극초단 펄스 레이저의 파장변환을 위한 이축 비선형 광학 결정에서 분산의 새로운 계산법

A novel method for calculation of dispersions in biaxial crystals for frequency conversion of short pulse lasers

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As laser pulse width becomes shortened from nanoseconds to femtoseconds, the effects caused by the dispersions of nonlinear optical mediums, such as group velocity mismatch and group velocity dispersion become considerably significant. The group velocity mismatch and group velocity dispersion are the major factors that lead to a decrease of frequency conversion efficiency and pulse spreading(1) for picosecond and femtosecond pulses. Much studies on the influence of the dispersions and the methods to ovecome them have been reported.(2, 3) For the detailed investigation of the dispersion effects, it is inevitable to calculate the dispersions in a nonlinear medium using Sellmeiers equation. However, calculation of the dispersions along the different crystal orientations involves a lot of complications, because the dispersions, in general, vary widely depending on the wave propagation direction in anisotropic crystals.

In the previous method(4), dispersion is obtained by direct differentiation of refractive indices with respect to wavelength. Since the refractive indices(5) for the two eigen modes in biaxial crystals are represented by a complicated function of the principal refractive indices as in eq. (1), calculation of the dispersion incurrs a great deal of complication, especially for higher-order terms.

$$n_{\pm}^2 = \frac{2}{-B \mp \sqrt{B^2 - 4C}} \qquad (1)$$

$$,where\ B = -s_x^2(b+c) - s_y^2(a+c) - s_z^2(a+b),$$
$$C = s_x^2 bc + s_x^2 ac + s_x^2 ab,$$
$$a = n_x^{-2},\ b = n_y^{-2},\ c = n_z^{-2}.$$

Here, $s_x = sin\theta cos\phi$, $s_y = sin\theta sin\phi$, and $s_z = \cos\theta$.

We propose a novel method that allows the dispersion in biaxial crystals to be estimated with much ease. The result is expressed in eq. (2), which is derived from the differentials of wave equation.

$$(K_i)^{\pm} = \frac{2}{-\mu \mp \sqrt{\mu^2 - 4\nu}} \qquad (2)$$

$$, where \quad \mu = -s_x^2(\beta+\gamma) - s_y^2(\alpha+\gamma) - s_z^2(\alpha+\beta),$$
$$\nu = s_x^2\beta\gamma + s_x^2\alpha\gamma + s_x^2\alpha\beta,$$
$$\alpha = \Omega_{ix}^{-2},\ \beta = \Omega_{iy}^{-2},\ \gamma = \Omega_{iz}^{-2}.$$

In Table 1, K_i and Ω_{ij}^2 (j = x, y, z) are shown for i = 1, 2, 3, where k is wave number and k_i's denote the dispersions of i-th order. As shown in eq. (2) and Table 1, our method provides much more efficient means of calculating the dispersions, especially for higher-order terms, because it only necessitates solving Fresnel's equation, and K_i's and Ω_{ij}^2's have recursive relations as in eqs. (3) and (4).

$$K_{i+1} = \frac{dK_i}{d\omega} \tag{3}$$

$$\Omega_{(i+1)j}^2 = \frac{d\Omega_{ij}^2}{d\omega} = \left(-\frac{\lambda^2}{2\pi c}\right)\frac{d\Omega_{ij}^2}{d\lambda} \tag{4}$$

Table 1. K_i and Ω_{ij}^2(j = x, y, z) for i = 1, 2, 3.

	i = 1	i = 2	i = 3
K_i	$2kk_1$	$2k_1^2 + 2kk_2$	$6k_1k_2 + 2kk_3$
Ω_{ij}^2	$-\frac{2\pi}{c}\frac{dn_j^2}{d\lambda}+\frac{4\pi}{\lambda c}n_j^2$	$\frac{2n_j^2}{c^2}-\frac{2\lambda}{c^2}\frac{dn_j^2}{d\lambda}+\frac{\lambda^2}{c^2}\frac{d^2n_j^2}{d\lambda^2}$	$-\frac{\lambda^4}{2\pi c^3}\frac{d^3n_j^2}{d\lambda^3}$

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