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Tip Position Command Tracking of a Flexible Beam Using Active Vibration Control

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Key Words : Internal Model Control (내부모델제어), Flexible Beam (유연지능보), Active Vibration Control (능동진동제어), Feedback Control (피드백제어), Feedforward Control (피드포워드제어).

ABSTRACT

The problem considered in this paper is that the tip position of a flexible cantilever beam is controlled to follow a command signal, using a pair of piezoelectric actuators at the clamped end. The beam is lightly damped and so the natural transient response is rather long, and also since the sensor and actuator are not collocated, the plant response is non-minimum phase. Two control strategies were investigated. The first involved conventional PID control in which the feedback gains were adjusted to give the fastest closed-loop response to a step input. The second control strategy was based on an internal model control (IMC) architecture. The control filter in the IMC controller was a digital FIR device designed to minimize the expectation of the mean square tracking error. The IMC controller designed for the beam was found to have very much reduced settling times to a step input compared with those of the PID controller.

1. Introduction

Recent developments in smart materials such as PZT ceramics and PVDF polymer films enable actuators and sensors to be integrated into smart structures, which can then be controlled actively.[1] The applications of such piezoelectric transducers have been investigated for smart structures for vibration rejection, using active vibration control (AVC), and sound radiation control of vibrating structures, using active structural acoustical control (ASAC) etc.[1] A new approach to tip position control of a flexible cantilever beam using PZT actuators is studied in this paper. This approach can be clearly distinguished from conventional vibration rejection controls in that the actuator is driven so that the beam tip follows a command signal. Lightly damped flexible structures, such as the beam considered here, can have a long transient response when moved suddenly. However, some mechanical systems consisting of flexible structures require high-speed and accurate tracking capabilities, such as robot manipulators in spacecrafts. In order to overcome the inherent long transient response of such structures, a feedforward control strategy could be used. The feedforward controller should anticipate the inverse dynamics of the plant within a specified bandwidth. The problem is made more difficult by the non-minimum phase behavior of the system response, caused by the non-collocation of sensor and actuator, and the dispersive property of flexible structures. The non-minimum phase zeros of the system response mean that an exact stable inverse cannot be achieved by direct inversion. A number of feedforward techniques have been developed to minimize the effect of unstable zeros [2], but an alternative formulation is presented here, in which a digital FIR filter is designed to minimize the mean-square tracking error.

A conventional analogue proportional, integral and derivative (PID) control technique could also be used for the problem, but the non-minimum phase behavior limits the maximum control gains before there is a danger of instability, resulting in a rather long closed-loop transient response. A different feedback controller architecture is investigated, which is known as internal model control (IMC).[3] The IMC architecture uses an internal model of the response of the system under control, the plant, and a control filter that can be designed to meet the control objectives of good tracking performance and robust stability. The IMC controller reduces to a feedforward system if the plant dynamics are known perfectly [3] and thus provides a connection with the earlier feedforward approach under nominal plant conditions. It is shown that although the feedback nature of the IMC controller can cause instability if the changes in the plant response are too large, the performance of the closed-loop system is very much better than that of an entirely open-loop, feedforward, system before this limit is reached.

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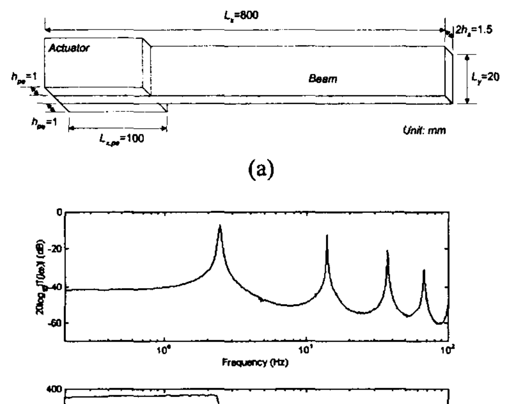
In most of the previous position control systems for flexible beams electrical motors have been the only actuator used, for example ref. (2). In this paper, however, a pair of integrated piezoceramic PZT actuators are used, which provide many attractive features such as light weight, high sensitivity, large bandwidth and distributed properties [4,5], although only limited motion is possible. A practical implementation of the IMC controller implemented with a digital signal processor has also been investigated.

