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Probability-Based Active Control Using Structure Energy
(구조물의 에너지를 이용한 확률에 기초한 능동제어)

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Key Words : active control; energy; Rayleigh probability distribution; Lyapunov controller

ABSTRACT

This paper presents active control algorithm using probability density function of structural energy. It is assumed that the structural energy under excitation has Rayleigh probability distribution. This assumption is based on the fact that Rayleigh distribution satisfies the condition that the structural energy is always positive and the occurrence probability of minimum energy is zero. The magnitude of control force is determined by the probability that the structural energy exceeds the specified target critical energy, and the sign of control force is determined by Lyapunov controller design method. Proposed control algorithm shows much reduction of peak responses under seismic excitation compared to LQR controller, and it can consider control force limit in the controller design. Also, chattering problem which sometimes occurs in Lyapunov controller can be avoided.

1. INTRODUCTION

During past few years, several passive, active, and hybrid response control methods have been developed to reduce the vibratory responses of civil engineering structures [1]. Most of them are based on linear optimal theory [2-6]. Linear control methods such as LQR, H_2 and H_∞ have been developed to guarantee the robustness and stability of actively-controlled structures. These methods yield control forces which are linearly proportional to the states of structures.

This paper presents a probabilistic approach to deal with control force saturation and chattering problem and to reduce peak responses under earthquake excitation. The structural energy can be used as an index to determine the effective control force, because the energy is regarded as an energy absorbing capacity of a structure during an earthquake excitation. However, an exact structural energy is difficult to define since limited responses are measured and there exists uncertainty in system modeling, unknown exogenous loadings and measurement noises. We assume that the structural energy follows Rayleigh probability distribution to consider this uncertainty. The probabilistic control force is determined based on the exceeding probability of target critical energy, which is specified previously. Since the probability has the value from zero to one, properly determined control forces are applied whose magnitude ranges are from zero to maximum control force. Rayleigh probability distribution of structural energy and various design parameters are suggested and they are incorporated into the design of the effective and robust controller, which considers the uncertainty and overcomes saturation and chattering problems.

To prove the effectiveness of the proposed method, earthquake analyses of single-story building and ten-story building are performed. El Centro earthquake

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is used and active mass damper is applied as controller. Its weight is 5% of total building weight.

2. CONTROL ALGORITHM

2.1 Rayleigh probability distribution [7-8]

It is assumed in this paper that the structural energy under excitation has the property of Rayleigh probability distribution based on the fact the structural energy is always positive and the occurrence probability of minimum energy is zero. Then, Rayleigh probability distribution is given as

$$p(E) = a(E - E_{min})\exp[-b(E - E_{min})^2 / 2] \quad (1)$$

where E and E_{min} denote current structural energy and minimum energy at time *t*, respectively. They are time-dependent and their exact expressions are $E(t)$ and $E_{min}(t)$. For the simplicity of the expressions, t is dropped herein. *a* and *b* are determined by the following probability distribution condition.

$$\int_{E_{min}}^{\infty} p(E)dE = 1 \quad (2)$$

$$\int_{E_{min}}^{\infty} Ep(E)dE = \bar{E} \quad (3)$$

Here, $\bar{E}$ means the structural mean energy. a and b are determined as

$$a = \frac{\pi}{2(\bar{E} - E_{min})^2} \quad (4)$$

$$b = \frac{\pi}{2(\bar{E} - E_{min})^2} \quad (5)$$

Substitution of a and b into Equation (1) yields the following probability distribution.

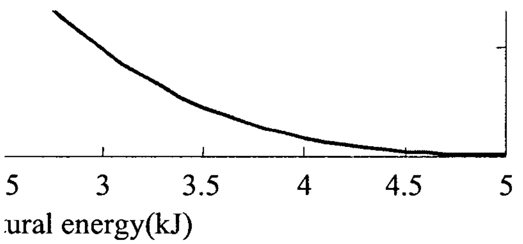

Figure 1. Raleigh probability distribution

$$p(E) = \frac{\pi(E - E_{min})}{2(\bar{E} - E_{min})^2}\exp[-\frac{\pi(E - E_{min})^2}{4(\bar{E} - E_{min})^2}] \quad (6)$$

As an example, Figure 1 shows Rayleigh probability distribution for the case of E_{min} = 0.5kJ and $\bar{E}$ = 2kJ.

