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FEM과 BEM을 사용한 소리굽쇠 특성 해석 및 설계

TUNING Fork Analysis and Design by FEM AND BEM

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KEYWORDS: 소리굽쇠(Tuning Fork), FEM, BEM, 모드해석(Modal Analysis), Sound Pressure Field

ABSTRACT

An unconstrained tuning fork with a 3-D model has been numerically analyzed by Finite Element Method (FEM) and Boundary Element Method (BEM). The first three natural frequencies were calculated by the FEM modal analysis. Then the trend of the change of the modal frequencies was examined with the variation of the tuning fork length and width. An formula for the natural frequencies-tuning fork length relationship were derived from the numerical analysis results. Finally the BEM was used for the sound pressure field calculation from the structural displacement data.

1. INTRODUCTION

The tuning fork was firstly invented in England by Royal trumpeter John Shore in 1711 [1]. The natural (modal) frequencies of a tuning fork depend on its material and structural fabrication. A numerical approach has been available only since Boundary Element Method (BEM) technique overcame its singularity problems [2]. There is no evidence of the application of BEM with Finite Element Method (FEM) to the analysis of a tuning fork. It is therefore very meaningful to apply BEM with FEM to this case because the mechanical behavior of the tuning fork may be predicted using the FEM-BEM numerical techniques.

2. BOUNDARY ELEMENT METHOD (BEM)

For sinusoidal steady-state problems, the Helmholtz equation, can be expressed as $\nabla^2\Psi + k^2\Psi = 0$, where Ψ is the acoustic pressure with time variation $e^{j\omega t}$, k (= ω /c) is the wave number and c is the sound speed, 340m/sec. In order to solve the Helmholtz equation in an infinite air media, a solution to the equation must not only satisfy the structural surface boundary condition, $\partial\Psi / \partial n = \rho_f \omega^2 a_n$ but also the radiation condition at infinity, $\lim_{|r|\to\infty} \oint (\partial\Psi / \partial r + jk\Psi)^2 dS = 0$ where a_n is normal displacement and ρ_f is air density. The Helmholtz integral equation derived from Green's second theorem provides such a solution for radiating pressure waves;

$$\oint \left(\Psi(q) \frac{\partial G_k(p,q)}{\partial n_q} - G_k(p,q) \frac{\partial \Psi(q)}{\partial n_q} \right) dS_q = \beta(p)\ \Psi(p) \quad (1)$$

where $G_k(p,q) = e^{-jkr} / 4\pi r$, $r = |p-q|$

In equation (1) p is any point in either the interior or the exterior and q is the surface point of integration. β (p) is

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the exterior solid angle at p. The acoustic pressure for the i^{th} global node, $\Psi(p_i)$, is expressed in discrete form, ($1 \leq i \leq ng$), as

$$\sum_{m=1}^{nt}\sum_{j=1}^{8}\left(\int_{S_m} N_j(q)\frac{\partial G(p_i,q)}{\partial n_q}dS_q\right)\Psi_{m,j} -$$

$$-\rho_f\omega^2\sum_{m=1}^{nt}\sum_{j=1}^{8}\left(\int_{S_m} N_j(q)G(p_i,q)n_q dS_q\right)a_{m,j} \qquad (2a)$$

$$= \sum_{m=1}^{nt}\sum_{j=1}^{8} A^i{}_{m,j}\Psi_{m,j} - \rho_f\omega^2\sum_{m=1}^{nt}\sum_{j=1}^{8} B^j{}_{m,j}a_{m,j} \qquad (2b)$$

$$\equiv ([A]-\beta[I])\{\Psi\} = +\rho_f\omega^2[B]\{a\} \qquad (2c)$$

where nt is total number of BEM elements and ng is total BEM nodes. q or p on a boundary element is interpolated from 8 quadratic nodes for each boundary element. When the impedance (system) matrices of equation (2c), [A] and [B], are computed, a singularity arises. At certain wave numbers the matrices become ill-conditioned. These wave numbers correspond to eigenvalues of the interior Dirichlet problem [3]. One approach to overcome the matrix singularity is to modified [A] and [B] of equation (2c) to provide a unique solution for the entire frequency range [4-5].

$$([A]-\beta[I]\oplus\alpha[C])\{\Psi\} = +\rho_f\omega^2([B]\oplus\alpha[D])\{a\} \qquad (3)$$

where

$$\alpha = \frac{-\sqrt{-1}}{k\cdot(\textit{Number of surface element adjacent a surface node})},$$

and [C] and [D] are also extra system matrices. The $\oplus$ symbol stands for coefficient matrix distribution between [A] and [C] and also between [B] and [D].