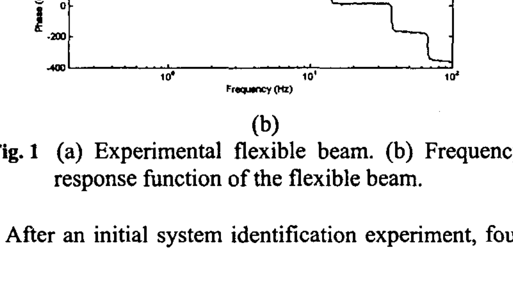
2. Beam Modelling

An experimental flexible beam of 800 mm long (L), 20 mm wide (B) and 1.5 mm thick (t) was constructed of aluminium strip, clamped at one end and free at the other end as shown in Fig. 1(a). A pair of 'Morgan Matroc' PZT 5H piezoceramic actuators, which are each 100 mm long (L_a), 20 mm wide, and 1 mm thick, was bonded on either side of the beam at the clamped end and driven out of phase so as to generate a bending moment.

For the detection of the beam tip motion, an inductive position sensor (Honeywell proximity sensor 924 series 30mm) was used. The frequency range of interest was 0 - 100Hz and the beam positioned vertically for the experiment. The input voltage and the tip deflection were measured with an HP3566A Signal Analyzer, and the input signal to the piezoceramic actuator was amplified to 100V by a PCB AVC 790 series power amplifier.



(a)



(b)

Fig. 1 (a) Experimental flexible beam. (b) Frequency response function of the flexible beam.

After an initial system identification experiment, four resonances were observed between 0 - 100Hz, at 2.37Hz, 13.92Hz, 37.25Hz, and 67.44Hz respectively as shown in Fig. 1(b). Their damping ratios were measured to be ζ_1=0.26%, ζ_2=0.33%, ζ_3=0.35%, and ζ_4=0.51%. The phase response shows a steep phase change of -180° at each resonance frequency. Fig. 1(b) also indicates that the plant is non-minimum phase, which is caused by the non-collocation of the actuator and sensor.

The measured steady state beam deflection was about 1.25mm for 100V step input to the piezoceramic actuators. The response of the beam to a step input is shown in Fig. 5(a), and the measured 95% settling time was about 125 seconds, which demonstrating the very lightly-damped nature of the beam.

A uniform cantilever beam with length L which has clamped-free ends boundary conditions is considered and it is subjected to a harmonic bending moment $M(x,t)$ at $x=L_a$. The tip deflection $y(L,t)$ of the beam is assumed by the superposition of the individual flexural mode as $y(L,t)=\sum_{n=1}^{\infty}B_n(t)\phi_n(L)$, where $B_n(t)$ is the nth flexural modal amplitude and $\phi_n(L)$ is the nth flexural mode shape at tip.

By considering the boundary conditions of the cantilever beam, a receptance form for the tip deflection due to the bending moment at $x=L_a$ can be derived as

$$\frac{y(L)}{M(L_a)}=\sum_{n=1}^{\infty}\frac{k_n\phi_n(L)\phi_n'(L_a)}{A\rho L[(\omega_n^2-\omega^2)+j2\zeta_n\omega_n\omega]}, \tag{1}$$

where k_n, ω_n, ζ_n are the nth flexural wavenumber, natural frequency, damping ratio respectively, and $\phi_n'(L_a)$ is the spatial derivative of $\phi_n(x)$ at $x=L_a$, and A is the sectional area of the beam.

A pair of PZT actuators can induce bending moment at $x=L_a$ when they are driven out-of phase. The relationship between the bending moment M induced by a pair of piezoactuators and input voltage V is given as [5]

$$M=\alpha V, \tag{2}$$

where the coefficient α is the gain of the piezoceramic actuator. Thus, the relationship between the input voltage V and the tip deflection $y(L)$ or the transfer function of the *plant* model can be written in the Laplace domain as

$$G(s)=\frac{y(L)}{V}=K\sum_{n=1}^{\infty}\frac{a_n}{s^2+2\zeta_n\omega_n s+\omega_n^2}, \tag{3}$$

where $a_n=\phi_n(L)\phi_n'(L_a)$ and $K=\alpha k_n/A\rho L$ is the gain of the plant. The zeros of the plant model *G(s)* depend on the coefficients a_n, which are determined by the phase relationship between the command input and the position output of the each resonant mode. In other words, the zeros, which have a direct effect on the overall stability of the control system [(6)], are dependent on the values of the mode shapes and the spatial derivatives of the mode shapes at L (location of sensor) and L_a (location of actuator). However, the poles are independent of the locations of the sensor and actuator since they correspond to the natural frequencies of the flexible beam system.

