퍼지구의 수렴성

Convergence of Fuzzy Spheres

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요약

원형함수를 사용하여 퍼지구를 정의하고 원형함수의 수렴을 이용하여 퍼지구의 수렴성을 측정하며 하나의 퍼지구는 특정한 일반 구로 수렴함을 보인다.

Abstract

The concept of a circularity function is used in defining fuzzy spheres The circularity function of a fuzzy sphere converses to one of crisp sphere as the fuzzy sphere shapes itself more and more like a crisp sphere.

I. Introduction

In this paper, we experiment with developing fuzzy geometry by limiting process of the notion of fuzzy sphere whose degree of circularity function is measured by a fuzzy set. The concept of a circularity function is also used in defining the union, intersection, and complement of fuzzy spheres. The circularity function of a fuzzy sphere converges to one of a crisp sphere as the fuzzy sphere shapes itself like a crisp sphere.

Let X be a nonempty crisp set. A fuzzy subset μ of X is a function from X into the closed unit interval $[0,1]$, that is, $\mu: X \to [0,1]$. In the literature, a fuzzy set may also be written as a set of ordered pairs: $\{(x, \mu(x)): x \in X\}$, where $\mu(x)$ is referred to as the membership function or grade of membership.

II. Fuzzy Spheres

Motivated by the concept of equations of motion we can specify surfaces in the three-dimensional xyz-space by using equations, $x = x(s,t)$, $y = y(s,t)$, $z = z(s,t)$, to express the coordinates of a point (x, y, z) on the surface as functions of auxiliary variables s and t. These are called parametric equations for the surface, and the variables s and t are called parameters. Let S be the surface consisting of all ordered triplets $(x(s,t), y(s,t), z(s,t))$ on the xyz-space, where $x(s,t), y(s,t)$ and $z(s,t)$ are continuous real valued functions defined on a closed two dimensional rectangular region R.

Let $P(s,t) = (x(s,t), y(s,t), z(s,t))$ for $(s,t) \in R$. The surface S is called a closed surface if for any two points on S, there exists a space curve C that connects the two points such that $C \subset S$. If a closed surface S does not intersect itself at any other point on the xyz

-space, then S is called a simple closed surface. Spheres and ellipsoids are typical examples of simple closed surfaces.

Definition 2.1. A function $\mu_{\tilde{S}}: R \rightarrow [0,1]$ is called a circularity function of S if

(1) there exits a function $f: \mathbb{R}\times\mathbb{R}\times\mathbb{R} \rightarrow [0,1]$ such that $\mu_{\tilde{S}}(s,t) = f(x(s,t), y(s,t), z(s,t))$ for all $(s,t) \in R$,

(2) $0 \leq \mu_{\tilde{S}}(s,t) \leq 1$ for $(s,t) \in R$,

(3) $\mu_{\tilde{S}}(s,t) = 1$ for all $(s,t) \in R$ when S is a sphere.

Clearly, $\mu_{\tilde{S}}(s,t)$ is a fuzzy subset of R. Intuitively, the circularity function $\mu_{\tilde{S}}(s,t)$ can be thought of as a numerical measure of the degree of circularity for simple closed surface on the xyz-space.

Definition 2.2. Let S be a simple closed surface on the xyz-space defined as above. A fuzzy sphere on the xyz-space is given by

$$\tilde{S} = \{((x(s,t), y(s,t), z(s,t)), \mu_{\tilde{S}}(s,t)) | (s,t) \in R\}$$

where

(1) $x(s,t), y(s,t)$, and $z(s,t)$ are continuous parametric functions on R that define S,

(2) $\mu_{\tilde{S}}(s,t)$ is a circularity function on S.

A fuzzy sphere is formed by a simple closed surface S together with a circularity functions $\mu_{\tilde{S}}(s,t)$, $(s,t) \in R$.