2.2 Mean and minimum of structural energy

The structural energy at time *t* consists of potential energy and kinetic energy and is written as

$$E = \frac{1}{2} z^T Q z \quad (7)$$

where

$$Q = \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix} \quad (8)$$

The information of all states is needed to find the exact structural energy. However, the number and type of sensors measuring the states are limited. It is required that all states be estimated from the measured states. In this paper, we use static observer to estimate the states. The static observer is regarded as an optimum observer for the cases that all states are of Gaussian distribution and measurement noises are very small [9]. The relationship between states and output measurement is given as

$$y = C_2 z \quad (9)$$

where C_2 location matrix which defines location of sensors and y is output measurement matrix.
Multiplication of the output matrix by gain matrix P gives the following equation.

$$\bar{z} = Py \quad (10)$$

where $\bar{z}$ means the estimated states.
The observer error e is defined below and the cost function is given as

$$e = z - \bar{z} \quad (11)$$

$$J = trace\{E[ee^T]\} \quad (12)$$

where E[·] is the function of mean.
Gain matrix *P* is obtained by minimizing the cost function and given by

$$P = SC_2[C_2 S C_2^T]^{-1} \quad (13)$$

where S means covariance matrix of states and is obtained by Lyapunov equation.

$$AS + SA^T + Q_2 = 0 \tag{14}$$

where Q_2 is covariance matrix of earthquake excitation. The estimated states $\bar{z}$ are regarded as conditional mean states, because they are obtained from measured states by minimizing the cost function. Hence, the mean of structural energy is described as a function of $\bar{z}$.

The estimated states $\bar{z}$ are regarded as conditional mean states, because they are obtained from measured states by minimizing the cost function. Hence, the mean of structural energy is described as a function of $\bar{z}$.

$$\bar{E} = \frac{1}{2}\bar{z}^T Q \bar{z} \tag{15}$$

States z can be separated as measured variable z_1 (= y) and unmeasured variable z_2. In this paper, the measurement signals are displacements or velocities which are parts of states. Then, the structural energy can be expressed in terms of z_1 and z_2.

$$E = \frac{1}{2}[z_1^T \; z_2^T]\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \frac{1}{2}(z_1^T q_{11} z_1 + z_2^T q_{21} z_1 + z_1^T q_{12} z_2 + z_2^T q_{22} z_2) \tag{16}$$

It is assumed that measured states are exactly same as real states and only unmeasured states should be determined by minimizing the structural energy. The value of z_{2min} which minimizes the structural energy can be determined by differentiating the energy with respect to z_2.

$$\frac{dE}{dz_2} = \frac{1}{2} q_{21} z_1 + \frac{1}{2} q_{12}^T z_1 + q_{22} z_2 \tag{17}$$

Equating the above equation to zero gives the expression of z_{2min} .

$$z_{2\min} = -\frac{1}{2} q_{22}^{-1}[q_{21} + q_{12}^T] z_1 \tag{18}$$

Finally the minimum of structural energy is expressed as

$$E_{\min} = \frac{1}{2}[z_1^T \; z_{2\min}^T]\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}\begin{bmatrix} z_1 \\ z_{2\min} \end{bmatrix} \tag{19}$$

2.3 Magnitude of Control force

The mean and minimum of structural energy are derived in the previous section. The probability distribution of structural energy is then described. The probability distribution of structural energy is located above $E_{\min}$ and around $\bar{E}$ as shown in Figure 1. Suppose that a control is needed in case that a structural energy is relatively large compared to a target critical energy. The target critical energy is denoted as E_c. The probability ρ of current structural energy exceeding E_c is the integration of the probability distribution from E_c to infinite value when E_c is not less than $E_{\min}$ and the probability is 1 whenever E_c is less than $E_{\min}$. The absolute control force is then proportional to the probability. Relevant probability and control force are given as

$$\rho = \begin{cases} \int_{E_c}^{\infty} p(E)dE = \exp\left\{-\frac{\pi(E_c - E_{\min})}{4(\bar{E} - E_{\min})}\right\}; & E_c \ge E_{\min} \\ 1 & ; E_c < E_{\min} \end{cases} \tag{20}$$

$$|u| = \rho u_{\max} \tag{21}$$

where u_{max} is an actuator generated maximum control force.