3. Controller Design

3.1 Analogue PID Feedback Controller

The block diagram of an analogue position control feedback system is shown in Fig. 2(a), in which $r(t)$ is the command signal, $G(s)$, $y(t)$, $e(t)$ and $u(t)$ are the plant, the output of the plant, the error signal, and the control signal respectively. A PID controller can be given as

$$H(s) = K_P + \frac{K_I}{s} + K_D s\,, \qquad (4)$$

where K_P is the proportional gain which can provide an electronic stiffness, K_I is the integral gain which removes the steady state tracking error, and K_D is the derivative gain which gives an active damping.

3.2 Digital IMC Feedback Controller

The IMC approach is a method of designing feedback control systems using the mathematical techniques developed for feedforward control. IMC can transform a feedback position control system into a system resembling a feedforward position control system. Consider the block diagram of a digital IMC controller for a sampled-time single input, single output control system as shown in Fig. 2(b).

The digital feedback controller $H(z)$ contains an internal model $\hat{G}(z)$ of the real plant $G(z)$ and the control filter $W(z)$. The frequency response of the entire feedback controller is

$$\frac{u(z)}{r(z)-y(z)} = H(z) = \frac{W(z)}{1-W(z)\hat{G}(z)}.\,, \qquad (5)$$

The response of the output $y(n)$ of the entire feedback control system to the command signal $r(n)$ can also be expressed as

$$\frac{y(z)}{r(z)} = \frac{G(z)H(z)}{1+G(z)H(z)} = \frac{W(z)G(z)}{1+W(z)[G(z)-\hat{G}(z)]}, \qquad (6)$$

If the plant model $\hat{G}(z)$ is a perfect representation of the plant $G(z)$ (i.e. $G(z)=\hat{G}(z)$) and $G(z)$ is stable, then the classical feedback system with controller $H(z)$ is internally stable if and only if $W(z)$ is stable [6] in which case $z(n)$ tends to zero in Figure 2(b) and the equivalent block diagram becomes entirely feedforward as shown in Fig. 2(c). The system output $y(n)$ is then $W(z)G(z)r(n)$, and thus the complementary sensitivity function is equal to $W(z)G(z)$ in this case. Thus, if $W(z)$ is the inverse of $G(z)$, then the output $y(n)$ will follow the command signal $r(n)$ perfectly. In practice this cannot be achieved with a stable $W(z)$ since $G(z)$ is non-minimum phase and so a least squares approximation to the inverse control be used.

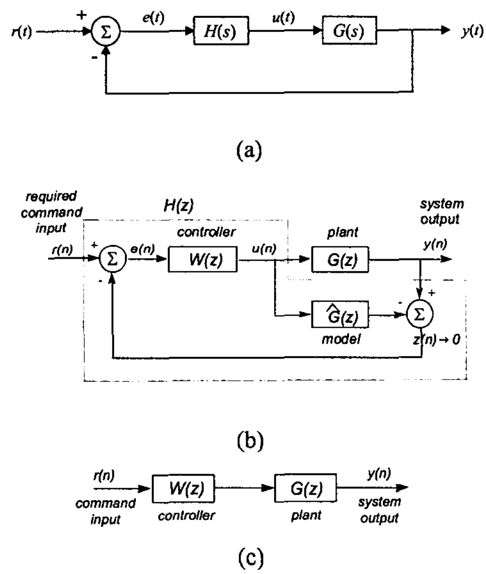
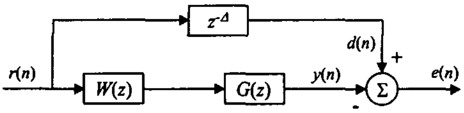


Fig. 2 Block diagrams of control stratigies. (a) Analogue PID feeback. (b) Digital IMC feedback. (c) Equivalent feedforward system to the IMC feedback when $\hat{G}(z) = G(z)$.

The more general problem of calculating the optimum performance of the feedforward system as shown in Fig. 3(a) is outlined below, when the command signal $r(n)$ is fed to an FIR feedforward digital filter $W(z)$, with I coefficients, whose output drives the digital plant $G(z)$ and the plant then produce the system output $y(n)$.

