinson, S. M. and Di Blasio, A., "On the Use of Orthogonal Polynomials in the Rayleigh-Ritz Method for the Study of the Flexural Vibration and Buckling of Isotropic and Orthotropic Rectangular Plates," *Journal of Sound and Vibration*, Vol. 108, No. 1, 1986, pp. 51-62.
16. Bhat, R. B., Laura, P. A. A., Gutierrez, R. G, Corinez, V. H. and Sanzi, H. C., "Numerical Experiments on the Determination of Natural Frequencies of Transverse Vibrations of Rectangular Plates, of Non-Uniform Thickness, " *Journal of Sound and Vibration*, Vol. 138, No. 2, 1990, pp. 205-219.
17. Zhou, D., "Natural Frequencies of Rectangular Plates Using a Set of Static Beam Functions in Rayleigh-Ritz Method," *Journal of Sound and Vibration*, Vol. 189, No. 1, 1996, pp. 81-87.
18. Reddy, J. N., *Theory and Analysis of Elastic Plates*, Taylor & Francis, London, 1999.
19. Kantorovich L. V. and Krylov, V. I., *Approximate Methods of Higher Analysis*, Interscience Publishers, New York, 1958.
20. Kerr, A. D., "An extended Kantorovich method for the solution of eigenvalue problems," *International Journal of Solids and Structures*, Vol. 5, 1969, pp. 559-572.
21. Yuan, S. and Jin, Y., "Computation of Elastic Buckling Loads of Rectangular Thin Plates Using the Extended Kantorovich Method," *Computers & Structures*, Vol. 66, No, 6, 1998, pp. 861-867.
22. Jones, R. and Milne, B. J., "Application of the Extended Kantorovich Method to the Vibration of Clamped Rectangular Plates," *Journal of Sound and Vibration*, Vol. 45, 1976, pp. 309-316.
23. Bhat, R. B., Singh, J. and Mundkur, G., "Plate Characteristic Functions and Natural Frequencies of Vibration of Plates by Iterative Reduction of Partial Differential Equation," *Journal of Vibration and Acoustics*, Vol. 115, 1993, pp. 177-181.
24. Bercin, A. N., "Free Vibration Solution for Clamped Orthotropic Plates Using the Kantorovich Method," *Journal of Sound and Vibration*, Vol. 196, No. 2, 1996, pp. 243-247.
25. Zienkiewicz, O. C., *The Finite Element Method*, 3rd., McGraw-Hill, London, 1977.
26. Cheung, Y. K, *Finite Strip Method in Structural Analysis*, Pergamon Press, Oxford, 1976.
27. Cocchi, G. M., "The Finite Strip Method in the Analysis of thin Plate Structures with Various Edge Restrains," *Computers & Structures*, Vol. 61, No. 2, 1996, pp. 303-313.
28. Friedrich, R., "Finite Strip Method: 30 Years A Bibliography (1968-1998)," *Engineering Computations*, Vol. 17, No. 1, 2000, pp. 92-111.
29. Cheung, Y. K. and Tham, L. G., "A Review of Finite Strip Method," *Progress in Structural Engineering and Materials*, Vol. 2, No. 3, pp. 359-368.
30. Doyle, J. F., *Wave Propagation in Structures: Spectral Analysis Using Fast Discrete Fourier Transforms*, 2nd Edition, Springer-Verlag, New York, 1997.
31. Bercin, A. N., "Eigenfrequencies of Rectangular Plate Assemblies," *Computers & Structures*, Vol. 65, No. 5, 1997, pp. 703-711.
32. Lee, U. and Lee, J., "Spectral-Element Method for Levy-Type Plates Subject to Dynamic Loads," *Journal of Engineering Mechanics*, Vol. 125, No. 2, 1999, pp. 243-247.
33. Lee, U., Kim, J. and Leung, A. Y. T., "Spectral Element Method in Structural Dynamics," *The Shock and Vibration Digest*, Vol. 32, No. 6, 2000, pp. 451-465.
34. Petyt, M., *Introduction to Finite Element Vibration Analysis*, Cambridge University Press, Cambridge, 1990.