Equation (20) shows that E_c and $\bar{E} - E_{\min}$ are important parameters in determining ρ. Larger difference of $\bar{E} - E_{\min}$ means slower variation of the probability distribution. As more states are measured, the difference becomes smaller. It means that a current structural energy E becomes more exact. For the cases where all states are measured, E is exactly found. Then all values of $\bar{E}, E_{\min}$ and E are same. For this deterministic case, the value of ρ becomes 1 when $\bar{E}$ is greater than E_c and zero when $\bar{E}$ is less than E_c. This means that the maximum control force is generated like bang-bang control for ρ = 1, and no control force for ρ = 0. However, even if all states are measured, the exact value of E is impossible to get because of inherent uncertainty in measurement and system modeling. Parameter α is suggested to consider the uncertainty and is defined as the larger value between $\bar{E} - E_{\min}$ and $\alpha_{\min}$.

$$\alpha = \max(\bar{E} - E_{\min}, \alpha_{\min}) \tag{22}$$

$\alpha_{\min}$ is a lower limit value of α and has two important tasks of considering uncertainty of structural energy and determining energy level where control force is not necessary. $\bar{E}$ less than $\alpha_{\min}$ means that the structural energy is small and control is not needed.
Hence, the magnitude of a control force is modified as

$$|u| = \begin{cases} \rho u_{\max} & ; \text{if } \bar{E} \geq \alpha_{\min} \\ 0 & ; \text{if } \bar{E} < \alpha_{\min} \end{cases} \tag{23}$$

where ρ is

$$\rho = \begin{cases} \exp\left\{-\dfrac{\pi(E_c - E_{\min})}{4\alpha}\right\}; & \text{if } E_c \geq E_{\min} \\ 1 & ; \text{if } E_c < E_{\min} \end{cases} \tag{24}$$

Equation (24) indicates that the probability ρ is 0.4559($= \exp(-\pi/4)$) for E_c equal to and α equal to $\bar{E} - E_{\min}$ and 1 for E_c less than $E_{\min}$. It means that 0.4559 times maximum control force is generated or maximum control force is generated depending on the corresponding probability. Furthermore, when $\bar{E}$ is less than $\alpha_{\min}$ means that the current structural energy is small compared to $\alpha_{\min}$, so that a control is not needed and control force is zero. From this analysis, E_c is recommended to have a small value in order to maximize the capacity of controller. We suggest a method for determining E_c as a linear function of $\overline{E_o}$ and E_{o_min}, which are mean and minimum structural energies, respectively, obtained from time history analysis of a structure without control under given or assumed earthquake excitation. Control designers can specify E_c considering structural properties, control efficiency and earthquake excitation by giving a proper value to design parameter β .

$$E_c = \beta \bar{E}_o + (1-\beta) E_{o\_\min} \tag{25}$$

2.4 Direction of control force

The magnitude of a control force is derived in Equation (23). The direction of a control force is determined based on Lyapunov controller design concept. Lyapunov controller is designed by taking the first derivative of Lyapunov function with respect to time. Structural energy E is regarded as Lyapunov function in this paper. The first derivative of E with respect to time is given as

$$\begin{aligned} \frac{dE}{dt} &= z^T Q \dot{z} = z^T Q(Az + Bu + H\ddot{x}_g) \\ &= -\dot{x}^T C \dot{x}^T + \dot{x}^T Du + \dot{x}^T E\ddot{x}_g \end{aligned} \tag{26}$$

