한국소음진동공학회 2003년도 춘계학술대회논문집, pp. 833~838.

탄성체로 인한 탄성파의 공명산란

ELASTIC WAVE RESONANCE SCATTERING FROM AN ELASTIC CYLINDER

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Key Words : elastic wave, resonance scattering, resonance scattering theory.

ABSTRACT

The problem of elastic wave resonance scattering from elastic targets is studied in this paper. A new resonance formalism to extract the elastic resonance information of the target from scattered elastic waves is introduced. The proposed resonance formalism is an extension of the works developed for acoustic wave scattering problems by the author. The classical resonance scattering theory computes reasonable magnitude information of the resonances in each partial wave, but the phase behaves in somewhat irregular way, therefore, is not clearly explainable. The proposed method is developed to obtain physically meaningful magnitude and phase of the resonances. As an example problem, elastic wave scattering from an infinitely-long elastic cylinder was analyzed by the proposed method and compared to the results by RST. In case of no mode conversion, both methods generate identical magnitude. However, the new method computes exact π radian phase shifts through resonances and anti-resonances while RST produces physically unexplainable phases. In case of mode conversion, in addition to the phase even magnitudes are different. The phase shifts through resonances and antiresonances obtained by the proposed method are not exactly π radians due to energy leak by mode conversion. But, the phases by the proposed method show reasonable and intuitively correct behavior compared to those by RST.

Nomenclature

A	scattering coefficient for P wave incidence and P wave scattering
B	scattering coefficient for P wave incidence and S wave scattering
F	scattering coefficient for S wave incidence and P wave scattering
G	scattering coefficient for S wave incidence and S wave scattering
K	modal charisteristics of the scatterer
$S_n^{\nu\sigma}$	scattering function
$S_n^{(r)res,\nu\sigma}$	resonance scattering function
X, Y, Z	Cartesian coordinates
a	radius of the cylinder
c_{p1}	P wave velocity in the medium
c_{s1}	S wave velocity in the medium
c_{p2}	P wave velocity in the cylinder
c_{s2}	S wave velocity in the cylinder
f	partial wave
i	$\sqrt{-1}$
k	wave number
z	acoustoelastic impedance
α	nondimensionalized frequency (= $k_p a$)
β	nondimensionalized frequency (= $k_s a$)
ε_n	Neumann function
θ	scattering angle
ρ_1	density of the medium material
ρ_2	density of the cylinder material

superscript

(r)	rigid background
res	resonance
pp	P wave incidence and P wave scattering
ps	P wave incidence and S wave scattering
sp	S wave incidence and P wave scattering
ss	S wave incidence and S wave scattering
rst	resonance scattering theory
ν	P or S wave
σ	P or S wave

subscript

n	normal mode number

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1. INTRODUCTION

Acoustic or elastic wave resonance scattering from elastic targets has been studied theoretically and experimentally in numerous papers and books (Refs. 1 ~ 9, 10, 14). Since late 1970's when resonance scattering theory (RST) was issued by applying the resonance theory of nuclear reactions to the problem of wave scattering from elastic bodies, it has become the standard theory to study acoustic and elastic wave resonance scattering problems. RST shows that the fluctuating behavior of the cross section for sound scattering from elastic bodies is caused by a linear superposition of the scatterer's elastic vibration and a smoothly-varying geometric background. According to the standard RST, the elastic resonance information of the scatterer was isolated by subtracting the proper background from each normal mode of the scattered wave. Using this background subtraction method, the magnitude of the resonance could reasonably be obtained. However, the behavior of the phase has remained unclear since it did not clearly exhibit π radians phase shifts through resonances although we expect that the phase of a resonance term should shift by π radians as the frequency passes through the resonance frequency. Some discussions were made on π phase jumps in the references 4 and 6. However, they mentioned only some trends showing phase jumps in the partial wave curve, not in the resonance curve, because the phase of the resonance curve obtained by the background subtraction method showed irregular behavior. In order to make sure that the resonance information is correctly extracted from scattered waves, one should be able to physically explain the behavior of phase as well as magnitude of the resonances.