Once $\{a\}$ and $\{\Psi\}$ are known, the acoustic pressure in the near field is determined by $\beta(p)=1$ of equation (1) for given values of surface nodal pressure and surface nodal displacement;

$$\Psi(p_i) = \sum_{m=1}^{nt}\sum_{j=1}^{8} A^i{}_{m,j}\Psi_{m,j} - \rho_f\omega^2\sum_{m=1}^{nt}\sum_{j=1}^{8} B^i{}_{m,j}\, a_{m,j} \qquad (4)$$

3. RESULTS

The particular structure considered was the unconstrained tuning fork shown in Fig. 1.

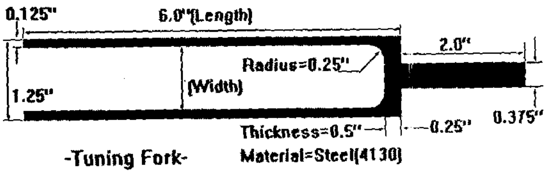
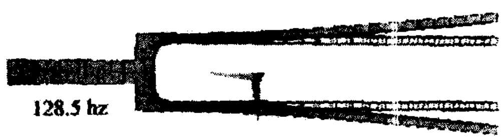


Fig. 1 3D tuning fork dimensions. Elements=550. Nodes=3934

Table 1 lists the material properties of air, steel and aluminum used in the calculations while Fig. 2 shows three modal shapes of the tuning fork. The pink frame is the undeformed shape of the tuning fork while the solid color shows the Von Mises stress with deformed shape.

Table 1 Material Properties

	Density (ρ) [Kg/m^3]	Young's Modulus (E) [N/m^2]	Poisson Ratio (ν)
Air	1.22	1.411E5	-
Steel	7822.9	2.0684E11	0.30
Aluminum	2703.8	6.9637E10	0.36



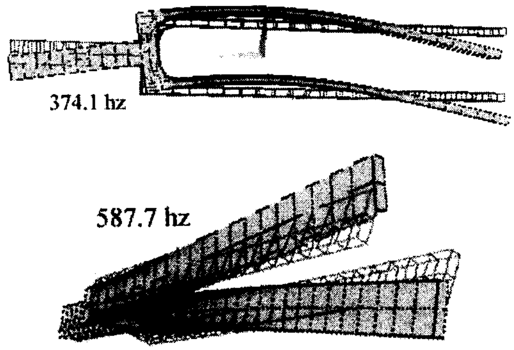


Fig. 2 Modal shape of tuning fork (Colour=Von Mises Stress) at 128.5 Hz (1st mode), 374.1 Hz (2nd mode), 587.7 Hz (3rd mode), Length=152.4 [mm], Width=25.4 [mm], Material=Steel(4130)

Fig. 3 shows the first three modal frequencies as functions of tuning fork length with a constant width.

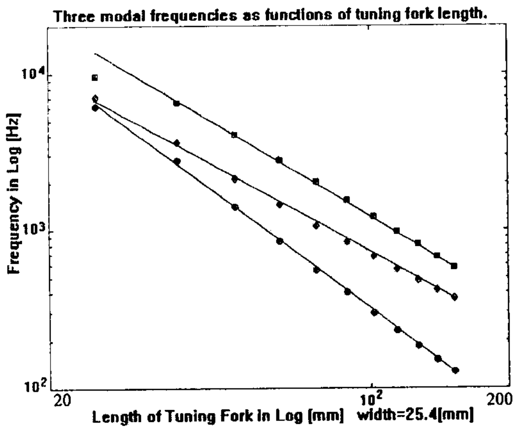


Fig. 3 The first three modal frequencies as functions of tuning fork length. Circle=1st mode, Diamond=2nd mode, Rectangle=3rd mode.