Equation (26) shows that damping matrix C decreases E at all times. When there is no earthquake excitation, the direction of control force, which decreases E, is given as

$$\text{sgn}(u^T) = -\text{sgn}(\dot{x}^T D) = -\text{sgn}(\lambda) \tag{27}$$

where sgn(.) is a sign function and $\dot{x}^T D$ is λ. This value equals 1 when the value in parenthesis is positive and - 1 when the value in parenthesis is negative. It is zero when the value in parenthesis is zero. λ is r vector denoting the number of controllers whose *i*th element is λ_i. Considering the magnitude and the direction of control forces, the *i*th control force u_i can be expressed as

$$u_i = -\text{sgn}(\lambda_i) H(|\lambda| - \varepsilon_o) \rho u_{\max} \tag{28}$$

in which $H(|\lambda| - \varepsilon_o)$ is the unit step function that removes undesirable chattering effect inside a boundary layer ε_0 and $|\cdot|$ denotes the norm of inside term.

3. NUMERICAL EXAMPLES

To prove the effectiveness of the proposed control algorithm, numerical analyses on SDOF and MDOF systems have been performed.

3.1 Earthquake analysis of SDOF system

Structural properties are same as those of the previous example. El Centro earthquake ground acceleration (1940, NS) is used as seismic excitation. Design parameters are chosen as E_c = 1918.1J, u_{max} = 16.75kN, $\alpha_{\min}$ =959.1J, and ε_o=0. E_c is calculated with β = -2 and $\alpha_{\min}$ is chosen as 20% of mean energy without control. Figure 2 shows the responses without control and with control by proposed algorithm and mean energies. Figure 3 shows that the controlled responses by proposed algorithm and LQR algorithm and their control forces. Weighting matrices of LQR algorithm are determined considering that maximum control force is same as u_{max} = 16.75kN so that the control performance of each algorithm can be compared. Figure 2(a) and (b) show relative displacements with respect to base displacement and absolute accelerations without control and with control by proposed algorithm. Figure 2(c) shows that mean energy with control by proposed algorithm is much smaller than that without control. The reduced mean energy results from the absorbed energy by control devices. Figure 3(a) and (b) show thecomparison of relative displacements and absolute accelerations by LQR and proposed algorithms. Figure 3(c) shows the comparison of control forces by LQR and proposed algorithms. Figure 3(c) shows that no chattering is found in the region of small responses and bang-bang control is found in the region of large responses. Table 1 shows that the responses by proposed algorithm are greatly reduced with same maximum control force. This observation indicates that the proposed algorithm designs nonlinear controller generating maximum control forces for large responses and no control forces for small responses. On the other hand, LQR algorithm designs linear controller generating control forces proportional to responses. Table 1 shows that the responses by proposed algorithm are greatly reduced with same maximum control force. This observation indicates that the proposed algorithm designs nonlonear controller generating maximum

Table 1. Response quantities of SDOF system under seismic excitation

Method		No control	LQR	Proposed
Control Force(kN)	rms	-	4.07	6.59
Displacement(cm)	peak	-	16.75	16.75
	rms	4.40	2.78	2.21
Velocity(cm/s)	peak	15.23	10.81	8.36
	rms	31.20	19.90	15.92
	peak	111.55	81.88	62.38
Acceleration	rms	214.95	135.89	108.04
	peak	743.11	527.73	408.96