A new method to extract the resonance information of the scatterer has been proposed for acoustic wave scattering problems in references 11~13. The newly proposed method and RST compute identical magnitude of the resonances from each partial wave. However, the phase obtained by the new method shows exact π radians phase shifts through the resonance and anti-resonance frequencies, which the standard RST does not show. And, the phase shifts by π radians through resonances occur gradually or abruptly depending on the resonance width, which is a physically meaningful and intuitive behavior. Even though both the new method and RST compute identical magnitude, they generate different total resonance spectra due to the phase difference in each partial wave. If a resonance-like peak is not accompanied with π radians phase shift, we can distinguish it from a scatterer's vibrational resonance. The correct computation of phase as well as magnitude of the resonances is important for the potential applications such as the remote target identification technique as well as for the understanding of the fundamental physics of the resonance scattering phenomena.

In this paper, we generalize the concept of the new resonance formalism, developed for acoustic wave scattering in the references 11~13 to elastic wave scattering to elastic targets. Applying the new resonance formalism, the problem of elastic wave scattering from an infinitely-long elastic cylinder is numerically solved. Due to mode conversion the new method and RST compute different magnitude and phase of the resonance from each partial wave. We will discuss that the new method computes physically meaningful and intuitive phase transitions through the resonances and anti-resonances compared with RST.

2. RESONANCE SCATTERING THEORY

Let us consider a plane elastic wave incident normally on an infinitely-long elastic (chosen as Tungsten carbide) cylinder imbedded in an elastic (chosen as Lucite) medium, as shown in Fig. 1. Resonance scattering theory (Refs. 4,14) claimed that the complicated shape of scattered wave is due to the linear superposition of the elastic resonance of the scatterer and the smoothly-varying geometric background. In this paper, the proper background is assumed as the rigid background because of the larger acoustic impedence of the cylinder material than the medium. Acoustic properties of the materials are shown in Table 1. A partial wave of scattered wave with the corresponding rigid backgrounds is shown in Fig. 2. Based on the standard RST, the resonances of the scatterer were obtained by subtracting the background from each partial wave as follows :

For P-wave (longitudinal wave) incidence case,

$$f_n^{(r)res,pp,rst} = \sqrt{\frac{2}{\pi i \alpha}}\,\varepsilon_n (A_n - A_n^{(r)}) \cos n\theta, \qquad (1a)$$

$$f_n^{(r)res,ps,rst} = \sqrt{\frac{2}{\pi i \beta}}\,\varepsilon_n (B_n - B_n^{(r)}) \sin n\theta. \qquad (1b)$$

For S-wave (shear wave) incidence case,

$$f_n^{(r)res,sp,rst} = \sqrt{\frac{2}{\pi i \alpha}}\,\varepsilon_n (F_n - F_n^{(r)}) \sin n\theta, \qquad (2a)$$

$$f_n^{(r)res,ss,rst} = \sqrt{\frac{2}{\pi i \beta}}\,\varepsilon_n (G_n - G_n^{(r)}) \cos n\theta, \qquad (2b)$$

where f_n is a partial wave for mode number n as defined by Solomon (1984) and ε_n is Neumann factor. The superscripts res and (r) stand for the resonance and the rigid background, respectively. The non-dimensionalized frequencies α, β and scattering coefficients A_n, B_n, F_n and G_n are defined in the same manner as Refs. 13 and 14. In Eqs. (1b) and (2a), a part of the energy of incident P (or S) wave is converted to S (or P) wave, which is known as mode conversion phenomena.

Figures 3 through 5 (dotted lines) show the resonance curves computed by Eqs. (1) and (2). The sharp peaks in magnitudes of these figures have been regarded as the vibrational resonances of the cylinder. We, however, note that in these figures the behavior of the phase obtained using Eq. (1) is not physically meaningful. As we pointed out in the reference 13, the phase of the resonances calculated by the standard RST does not clearly exhibit π radians phase shifts through a resonance although it is well known that the phase of a resonance should change by π radians, varying gradually or abruptly depending on the resonance width, as the frequency passes through the resonance frequency. Therefore, we can argue that Eq. (1) may not correctly isolate the resonances of the scatterer although the magnitude plots show resonance-like features.