Table 2 Modal frequencies with different tuning fork width Length=152.4 [mm], Material=Steel(4130)

Width [mm]	Frequency [Hz]		
	1st mode	2nd mode	3rd mode
50.8	93.5	198.9	525.2
38.1	125.6	339.5	559.5
25.4	128.5	374.1	587.7
12.7	128.6	391.2	576.7
6.3	124.2	395.2	525.5

Table 2 shows that the change (50%~150% with reference of 25.4 mm width) of the tuning fork width affects the modal frequencies within 10% for the same length 152.4 mm. Fig. 4 shows the modal frequencies of the tuning fork with different materials. The two curves virtually overlap. These results show that the length of the tuning fork mainly affects the natural frequency of the tuning fork as far as metallic materials are used.

Finally Fig. 5 shows the acoustic pressure directivity pattern at 1m away from the tuning fork at 128.5 Hz (1st mode).

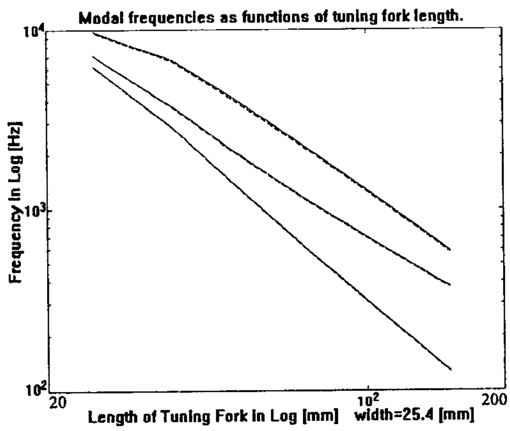


Fig. 4 The first three modal frequencies as functions of tuning fork length as Fig. 3. Continuous Lines are Steel. Dashed Lines are Aluminum

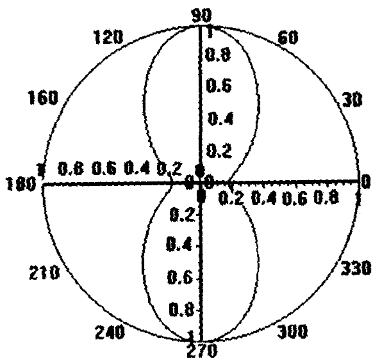


Fig. 5 Acoustic pressure directivity pattern at 1m away from the tuning fork. At 128.5 Hz (1st mode), Length = 152.4 mm, Width = 25.4mm, Material = Steel(4130)

4. CONCLUSION

It is concluded that the length of the tuning fork mainly affects the natural frequencies of the tuning fork as far as metallic materials are used. The small width variation of the tuning fork does not much produce differences in modal frequencies.

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REFERENCES

(1) Alexander J. Ellis. "On the History of Musical Pitch," *J. of the Society of Arts*, March 5, 1880. Reprinted in *Studies in the History of Music Pitch*, Amsterdam: Frits Knuf, p. 44. Ellis measured the pitch of the Streicher fork at A=442.78, 1968.

(2) M.R. Bai, "Application oF BEM(boundary element method)-based acoustic holography to radiation analysis of sound sources with arbitrarily shaped geometries", J. Acoust. Soc. Am. **92**(1), 533-549(1992)

(3) L.G. Copley, "Fundamental results concerning integral representations in acoustic radiation", J. Acoust. Soc. Am. **44**, 28-32(1968)

(4) D.T.I. Francis, "A boundary element method for the analysis of the acoustic field in three dimensional fluid-structure interaction problems", Proc. Inst. of Acoust., **12**, Part 4, 76-84 (1990)

(5) H.A. Schenck, "Improved integral formulation for acoustic radiation problems", J. Acoust. Soc. Am. **44**, 41-58(1968)