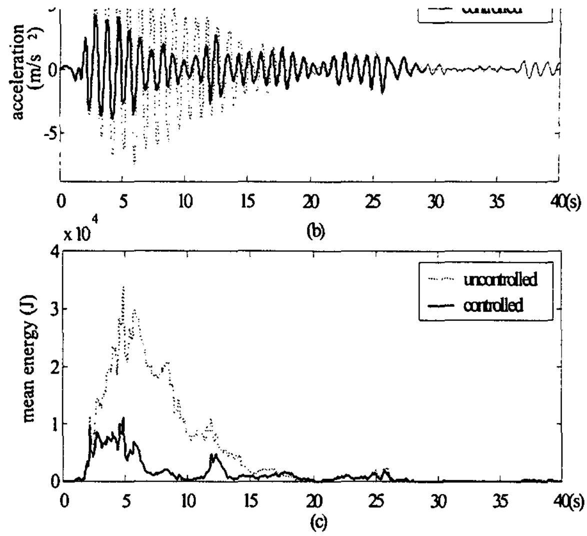

Figure 2. Responses and mean energy of SDOF system

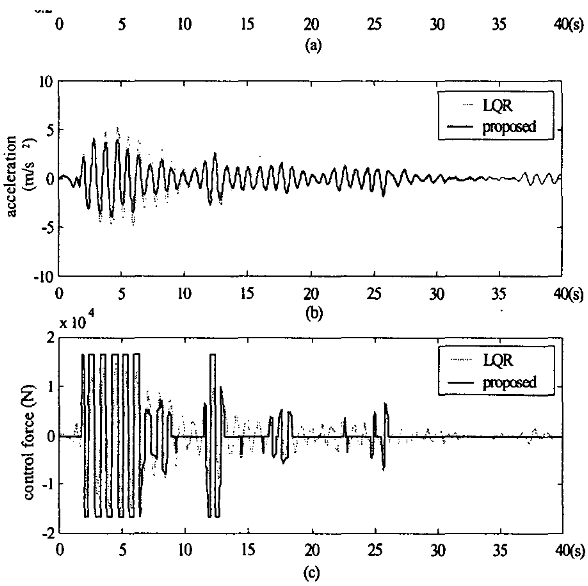

Figure 3 Responses and control force of SDOF system

control forces.for large responses and no control forces for small responses. On the other hand, LQR algorithm designs linear controller generating control forces proportional to responses.

3.2 Earthquake analysis of MDOF system

Earthquake analysis of a ten-story building has been performed. Ground motion of El Centro earthquake acceleration data (1940, NS) is applied as seismic excitation. The mass of each story is 50ton, and stiffness is $k_i(i=1,2,3,4)$ = 15×10^6kN/m, $k_j(j=5,6,7)$= 10.5×10^6 kN/m, $k_k(k=8,9,10)$ = 7.35×10^6kN/m and modal damping ratio is 2%. Design parameters are chosen as E_c =5059.4J, u_{max} = 84.16kN, α_{min}=2370.1J, and ε_0=0.2. E_c is determined with β = -1 and α_{min} is set to be 5% of mean of structural energy without control. It is assumed that the displacement and velocity of top floor where one AMD is located can be measured. To compare the control performance of proposed algorithm and LQR algorithm, weighting matrices of LQR algorithm are chosen to have the same maximum control force as that of the proposed algorithm.

Figure 4(a) and (b) show relative displacement of top floor with respect to base displacement and absolute acceleration of top floor without control and with control by proposed algorithm. Figure 4(c) shows that mean energy with control by proposed algorithm is much smaller than that without control. The responses and control forces by proposed algorithm and LQR algorithm are investigated in Figure 5. Maximum responses of displacement are much reduced in proposed algorithm. Accelerations are not greatly reduced comared to displacements. Figure 5(a) and (b) show that control efficiency of the proposed algorithm is better than that of LQR algorithm. The comparison of the control forces by both algorithms are shown Figure 5(c) and the direction of both control forces are nearly same. Similarly, in the analysis of SDOF system, the magnitude of control forces by proposed algorithm is generally larger than the control forces by LQR algorithm in the region of large responses and zero in the region of small responses showing no chattering phenomena. The various evaluation indices are considered to investigate the performance of the proposed algorithm. They are dimensionless and given in Equations (29) and (30). J_1, J_2, and J_3 represent maximum rms values of the relative displacement of each floor with respect to the base displacement, inter-story drifts, and absolute floor accelerations, which are normalized to the correspondent rms responses of uncontrolled case. J_4 represents rms values of control force by proposed algorithm normalized to those by LQR algorithm. Second performance evaluation indices, J_5, J_6, and J_7 represent maximum values of the relative