3. NEW METHOD FOR EXTRACTING THE ELASTIC RESONANCE INFORMATION

In this section, we propose a new method to extract the resonance information, of which both magnitude and phase are physically meaningful, for elastic wave scattering problems. This investigation provides a generalization of the previous research (Refs. 11~13) on acoustic wave scattering for elastic wave scattering problems.

The scattering functions can be expressed as

$$S_n^{\nu\sigma} = S_n^{(r)\nu\sigma} S_n^{(r)*} = S_n^{(r)\nu\sigma} \left(\frac{z_n^{(2)\nu\sigma} - K_n}{z_n^{(1)\nu\sigma} - K_n} \right), \quad (3)$$

,where the z's are acoustoelastic impedances and K_n is related to modal characteristics of the scatterer, which are ratios of 2 x 2 determinants. The superscripts ν and σ denote p or s , which represents P or S wave, respectively. $S_n^{(r)\nu\sigma}$ is the scattering function corresponding to the rigid cylinder. In Eq. (3), we can see that $S_n^{\nu\sigma}$ is the product of the background $S_n^{(r)\nu\sigma}$, and the remaining term $S_n^{(r)*}$ which includes the resonance information of the elastic target. However, $S_n^{(r)*}$ is not a pure resonance form because it contains the modal property information K_n in both numerator and denomenator. Thus, $S_n^{(r)*}$ contains a real unit constant which hides resonances unless it is removed. The unit constant contained in $S_n^{(r)*}$ should be subtracted in order to obtain the resonances because adding a constant term to a complex quantity changes both magnitude and phase of the original complex quantity. $S_n^{(r)*}$ may be written as

$$S_n^{(r)*} = \frac{z_n^{(2)\nu\sigma} - K_n}{z_n^{(1)\nu\sigma} - K_n} = \frac{z_n^{(2)\nu\sigma} - z_n^{(1)\nu\sigma}}{z_n^{(1)\nu\sigma} - K_n} + 1$$

$$= S_n^{(r)res,\nu\sigma} + 1, \quad (4)$$

,where $S_n^{(r)res,\nu\sigma} (= \frac{S_n^{\nu\sigma}}{S_n^{(r)\nu\sigma}} - 1)$ is defined as the resonance scattering function which consist purely of resonance information of the scatterer.

By the definition in Eq. (4), the resonance scattering functions can be written as

$$S_n^{(r)res,pp} = \frac{z_n^{(2)pp} - z_n^{(1)pp}}{z_n^{(1)pp} - K_n} = \frac{S_n^{pp}}{S_n^{(r)pp}} - 1 = 2\frac{A_n - A_n^{(r)}}{1 + 2A_n^{(r)}}, \quad (5a)$$

$$S_n^{(r)res,ps} = \frac{z_n^{(2)ps} - z_n^{(1)ps}}{z_n^{(1)ps} - K_n} = \frac{S_n^{ps}}{S_n^{(r)ps}} - 1 = \frac{B_n - B_n^{(r)}}{B_n^{(r)}}, \quad (5b)$$

$$S_n^{(r)res,sp} = \frac{z_n^{(2)sp} - z_n^{(1)sp}}{z_n^{(1)sp} - K_n} = \frac{S_n^{sp}}{S_n^{(r)sp}} - 1 = \frac{F_n - F_n^{(r)}}{F_n^{(r)}}, \quad (5c)$$

$$S_n^{(r)res,ss} = \frac{z_n^{(2)ss} - z_n^{(1)ss}}{z_n^{(1)ss} - K_n} = \frac{S_n^{ss}}{S_n^{(r)ss}} - 1 = 2\frac{G_n - G_n^{(r)}}{1 + 2G_n^{(r)}}. \quad (5d)$$

