

FE MODEL UPDATING OF ROTOR SHAFT USING OPTIMIZATION TECHNIQUES

최적화 기법을 이용한 로터 축 유한요소모델 개선

Yong Han Kim*, Fu Zhou Feng*, Bo-Suk Yang†

김용한*, Fu Zhou Feng*, 양보석†

Key Words : Model Updating, Hybrid Optimization Algorithm, Genetic Algorithm, Simulated Algorithm.

ABSTRACT

Finite element (FE) model updating is a procedure to minimize the differences between analytical and experimental results, which can be usually posed as an optimization problem. This paper aims to introduce a hybrid optimization algorithm (GA-SA), which consists of a Genetic algorithm (GA) stage and an Adaptive Simulated Annealing (ASA) stage, to FE model updating for a shrunk shaft. A good agreement of the first four natural frequencies has been achieved obtained from GASA based updated model (FEgasa) and experiment. In order to prove the validity of GA-SA, comparisons of natural frequencies obtained from the initial FE model (FEinit), GA based updated model (FEga) and ASA based updated model (FEasa) are carried out. Simultaneously, the FRF comparisons obtained from different FE models and experiment are also shown. It is concluded that the GA, ASA, GA-SA are powerful optimization techniques which can be successfully applied to FE model updating, the natural frequencies and FRF obtained from all the updated models show much better agreement with experiment than that obtained from FEinit model. However, FEgasa is proved to be the most reasonable FE model, and also FEasa model is better than FEga model.

Nomenclature

| | |
|---------------|---|
| p | : updating parameters |
| d | : effective stiffness diameter |
| ω_{Ai} | : the i th analytical natural frequency |
| ω_{Xi} | : the i th experimental natural frequency |
| $f(p)$ | : objective function |
| P_c | : crossover probability in GA |
| P_m | : mutation probability in GA |

1. Introduction

FE model based dynamic analysis has been widely used to predict the dynamic characteristics of structures and rotating machinery. However, the results obtained from an FE model often differ from the experimental results from a vibration or modal test. This disparity is often ascribed to modeling errors and experimental errors. The experimental errors can be controlled to some allowable extent by applying high accuracy sensors, reliable data acquisition, and well-developed measuring and extraction methods [1,2]. While modeling errors can be corrected to some extent by processing records of

dynamic properties from experimental data, e.g. natural frequencies and mode shapes, the correcting process is often called model updating.

Since about 1970s, there has been a continuous stream of publications addressing the problem of improving analytical dynamic models through the use of experimental data. The well-known survey on model updating research in structural dynamics written by Mottershead and Friswell lists well over 200 such publications between 1970 and 1993 [3]. To date, many model updating methods have been proposed, which can be broadly classified into two groups: direct methods and parametric methods, based on whether there is any adjustment on mass and stiffness matrices directly or not. It has been shown that direct method are not appropriate to model updating as the directly updated elements of mass and stiffness matrices are not physically meaningful, although the resulting updated matrices can reproduce the measured modal data exactly [4,5]. There are many approaches to model updating via parametric changes to an existing FE model [6]. One of the most important and recently developed approaches is to incorporate optimization algorithm into model updating, i.e., the model updating can be posed as an optimization problem. Recently, two new approaches to optimization have been developed independently: GA and SA. Both of these algorithms are probabilistic search algorithms that are capable of finding globally optimum results to complicated optimization problems. SA is a generalization on the random walk optimization algorithm,

† School of Mech. Eng., Pukyong Nat. Univ., Busan, Korea
E-mail : bsyang@pknu.ac.kr
Tel : (051) 620-1604, Fax : (051) 620-1405

* School of Mech. Eng., Pukyong Nat. Univ., Busan, Korea

which consists of simply ‘wandering’ about the search space for a given number of iterations, and selecting the best point visited as the global minimum. While GA is an optimization algorithm based on analogy with natural evolution [7]. Although GA and SA have different background, they are both probabilistic search algorithms that are capable of finding globally solutions to complicated optimization problem, and they are robust.