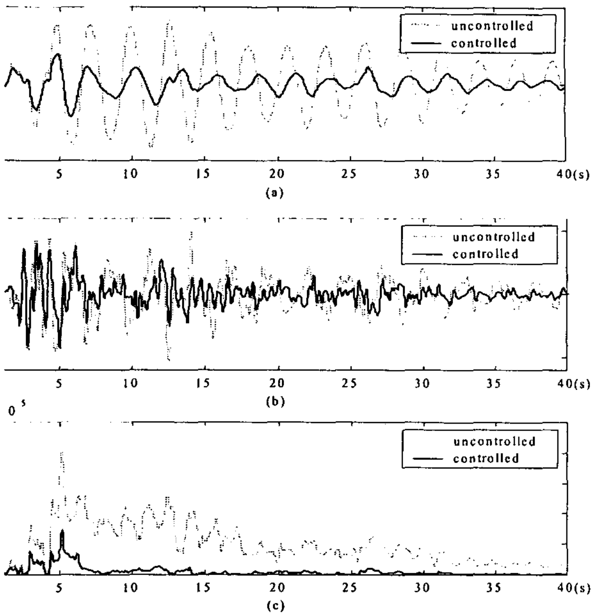

Figure 4. Responses and mean energy of MDOF system

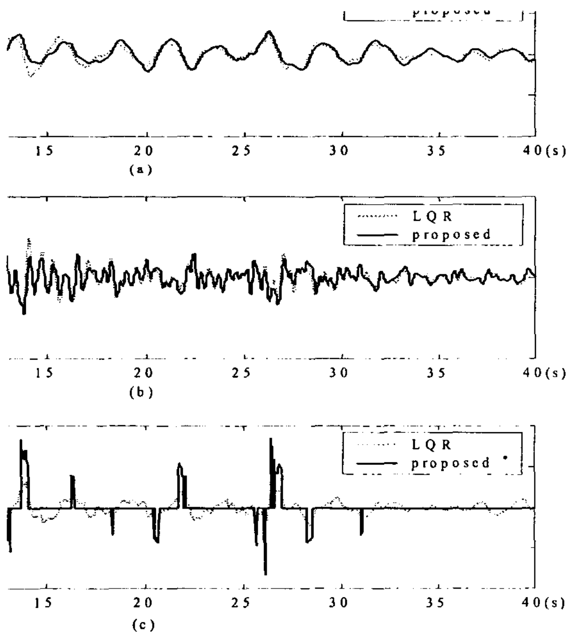

Figure 5. Responses and control force of MDOF system

displacements of each floor with respect to base displacement, inter-story drifts and absolute floor accelerations which are normalized to the correspondent peak responses of uncontrolled case. J_8 represents maximum control force by proposed algorithm normalized to the one of LQR algorithm.

$$J_1 = \max_{i=1,\ldots,10}\left\{\frac{\sigma_{x_i}}{\sigma_{x_{io}}}\right\},\quad J_2 = \max_{i=1,\ldots,10}\left\{\frac{\sigma_{x_i - x_{i-1}}}{\sigma_{x_{io} - x_{(i-1)o}}}\right\}$$

$$J_3 = \max_{i=1,\ldots,10}\left\{\frac{\sigma_{\dot{x}_i}}{\sigma_{\dot{x}_{io}}}\right\},\quad J_4 = \frac{\sigma_u}{\sigma_{u_{LQR}}} \tag{29}$$

$$J_5 = \max_{i=1,\ldots,10}\left\{\frac{|x_i|}{|x_{io}|}\right\},\quad J_6 = \max_{i=1,\ldots,10}\left\{\frac{|x_{i-i-1}|}{|x_{(i-i-1)o}|}\right\}$$

$$J_7 = \max_{i=1,\ldots,10}\left\{\frac{|\ddot{x}_i|}{|\ddot{x}_{io}|}\right\},\quad J_8 = \max\left\{\frac{|u|}{|u_{LQR}|}\right\} \tag{30}$$

Table 2 shows the comparison of each performance evaluation index with control and without control. The rms of control force by proposed algorithm is much larger than that by LQR algorithm.(See J_4 in Table 2) On the other hand, the peak value of control force by proposed algorithm is same as that by LQR algorithm. It is noted that the peak and rms responses are greatly reduced with same maximum control force of LQR algorithm. This means that the proposed algorithm is more suitable for the reduction of peak and rms responses under the constraint of maximum control force.