From Eq. (5), the relationship among the scattering coefficient, background and the resonance scattering function may be expressed as

$$A_n = A_n^{(r)} + \tfrac{1}{2} S_n^{(r)res,pp} + A_n^{(r)} S_n^{(r)res,pp}, \quad (6a)$$

$$B_n = B_n^{(r)} + B_n^{(r)} S_n^{(r)res,ps}, \quad (6b)$$

$$F_n = F_n^{(r)} + F_n^{(r)} S_n^{(r)res,sp}, \quad (6c)$$

$$G_n = G_n^{(r)} + \tfrac{1}{2} S_n^{(r)res,ss} + G_n^{(r)} S_n^{(r)res,ss}. \quad (6d)$$

Note that Eq.(6) states that in each Rayleigh normal mode (partial wave) the resonance interacts with the background as a product term. This means that Eq.(5) clearly shows that the scattering in each normal mode is not just a simple summation of the background and resonance as claimed by RST, but includes a product interaction term between the resonance and background.

Using the same rationale in the reference 13, and considering Eq. (5) and the following relationships

$$A_n = \frac{1}{2}(S_n^{pp} - 1),\ B_n = \frac{1}{2}S_n^{ps},\ C_n = -\frac{1}{2}S_n^{sp},$$

$$\text{and } D_n = \frac{1}{2}(S_n^{ss} - 1), \tag{7}$$

the new expressions to compute the resonances can be written as

$$f_n^{(r)res,pp,new} = \sqrt{\frac{2}{\pi i \alpha}}\varepsilon_n \frac{1}{2} S_n^{(r)res,pp} \cos n\theta$$
$$= \sqrt{\frac{2}{\pi i \alpha}}\varepsilon_n \frac{A_n - A_n^{(r)}}{1 + 2A_n^{(r)}} \cos n\theta, \tag{8a}$$

$$f_n^{(r)res,ps,new} = \sqrt{\frac{2}{\pi i \beta}}\varepsilon_n \frac{1}{2} S_n^{(r)res,ps} \sin n\theta$$
$$= \sqrt{\frac{2}{\pi i \beta}}\varepsilon_n \frac{1}{2}\frac{B_n - B_n^{(r)}}{B_n^{(r)}} \sin n\theta, \tag{8b}$$

$$f_n^{(r)res,sp,new} = \sqrt{\frac{2}{\pi i \alpha}}\varepsilon_n \frac{1}{2} S_n^{(r)res,sp} \sin n\theta$$
$$= \sqrt{\frac{2}{\pi i \alpha}}\varepsilon_n \frac{1}{2}\frac{F_n - F_n^{(r)}}{F_n^{(r)}} \sin n\theta, \tag{8c}$$

$$f_n^{(r)res,ss,new} = \sqrt{\frac{2}{\pi i \beta}}\varepsilon_n \frac{1}{2} S_n^{(r)res,ss} \cos n\theta$$
$$= \sqrt{\frac{2}{\pi i \beta}}\varepsilon_n \frac{G_n - G_n^{(r)}}{1 + 2G_n^{(r)}} \cos n\theta. \tag{8d}$$

Using Eq. (8), the resonance information, which is mixed with the background as seen in Eq. (6), can be isolated. The only difference between Eqs. (1) and (8) is the existence of the denominator which is equal to the rigid background scattering function. This difference is consistent with acoustic wave scattering problem. However, unlike acoustic wave scattering, the background scattering functions in the denominators of Eq. (8) are not unitary when mode conversion occurs. Therefore,both magnitude and phase of the resonances computed by Eqs. (1) and (8) are generally different.

4. NUMERICAL ANALYSIS AND DISCUSSION

Let us consider the backscattering ($\theta = \pi$) for PP and SS cases. For PS case, we plot the values in Eqs. (1) and (8) divided by $\sin n\theta$ because there is no backscattering response.