In this paper, a hybrid optimization algorithm, combining GA and SA (GA-SA), is employed to the model updating for a shrunk shaft, which is a shaft that assembled with a mass disk or cylinder by using shrinkage fit method. In the updating process, the updating parameters are selected as effective stiffness diameters of the shaft segment with cylinder, and the objective function is defined as the sum-square difference of natural frequencies obtained from FE model and experiment. Then, natural frequencies obtained from the updated FE models based on GA, SA and GA-SA are compared with those obtained from experiment, the results show that difference of natural frequencies obtained from GA-SA based updated model and experiment is the least among the three updated models. In order to verify the validity of FEgasa model, a modal experiment is carried out, and substituting the updated parameters to FEinit model, the FRFs are obtained from FEinit model, FEga model, FEasa model and FEgasa model are obtained. From the comparison, it can be concluded that GA and SA are both powerful optimization techniques which can be successfully applied to FE model updating, but GA-SA based updated model can predict natural frequencies and FRF that match the experiment much better than GA and SA based updated model.

2. Model updating process

2.1 GA-SA algorithm

GA is based on the principles of genetic and natural selection, Darwin’s “survival of the fitness” strategy. In natural evolution, members of population compete with each other to survive and reproduce successfully. If the genetic makeup of an individual member of a population gives that individual an advantage over its rivals, then it is more likely to breed successfully. As a result, the combinations of genes that confer this advantage are likely to spread across the population. In this way, the population continuously adapts to its environment and in some sense improves its “fitness” [4]. The central theme of the research on GAs has been the robustness, and the balance between the efficiency and the efficacy necessary for survival in many different environments. GAs are computationally simple, yet powerful, and are not limited by assumptions about the search space [8]. However, a GA cannot guarantee that the solution will converge to the optimum, it tries to find the optimum, i. e., it works towards an improvement. And GA also

depends heavily on crossover and mutation probability. Since the probability of mutation is much smaller than the probability of crossover, GA often lacks a hill-climbing capability.

SA was derived from an analogy with the annealing process of material physics by Kirkpatrick in 1983. It is well known that certain materials have multiple stable states which have differing molecular distributions and energy levels. The annealing process consists of heating the substance until it is molten, then slowly and discretely lowering the temperature. The substance is allowed to reach thermal equilibrium at each temperature. Eventually the temperature is lowered until the material freezes. If the temperature is lowered sufficiently slowly, the annealing process can always pick out the global minimum energy state from the almost unlimited number of possible states [5]. The central theme of SA is to define a cooling schedule and neighborhood function.

Various researchers have suggested differing implementations of SA algorithm. In particular, the most thoroughly investigated SA software is the ASA code introduced at the web site (<http://www.ingber.com/>). ASA is developed to statistically find the best global solution for a nonlinear constrained non-convex cost-function over a D-dimensional space. This algorithm permits and annealing schedule for “temperature” decreasing exponentially in annealing time. The introduction of re-annealing also permits adaptation to changing sensitivities in the multi-dimensional parameter space [9].

It is true that GA and SA are both probabilistic search algorithms capable of finding the global minimum amongst many local minima. However, empirically, GA often lacks a hill-climbing capability, and it doesn’t tend to work well when the objective function is a huge multimodal function or a highly coupled function, e.g. the banana function [8]. Since SA has a statistical hill-climbing capability and the solution state cannot stay at a fixed point for a long time. So, for a particular problem, if GA is applied in the first step to get a relatively best solution, then taking this best solution as initial solution of SA, no matter what kind of shape of the solution space for this problem, the final solution must be better than that obtained from a single GA or SA. In this paper, GA-SA is introduced to FE model updating for a shrunk rotor shaft. Comparing GA-SA with a single GA and SA, it has been verified that it has at least advantages in two aspects, GA-SA can not only overcome demerit of GA, but also increase the probability of finding the global optimum.

2.2 Updating parameters

One of the most important but difficult issues in FE model updating is the selection of updating parameters. If the selected parameters are inadequate, the updated model becomes unsatisfactory or unrealistic. However, too many updating parameters might often cause an ill-conditioned numerical problem, so the number of

updating parameters should be kept as small as possible. Generally, updating parameters should be selected with the aim of correcting modeling errors, and the selected parameters should be sensitive to modal properties. There are many methods for selection of updating parameters, such as sensitivity analysis [10,11], error localization algorithm [11], etc. However, in a practical application, updating parameters are often selected as global, sub-structural or local material or geometric parameters. Many researches are focused on material properties updating because these parameters are often unknown or partially known, while the knowledge of geometric parameters of the structure can be obtained easily, although they might be simplified in the model [4, 6,12,13]. One of the strategies to ensure that only meaningful corrections are made after the updating process is to select updating parameters on the basis of engineering judgment about the possible locations of modeling error or material properties variation in a structure [12]. As rules of thumb, for the case of a rotor with or without mass disks, parameters, e.g. Young's modulus and mass density of each element in FE model, effective stiffness diameters of the shaft segment with mass disks can be selected as updating parameters. After careful consideration, in this paper, updating parameters are selected as the effective stiffness diameter of the shaft segment with mass disk. Of course, some other parameters, e.g. mass distribution of the motor, have been tried but the mentioned ones turned out to be the most effective, so we use these parameters in our current research.