Table 2. Control performance of MDOF system

	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
propo-sed	0.308	0.504	0.616	1.607	0.505	0.576	0.796	1.000
LQR	0.390	0.666	0.637	1.000	0.651	0.775	0.877	1.000

3.3 Number of measurement signals

Rayleigh probability distribution of a current structural energy at motion is derived with mean and minimum structural energies shown in Equation (6). If more signals can be measured, more exact structural energy is obtained. It means that exactness of energy depends on the number of measurement signals. Figure 6 shows various Rayleigh probability distributions of Equation (6) depending on the number of measurement signals.

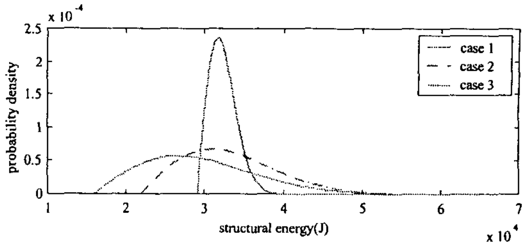

Figure 6. Rayleigh Probability Distribution According to the Number of Measurement Signals

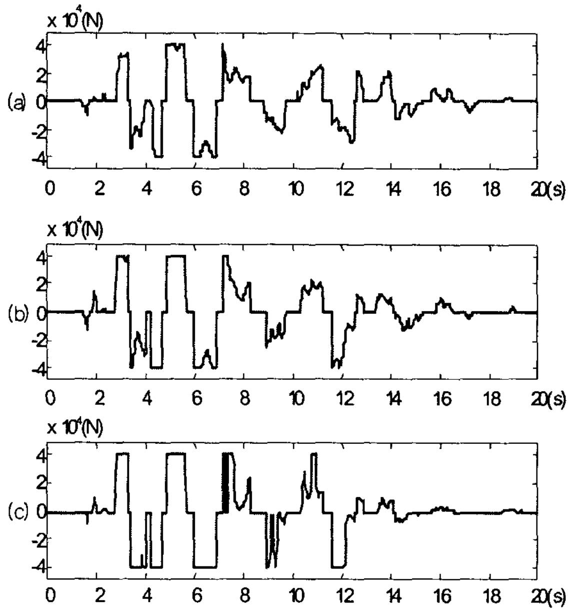

Figure 7. The Influence of Number of Measurement Signals on Control Force

(a) case 1, (b) case 2, (c) case 3

These distributions are based on the values of $\bar{E}$ and $E_{\min}$. Three cases of measurement locations are investigated: (case 1) the base and the 10th story; (case 2) the base, the 3rd story, the 7th story, and the 10th story; (case 3) the base and each story. The displacement and velocity of measurement locations are measured. $\alpha_{\min}$ is set to be 5% of mean of structural energy without control. It is found in Figure 6 that if more signals are measured, more precise values of mean and minimum structural energies are obtained and the probability distribution becomes more narrow. The corresponding control forces are shown in Figure 7