For PP case, mode conversion does not occur for the breathing mode n = 0. Therefore, in Fig. 3(a) we obtain identical magnitude by the standard RST [Eq. (1)] and new [Eq. (8)] method. This is because the denominator $1 + 2A_n^{(r)}$, which is $S_n^{(r)pp}$, in Eq. (8a) is unitary. However, their phases are significantly different because $S_n^{(r)pp}$ has its own phase shift. While the phase obtained by RST does not behave in a physically meaningful way, the new method generates phase shifts of exact π radians through the resonances and anti-resonances. For $n \geq 1$, due to the energy leak by mode conversion, $S_n^{(r)pp}$ in the denominator in Eq. (8a) is not unitary. Therefore, even magnitudes computed by the two methods are different as can be seen in Fig. 3(b) for n=2. In Fig. 3(b), the phase shifts through resonances and anti-resonances computed by the new method are not π radians because of the energy leak. However, the behavior of phase computed by the new method looks physically more meaningful and intuitive. This kind of phase behavior is also found in a damped mechanical vibration system.

For SS case, as shown in Fig. 4 we have similar results to PP case.

For mode converted cases such as S (P) wave scattering with P (S) wave incidence, we can observe the same trend with PP or SS case, except that there is no n=0 mode. Fig. 5 compares the resonances computed by the two methods for PS case, n=2 mode.

Due to the difference in the phase of each partial wave, the summed resonance spectra is significantly different. For example, Fig. 6 compares summed resonance spectra computed by the two methods, which are $\sum_{n=0}^{30} f_n^{(r)res,pp,new}$ and $\sum_{n=0}^{30} f_n^{(r)res,pp,rst}$, respectively.

5. CONCLUSIONS

A new method for extracting the elastic resonance information of the elastic scatterer from scattered waves is proposed and numerically applied for elastic wave resonance scattering from an elastic cylinder. The concept of the resonance scattering function consisting purely of the resonance information, which was originally developed for acoustic wave scattering, has been extended for elastic wave scattering. For a non-mode conversion case, both the new method and the standard RST compute identical magnitude of the resonances from each partial wave. However, their phases are significantly different. The new method generates exact π radians phase shifts through the resonances and anti-resonances which the standard RST does not obtain. For a mode conversion case, the two methods produce different magnitude and phase due to the leak of the incident wave energy. The new method computes physically more meaningful and intuitive phase transitions through the resonance and anti-resonance. Based on these facts the new method more properly extracts the resonance information from scattered waves than the classical resonance scattering theory.

Acknowledgement

This work was supported by Brain Korea 21 project in 2003.

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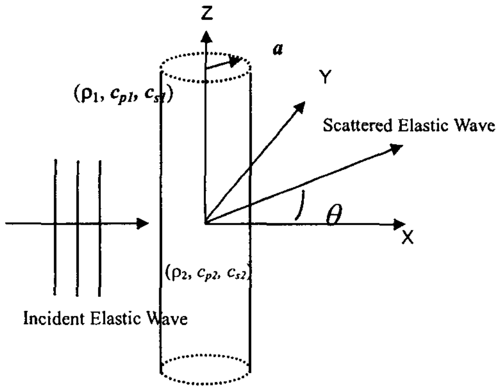


FIG. 1 Geometry of elastic wave scattering from an elastic cylinder

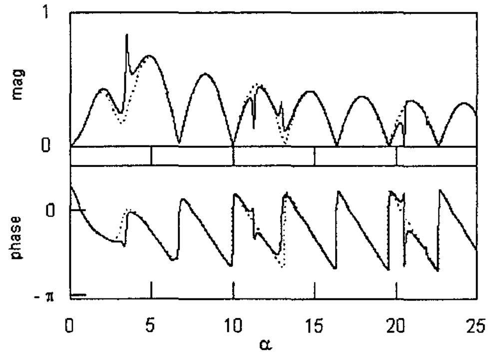
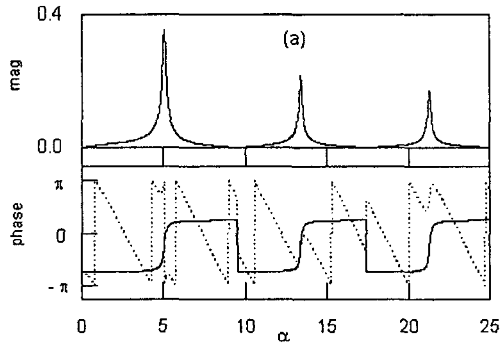


FIG 2. Magnitude and phase of *n* th scattered partial wave (solid line) for a Tungsten Carbide cylinder imbedded in Lucite matrix and the rigid background (dotted line), for *PP* case, *n*=2.