The desired signal $d(n)$ is equal to the command signal $r(n)$ delayed by Δ samples. Such a *modeling delay* is not generally used in control systems since the required signal may not be known in advance in all applications, Δ will initially be taken to be zero. In some applications, however, such as when the plant is required to execute a repetitive motion for example, the required signal is known in advance, and considerable improvements in performance can be obtained with a suitable choice of Δ, which is known as the modeling delay in the signal processing literatures.[7]



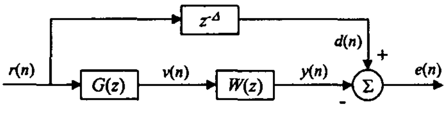


Fig. 3 (a) Block diagram of a feedforward control system to track a setpoint command. (b) Rearrangement of block diagram for the design of the optimal control filter $W(z)$.

The error signal $e(n)$ can be given by subtracting the system output $y(n)$ from the desired signal $d(n)$ as

$$e(z) = d(z) - W(z)G(z)r(z), \tag{7}$$

However, if the control filter $W(z)$ is fixed and is linear time invariant, the order of the blocks $G(z)$ and $W(z)$ can be reversed, as shown in Figure 3(b), to generate a signal $v(n)$ which is equal to the command signal filtered by the digital plant $G(z)$.

Since the control filter is a FIR filter device the error signal can now be expressed as [8]

$$e(n) = d(n) - \mathbf{w}^T \mathbf{v}(n), \tag{8}$$

where $\mathbf{w} = [w_0 \cdots w_{I-1}]^T$, w_i is the ith coefficient of the control filter $W(z)$ and $\mathbf{v}(n) = [v(n) \cdots v(n-I+1)]^T$. Assuming that the reference signal is random each w_i can be adjusted to minimize a cost function J_1 equal to the expectation of square values of the error signals $e(n)$ and so

$$J_1 = E[e^2(n)], \tag{9}$$

The expectation of the squared error signals can now be written as

$$E[e^2(n)] = \mathbf{w}^T \mathbf{A} \mathbf{w} - 2\mathbf{w}^T \mathbf{b} + c, \tag{11}$$

where c is the scalar $E[d^2(n)]$, $\mathbf{b}$ is the vector of cross-correlation function between $\mathbf{v}(n)$ and $\mathbf{d}(n)$, $\mathbf{b} = E[\mathbf{v}(n)d(n)]$ and $\mathbf{A}$ is a Toeplitz matrix of auto-correlation function of v as $\mathbf{A} = E[\mathbf{v}(n)\mathbf{v}^T(n)]$. If $\mathbf{A}$ is not singular, the matrix equation can be solved for the optimal, Wiener, set of filter coefficients $\mathbf{w}_{opt}$ which will produce a minimum error signal as

$$\mathbf{w}_{opt} = \mathbf{A}^{-1}\mathbf{b}, \tag{12}$$

This Wiener filter can then be readily calculated from the cross-correlation vector and the auto-correlation matrix.

The numerical stability of the solution of equation (12) depends on the conditioning of the matrix $\mathbf{A}$, because this optimal Wiener solution depends on its inverse.

The conditioning may be improved by modifying the cost function to add a regularization term that is proportional to the expectation of the squared values of the filter coefficients[9], so that $J_2 = E[e^2(n)] + \beta \mathbf{w}^T \mathbf{w}$, in which the coefficient weighting or regularization parameter β improves the condition number of the $\mathbf{A}$ matrix to be inverted.