2.3 Objective function

Since FE model updating is to minimize the differences between the experimental and analytical modal data by adjusting updating parameters, an objective function should be defined firstly. An obvious first choice is the sum-squared difference between the natural frequencies, mode shapes or FRFs obtained from experiment and analytical FE model. In this paper, the objective function is defined as the sum-squared difference of natural frequencies obtained from analytical model and experiment, represented in Eq. (1).

$$f\{p\} = \sum_i (\omega_{Ai} - \omega_{Xi})^2 \quad (1)$$

3. Applications

3.1 The shrunk shaft model

A shrunk shaft is a shaft that assembled with a cylinder by using shrinkage fit method. Figs. 1 and 2 show the structure and initial FE model of the considered shrunk shaft, respectively [14].

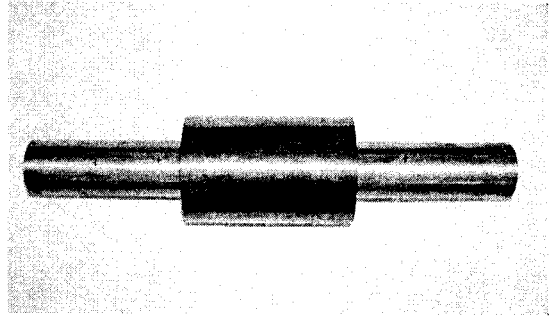


Fig. 1 Structure of the shrink shaft

Table 1 gives the detail geometric and material property parameters of the shrunk shaft. Generally, for a shaft, the stiffness diameter of each shaft segment equals its practical geometric diameter, but due to the assembled interference of the shrunk shaft, the stiffness of shaft segment with cylinder cannot be exactly calculated with their geometric diameters in FEM, thus a concept of effective stiffness diameters is employed here. Although there are some empirical formulas for calculating the effective stiffness diameters of special shaft segments, e.g. step element, shaft segment with mass disk, the results is usually not very good. So, here the updating parameters are selected as effective stiffness diameters, $d_6 \sim d_{11}$, which are indicated in Fig. 2. Considering the symmetric characteristics of the shaft, assumed that $d_6 = d_{11}$, $d_7 = d_{10}$ and $d_8 = d_9$.

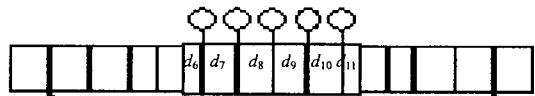


Fig. 2 FE model of the shrunk shaft

Table 1 Shaft parameters, length of each element and diameters (from the left end to the right in Fig. 1).

| Element No. | Shaft length/diameter(mm) | Element No. | Shaft length/diameters (mm) |
|-------------|---------------------------|-------------|-----------------------------|
| 1 | 30/40 | 9 | 26/45 |
| 2 | 30/40 | 10 | 26/45 |
| 3 | 30/40 | 11 | 13/45 |
| 4 | 20/40 | 12 | 20/40 |
| 5 | 20/40 | 13 | 20/40 |
| 6 | 13/45 | 14 | 30/40 |
| 7 | 26/45 | 15 | 30/40 |
| 8 | 26/45 | 16 | 30/40 |

$E = 211 \text{ GN/m}^2$ $\rho = 7800 \text{ kg/m}^3$

3.2 Comparison of updated results

FE models for rotor shafts considered here are programmed in Matlab6.0. The tested shaft is supported using steel piano strings. Two high accuracy acceleration sensors, amplifier chargers and Medallion software are employed to data acquisition and modal analysis. In the modal testing, 5 times average on the measured

frequency response function (FRFs) data is used to reduce the random experimental error. And the experimental natural frequencies are identified using FRF data by means of rational polynomial method.