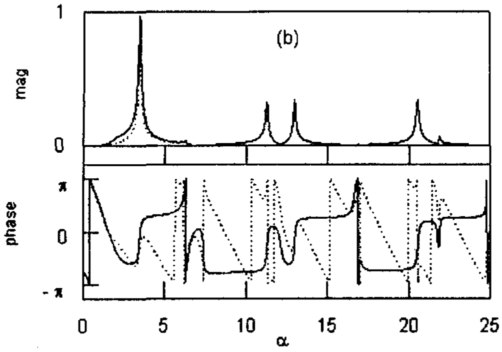


FIG 3. Comparison of magnitude and phase of isolated resonances by the new method (solid line) and RST(dotted line) for n th scattered partial wave for a Tungsten Carbide cylinder imbedded in Lucite matrix for *PP* case, (a)n=0, (b)n=2.

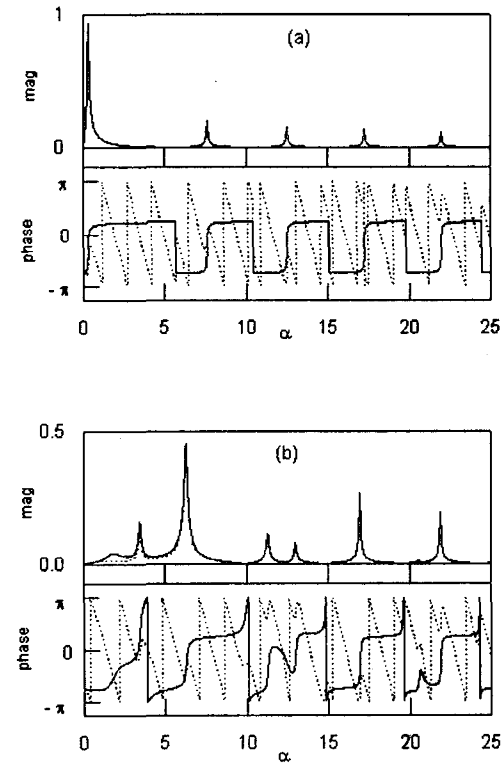


FIG 4. Comparison of magnitude and phase of isolated resonances by the new method (solid line) and RST (dotted line) for n th scattered partial wave for a Tungsten Carbide cylinder imbedded in Lucite matrix. for *SS* case, (a)n =0,(b)n=2.

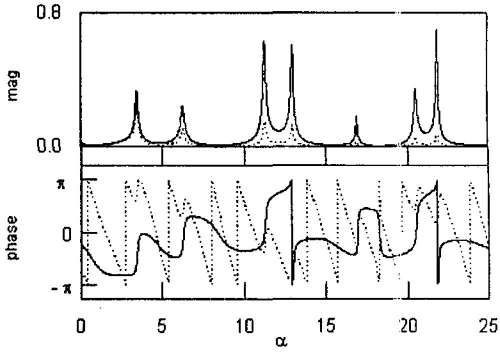


FIG 5. Comparison of magnitude and phase of isolated resonances by the new method (solid line) and RST (dotted line) for n th scattered partial wave for a Tungsten Carbide cylinder imbedded in Lucite matrix for *PS* case, n = 2.

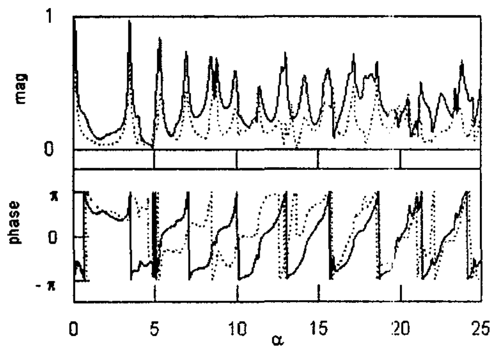


FIG 6. Summed resonance spectra ($\sum_{n=0}^{30} f_n^{(r\,res,pp)}$) by the new method (solid line) and RST (dotted line) for *PP* case.