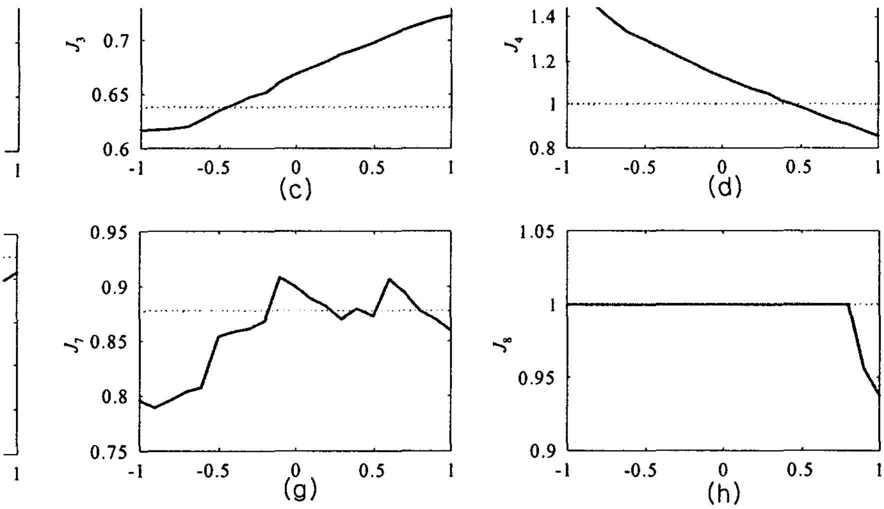

Figure 8. The Influence of β on Perfomance Indices (---:LQR, —:proposed)

when E_c is 35700J and u_{max} is 40kN. E_c is calculated by time history analysis under previously used El Centro earthquake for case 1 and with β =0. Three cases of measurement locations give the findings that larger control forces are applied and the executing time of maximum control force is increased when more signals are measured. This results from the fact that control forces are calculated linearly and proportionally to the probability that current structural energy exceeds the critical energy. Once again, this means that if more signals are measured, the more exact values of magnitude and generating time of control forces can be found, thus converting most of the capacity of controller to maximum control force.

3.4 Design of β

Control force is sensitive to the value of E_c which is dependent on β specified by a control designer. Therefore, it is important to determine the proper value of β. Equation (25) shows that smaller value of β decreases the value of E_c and thus, increases control force linearly proportional to the probability of exceeding E_c. Figure 8 shows the variation of performance indices of Equations (29) and (30) according to the variation of β from -1 to 1 for a ten-story building of case 1 which is previously analyzed. Performance indices by LQR and proposed algorithms are compared. Figure 8(a)-(c) and (e)-(g) show that performance indices by proposed algorithm are much lower than those by LQR in the region of small values of β, where the value of Ec becomes small so that the probability of exceeding Ec increases and larger control force is applied. Also, it can be known that the control effect becomes poor with increasing β, which is natural phenomenon resulting from less control efforts shown in Figure 8(d) and (h). However, the value of E_c becomes small for very small value of β and the exceeding probability of current structural energy is almost equal to 1, resulting in ideal bang-bang control. Since this effect can cause chattering problem, the proper choice of β is recommended. The absolute floor acceleration performance index J_7 show the interesting irregular shape whereas other indices shows the monotonic increasing tendency with increasing β. Figure 8(h) shows that maximum control force by proposed algorithm doesn't exceed the one by LQR algorithm.

4. CONCLUSIONS

This paper suggests a new control algorithm based on the probability distribution of structural energy. The structural energy under seismic excitation is assumed to follow Rayleigh probability distribution and its distribution is derived with mean and minimum structural energies. Control force is calculated according to the probability that a current structural energy exceeds the specified target critical energy. Its effectiveness has been proved using earthquake analyses

on single-degree-of-freedom system and multi-degree-of-freedom system. It is shown that the proposed algorithm can consider the saturation problem of control force in designing a controller and prevent the chattering problem of bang-bang control. Since the control force is designed to be linearly proportional to the probability of structural energy exceeding the target critical energy, which a control designer specifies, the control force cannot be greater than the maximum control force. Also, as the control force becomes zero below a certain specified value, the chattering can be prevented. The direction of control force is determined according to Lyapunov controller design method.
Comparing the efficiency of the proposed algorithm with LQR algorithm whose control force is proportional to structural responses, the proposed algorithm shows an effect of reducing the maximum non-stationary responses under earthquake excitation even with same maximum control force. The effects of the number of measurement signals on the probability density function of structural energy are investigated. Design parameter β which determines the target critical energy, E_c, is suggested and its effect on the control force has been discussed. The results show that it is important to select proper values of β.

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