The natural frequencies obtained from the FE_{init} model and experiments are listed in the 2nd column and last column of Table 4. Note that the other higher natural frequencies cannot be measured due to the limited frequency range in modal analysis. It is obviously that in higher frequency range, the difference is relatively great, especially the 3rd mode frequency, the absolute frequency difference is about 613 Hz. That is to say, the parameters, $d_6 \sim d_{11}$, should be updated with optimization method so as to minimize the difference of natural frequencies.

In the model updating process, the initial value of updating parameters and parameters related to the GA algorithm and ASA algorithm are listed in Table 2.

Tables 3 and 4 give the final solution obtained by different optimization algorithm and natural frequencies obtained from FE_{init}, FE_{ga}, FE_{asa}, FE_{gasa} models, respectively.

Comparing the natural frequencies obtained from all the updated models and experiment, there is a great improvement on difference of the 3rd mode frequency, however, there are still some differences on the 1st and 2nd natural frequencies. It is obvious that FE_{gasa} model seems to be the best model that can match the experimental natural frequencies closely in the lower frequency. That is to say, the difference of the first four

Table 2 Updating parameters and optimization algorithm related parameters.

| Parameter item | Parameter value/range |
|--------------------------------------|-------------------------------------|
| p_1 ($d_6 = d_{11}$) mm | [45 75] |
| p_2 ($d_7 = d_{10}$) mm | [45 75] |
| p_3 ($d_8 = d_9$) mm | [45 75] |
| Young's modulus (GPa) | 211 |
| Material density (kg/m^3) | 7800 |
| No. of Generation in GA | 20 |
| No. of population in GA | 150 |
| P_c | 60% |
| P_m | 5% |
| ASA related parameters | Default in ASA source code [16] |
| GA-SA related parameters | The same as that used in GA and ASA |

Table 3 Comparison of final solutions obtained by GA, ASA and GA-SA based model updating.

| | p_1 ($d_6 = d_{11}$) (mm) | p_2 ($d_7 = d_{10}$) (mm) | p_3 ($d_8 = d_9$) (mm) |
|----------------|-------------------------------------|-------------------------------------|----------------------------------|
| Initial model | 65.0 | 65.0 | 65.0 |
| GA solution | 56.0 | 74.6 | 74.9 |
| ASA solution | 48.36 | 75.0 | 75.0 |
| GA-SA solution | 47.9 | 75.0 | 75.0 |

Table 4 Natural frequencies of the initial FE model, updated model and experiment

| | Natural frequency (Hz) | | | | Exp. |
|----|--------------------------|------------------------|-------------------------|--------------------------|------|
| | FE _{init} model | FE _{ga} model | FE _{asa} model | FE _{gasa} model | |
| 1 | 1557 | 1611 | 1588 | 1584 | 1556 |
| 2 | 2915 | 2914 | 2895 | 2890 | 2845 |
| 3 | 6472 | 7021 | 7037 | 7028 | 7085 |
| 4 | 8906 | 8979 | 8872 | 8852 | 8846 |
| 5 | 11967 | 13203 | 13373 | 13368 | - |
| 6 | 16630 | 17804 | 17797 | 17773 | - |
| 7 | 19752 | 20047 | 19885 | 19848 | - |
| 8 | 21178 | 22924 | 23070 | 23053 | - |
| 9 | 24709 | 28090 | 28247 | 28218 | - |
| 10 | 26112 | 30025 | 30192 | 30156 | - |

natural frequencies between that obtained from FE_{gasa} model and experiment is the least. From the viewpoint of the optimization, the better the value of objective function is, the better the model is. So, seeing from the comparison in Table 4, it can be concluded that all the updated models are much better than FE_{init} model, but FE_{gasa} model is better than FE_{asa} model, while FE_{asa} model is better than FE_{ga} model.

4. Verification

In order to verify the validity of GA-SA, the updated parameters obtained from GA, ASA and GA-SA are substitute to the initial FE model, then FRFs are carried out and compared, which are plotted in Fig. 3. Obviously, the FRF obtained from FE_{gasa} model shows the best agreement with experimental FRF than that obtained from FE_{init} model, FE_{ga} model, especially in the lower frequency range. However, there are still some differences in the higher frequency range. It seems that FE_{asa} model have the same good performance as FE_{gasa} model, but FE_{gasa} model shows much better agreement at each natural frequency with experiment than that obtained by FE_{asa} model.