4. Experimental Results

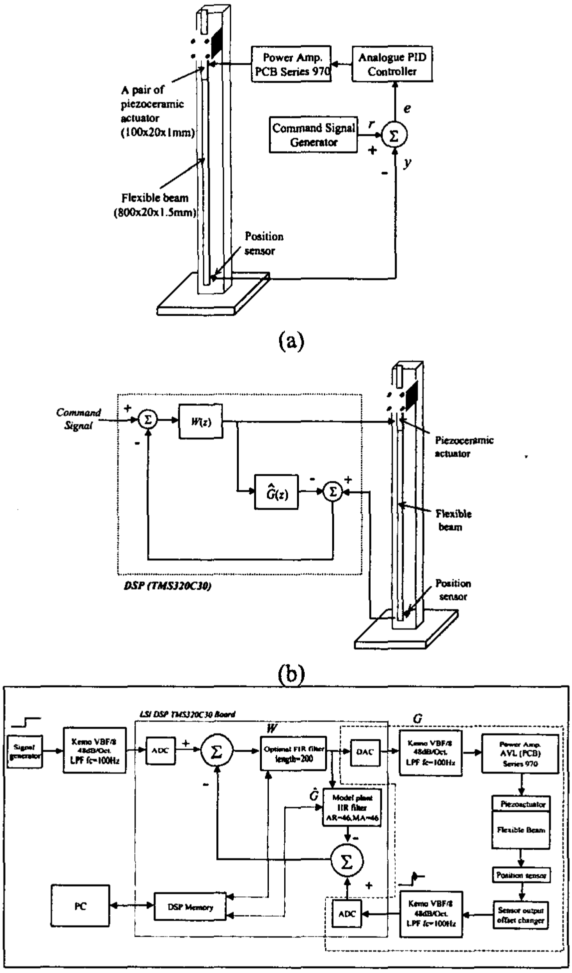
Two position-control experiments for the flexible beam were performed, with either an analogue PID feedback controller or a digital IMC feedback controller.

In the analogue PID control experiment as shown in Fig. 4(a), the three control gains in equation (6) were determined by manual tuning to be K_P=0.4, K_I=1.6 and K_D= 0.0004.

Although there has been a number of *ad-hoc* tuning rates for the PID controller such as Ziegler-Nicholas [10], most of which assume a well-damped plant response, there is no analytic method of adjusting the three parameters to obtain the shortest transient response. In this work a trial and error approach was thus adopted to obtain the exact values of the gains about their final settings.

Fig. 5(b) shows the measured closed-loop step response with the PID control, which settles within 95% of the command position at about 75 seconds. The step response before control was about 125 seconds as plotted in Fig. 5(a).

The measured step response with the PID controller shows that it follows the command gradually with limited overshoot using an electronic stiffness (by K_P and K_D of the controller) created by the piezoactuators. It settles precisely to the command position by the action of the gain K_I as can be seen from Fig. 5(b).



(c)

Fig. 4 Implementation of the control systems. (a) Analogue PID feedback control. (b) Digital IMC feedback controll. (c) Details of IMC architecture.

In the real-time IMC feedback control system, a digital signal processor (DSP) board (Loughborough Sound Images TMS320C30 PC system board) with 16 bit ADC/DAC was used.

The digital plant model $\hat{G}(z)$ for the sampled-time plant $G(z)$ has been defined to include the responses of a DAC, an antialiasing low-pass filter, a power amplifier, the physical flexible beam with a pair of piezoactuators, a tip position sensor, a reconstruction low-pass filter and an ADC as shown in Fig. 4(b).

The sampling frequency was 300 Hz and the cut-off frequency in the low-pass filters was 100 Hz. Thus the plant model $\hat{G}(z)$ on which the digital design is based must contain the time delay caused by the DSP computation time and the low-pass filters' delay, as well as the pure delay in the non-minimum phase flexible beam.

In the real-time control experiment, the IMC feedback control designed has been implemented based on the block diagram in Fig. 4(b). The details of the IMC control implementation with the DSP is illustrated in Fig. 4(c). Both the plant model $\hat{G}(z)$ with an IIR filter and the 200-coefficient control filter w_{opt} designed with Brownian noise input were implemented inside the DSP board.

The upper and the lower graphs in Fig. 5(c) represent the measured real-time closed-loop step responses from 0 - 10000 samples (about 0 - 33 seconds) and 0 - 500 samples (about 0 - 1.7 seconds) respectively. The closed-loop step response of the IMC feedback control settled within 95% of command position at about 10.93 seconds.

The step response with the IMC control in Fig. 5(c) shows a sort of ringing motion at about 2.4Hz, which makes the tip positioning difficult. This is probably caused by the misalignment in frequencies and magnitude differences between the anti-resonances of the control filter $W(z)$ and the resonances of the actual plant response $G(z)$, because of plant model was not perfect, $G(z) \neq \hat{G}(z)$. The misalignment at the first resonance frequency was dominant in the measured closed-loop step response with a ringing at about 2.4Hz.