If S is a crisp sphere, then the corresponding fuzzy sphere must have the maximum degree of circularity, that is, $\mu_{\tilde{S}}(s,t) = 1$, for $(s,t) \in R$. $\mu_{\tilde{S}}(s,t)$ can also be used as a measure in comparing two fuzzy spheres.

Definition 2.3. Let $\tilde{S}_1$ and $\tilde{S}_2$ be two fuzzy spheres with circularity functions $\mu_{\tilde{S}_1}(s,t)$ and $\mu_{\tilde{S}_2}(s,t)$, respectively. Then

(1) $\tilde{S}_1$ is called a fuzzy subsphere of $\tilde{S}_2$, written as $\tilde{S}_1 \leqslant \tilde{S}_2$, if $\mu_{\tilde{S}_1}(s,t) \leq \mu_{\tilde{S}_2}(s,t)$ for all $(s,t) \in R$.

(2) $\tilde{S}_1$ is said to be equal to $\tilde{S}_2$, written as $\tilde{S}_1 \simeq \tilde{S}_2$, if $\tilde{S}_1 \leqslant \tilde{S}_2$, $\tilde{S}_2 \leqslant \tilde{S}_1$, that is, $\mu_{\tilde{S}_1}(s,t) \leq \mu_{\tilde{S}_2}(s,t)$ and $\mu_{\tilde{S}_2}(s,t) \leq \mu_{\tilde{S}_1}(s,t)$ for all $(s,t) \in R$.

We can find that the relation $\simeq$ is an equivalence relation and if S_0 is a crisp sphere in the xyz-space, then $\tilde{S} \leqslant \tilde{S}_0$ for any simple closed surface S on the xyz-space. In other words, $\tilde{S}_0$ is the strongest fuzzy sphere.

Let S_1 be a simple closed surface given parametrically in terms of ordered triplets $(x_1(s,t), y_1(s,t), z_1(s,t))$ for $(s,t) \in R$, where $x_1(s,t), y_1(s,t)$, and $z_1(s,t)$ are real valued continuous functions on R, and let $\tilde{S}_1$ be the fuzzy sphere that is formed by S_1 with the circularity function $\mu_{\tilde{S}_1}(s,t)$. Likewise, let S_2 be a simple closed surface given parametrically in terms of ordered triplets $(x_2(s,t), y_2(s,t), z_2(s,t))$ for $(s,t) \in R$, where $x_2(s,t), y_2(s,t)$, and $z_2(s,t)$ are real valued continuous functions on R, and let $\tilde{S}_2$ be the fuzzy sphere that is formed by S_2 with the circularity function $\mu_{\tilde{S}_2}(s,t)$. Assume that the union and intersection of S_1 and S_2 are also simple closed surfaces.

(1) The union of $\tilde{S}_1$ and $\tilde{S}_2$, denoted by $\tilde{S}_1 \cup \tilde{S}_2$, is defined to be the fuzzy sphere

$$\tilde{S}_1 \cup \tilde{S}_2 = \{((x^*(s,t), y^*(s,t), z^*(s,t)), \mu_{\tilde{S}_1 \cup \tilde{S}_2}(s,t)) \mid (s,t) \in R\}$$

where the circularity function $\mu_{\tilde{S}_1 \cup \tilde{S}_2}(s,t)$ is given by

$$\mu_{\tilde{S}_1 \cup \tilde{S}_2}(s,t) = (\mu_{\tilde{S}_1} \vee \mu_{\tilde{S}_2})(s,t) = \mu_{\tilde{S}_1}(s,t) \vee \mu_{\tilde{S}_2}(s,t)$$

and $(x^*(s,t), y^*(s,t), z^*(s,t))$ denotes the parametric representation of $S_1 \cup S_2$.