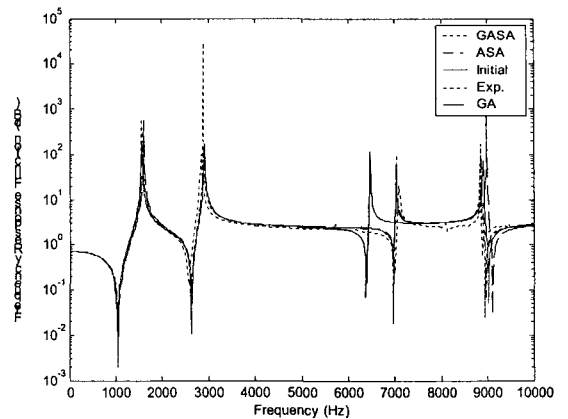


Fig. 3 Comparison of FRF obtained from updated FE model and experiment

5. Conclusions

In this paper, a hybrid optimization technique termed as GA-SA based FE model updating for a shrunk shaft, a typical structure in the research area of rotor dynamics, has been explored. From the results achieved in this paper, for an initial FE model, effective stiffness diameters of some special shaft segments can be updated to predict natural frequencies and FRFs that can match the experimental results more exactly. A good agreement of the first four order natural frequencies has been achieved obtained from FEgasa model and experiment. In order to prove the validity of GA-SA, comparisons of natural frequencies obtained from the FEinit model, FEga model and FEasa model are carried out. Simultaneously, the FRF comparisons obtained from different FE models and experiment are also shown. It is concluded that the GA, ASA, GA-SA are powerful optimization techniques which can be successfully applied to FE model updating, the natural frequencies and FRF obtained from all the updated models show much better agreement with experiment than that obtained from FEinit model. However, FEgasa is proved to be the most reasonable FE model, and also FEasa model is better than FEga model. In short, the GA and SA are both powerful optimization techniques which can be successfully applied to FE model updating, but their combination, termed as GA-SA, can achieve much better results than a single GA or ASA.

References

- (1) N.M. Maia and J. M. M. e Silva, 1997, Theoretical and experimental modal analysis, England: Research Studies Press.
- (2) D.J. Ewins, 2000, Modal testing: Theory, Practice and Application. England: Research Studies Press.
- (3) J.E. Mottershead, M.I. Friswell, 1993, Model updating in structural dynamics: a survey, Journal of Sound Vibration 162, pp. 347~375.
- (4) R. I. Levin and N. A. J. Lieven, 1998, Dynamic finite element model updating using simulated annealing and genetic algorithms, Mechanical Systems and Signal Processing 12(1), pp. 91~120.
- (5) Gyeong-Ho Kim, Youn-Sik Park, An improved updating parameter selection method and finite element model update using multi-objective optimization technique, Mechanical Systems and Signal Processing (in press).
- (6) M.I. Friswell and J.E. Mottershead, Best practice in finite element model updating, International Forum on Aeroelasticity and Structural Dynamics 2, 57-1~57-11.
- (7) Holland, J. H, Adaptation in natural and artificial system, 1975, The University of Michigan Press, Michigan.
- (8) I. K Jeong and J. J Lee, 1996, Adaptive Simulated Annealing Genetic Algorithm for System Identification, Engng Applic. Artif. Intell. 9 (5), pp. 523~532,
- (9) Y.H. Kim, B.S. Yang, Y.C. Kim, Bearing Parameters Identification Using Hybrid Optimization Algorithm, 2003, The 32nd International Congress and Exposition on Noise Control Engineering, Jeju, Korea.
- (10) Q.W. Zhang, C.C. Chang, etc., 2000, Finite element model updating for structures with parametric constraints, Earthquake Engineering and Structural Dynamics 29, pp. 927~944.
- (11) K. Bohle and C.P. Fritzen, 2003, Results obtained from minimizing natural frequency and max-value errors of a plate model, Mechanical Systems and Signal Processing 17(1), pp. 55~64.
- (12) S.V. Modak, T.K. Kunard and B.C. Nakra, 2002, Use of updated finite element model for dynamic design, Mechanical Systems and Signal Processing 16(2-3), pp. 303~322.
- (13) S.V. Modak, T.K. Kundra, etc, 2000, Model updating using constrained optimization, Mechanics Research Communications 27, pp. 543~551.
- (14) Y. C. Kim etc, 2003, Stiffness effect of fitting interference for a shrunk rotor, Proceeding of KSNVE Spring Annual Meeting, pp. 319~324