The misalignment of the resonance frequencies of $G(z)$ and the anti-resonance frequencies of $W(z)$ may have been caused by slight changes in the plant dynamics. The actual plant dynamics $G(z)$ could be perturbed by lots of reasons. The variation of the plant model $\hat{G}(z)$ at over the course of the real-time control experiment showed that the mean value of the first natural frequencies was 2.3763 Hz and the standard deviation was 0.0018 Hz. This amount of variation in the first natural frequency does not destabilize the control system, but can cause the observed ringing in the closed-loop step response.

This frequency misalignment problem in the IMC feedback can make a very-lightly damped system unstable if it is too large. The system will only be stable if the error in the estimate of the natural frequency is less than about half the bandwidth of the resonance. The condition of stability with an IMC position control architecture for a flexible structure can thus be approximated by

$$\left| f_n - \hat{f}_n \right| < f_n \zeta_n, \qquad (15)$$

where f_n and $\hat{f}_n$ are the nth natural frequencies of the actual plant $G(z)$ and plant model $\hat{G}(z)$, and ζ_n is the nth damping ration of the actual plant.

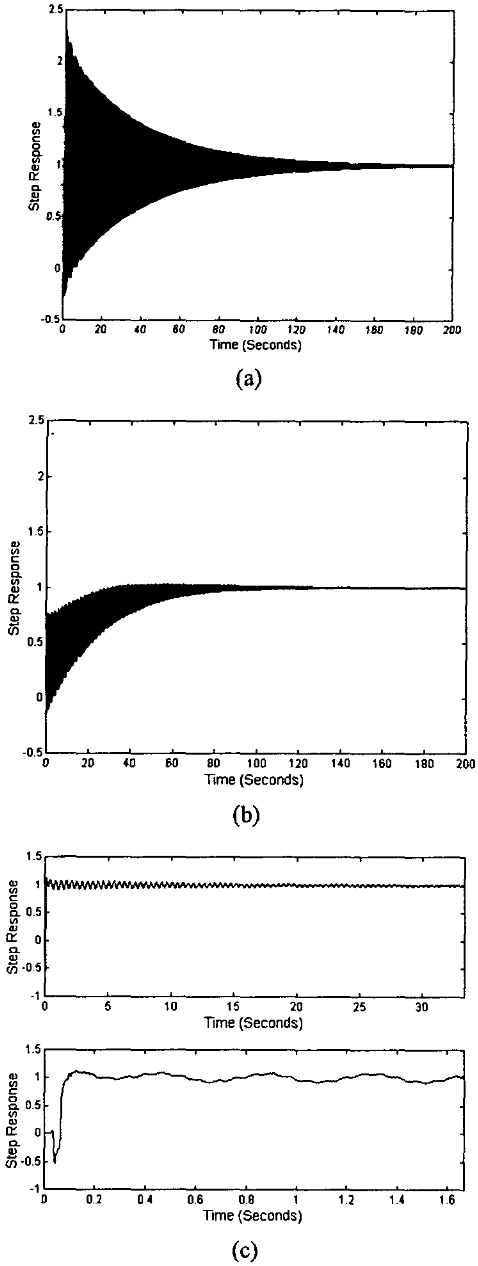


Fig. 5 Measured step responses. (a) Before control. (b) After analogue PID feedback control. (c) After digital IMC feedback control.

5. Conclusions

This paper describes the design and implementation of an active position controller using internal model control (IMC) for setpoint tracking control of a smart flexible beam. The smart flexible cantilever beam was excited by a pair of piezoceramic actuators. The objectives of the position controller for the smart flexible beam were to eliminate its long natural response, due to the very low damping ratio of the beam material, and to maintain a low steady state error by inverse control of the non-minimum phase plant. The optimum performance of a feedback control system was obtained by using quadratic optimization techniques based on the minimization of the mean square tracking error.

The performance and stability of the digital IMC feedback controller were compared with those of the analogue PID feedback controller. The digital IMC feedback control showed much better performance in settling time than that of the analogue PID feedback control. The analogue PID feedback controller was very robust but gave poor performance. In a practical implementation of the IMC position control, even a very small misalignment between the natural frequency of the actual plant and the anti-resonance frequency of the control filter could cause a ringing motion and a reduced tracking performance. However, a much faster settling was achieved with real-time digital IMC control than with analogue PID control.

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