(2) The intersection of $\tilde{S}_1$ and $\tilde{S}_2$, denoted by $\tilde{S}_1 \cap \tilde{S}_2$, is defined to be the fuzzy sphere

$$\tilde{S}_1 \cap \tilde{S}_2 = \{((x_*(s,t), y_*(s,t), z_*(s,t)), \mu_{\tilde{S}_1 \cap \tilde{S}_2}(s,t)) \mid (s,t) \in R\}$$

where the circularity function $\mu_{\tilde{S}_1 \cap \tilde{S}_2}(s,t)$ is given by

$$\mu_{\tilde{S}_1 \cap \tilde{S}_2}(s,t) = (\mu_{\tilde{S}_1} \wedge \mu_{\tilde{S}_2})(s,t) = \mu_{\tilde{S}_1}(s,t) \wedge \mu_{\tilde{S}_2}(s,t)$$

and $(x_*(s,t), y_*(s,t), z_*(s,t))$ denotes the parametric representation of $S_1 \cap S_2$.

(3) The complement of $\tilde{S}$, denoted by $\tilde{S}^c$, is defined to be the fuzzy sphere

$$\tilde{S}^c = \{((x(s,t), y(s,t), z(s,t)), \mu_{\tilde{S}^c}(s,t)) \mid (s,t) \in R\}$$

with the circularity function $\mu_{\tilde{S}^c}(s,t)$ given by $\mu_{\tilde{S}^c}(s,t) = 1 - \mu_{\tilde{S}}(s,t)$

For $n = 1, 2, \cdots$, let S_n be a simple closed surface given parametrically in terms of ordered triplets $(x_n(s,t), y_n(s,t), z_n(s,t))$ for $(s,t) \in R$, where $x_n(s,t), y_n(s,t)$, and $z_n(s,t)$ are real valued continuous functions on R, and let $\tilde{S}_n$ be the fuzzy sphere that is formed by S_n with the circularity function $\mu_{\tilde{S}_n}(s,t)$. Assume that the union and intersection of S_n is also simple closed surfaces. Then the union of a sequence $\{\tilde{S}_n\}$ of fuzzy spheres, denoted by $\bigcup_{n=1}^{\infty} \tilde{S}_n$, is defined as the fuzzy sphere

$$\bigcup_{n=1}^{\infty} \tilde{S}_n = \{((x^*(s,t), y^*(s,t), z^*(s,t)), \mu_{\cup \tilde{S}_n}(s,t)) \mid (s,t) \in R\}$$

where the circularity function $\mu_{\cup \tilde{S}_n}(s,t)$ is given by

$$\mu_{\cup \tilde{S}_n}(s,t) = (\bigvee_{n=1}^{\infty} \mu_{\tilde{S}_n})(s,t) = \bigvee_{n=1}^{\infty} \mu_{\tilde{S}_n}(s,t)$$

and $(x^*(s,t), y^*(s,t), z^*(s,t))$ denotes the parametric representation of $\bigcup_{n=1}^{\infty} \tilde{S}_n$. Also, the intersection of a sequence $\{\tilde{S}_n\}$ of fuzzy spheres, denoted by $\bigcap_{n=1}^{\infty} \tilde{S}_n$, is defined as the fuzzy sphere

$$\bigcap_{n=1}^{\infty} \tilde{S}_n = \{((x_*(s,t), y_*(s,t), z_*(s,t)), \mu_{\cap \tilde{S}_n}(s,t)) \mid (s,t) \in R\}$$

with the circularity function $\mu_{\cap \tilde{S}_n}(s,t)$ given by

$$\mu_{\cap \tilde{S}_n}(s,t) = (\bigwedge_{n=1}^{\infty} \mu_{\tilde{S}_n})(s,t) = \bigwedge_{n=1}^{\infty} \mu_{\tilde{S}_n}(s,t)$$

and $(x_*(s,t), y_*(s,t), z_*(s,t))$ denotes the parametric representation of $\bigcap_{n=1}^{\infty} \tilde{S}_n$.

Theorem 2.4. [8] Let $\tilde{S}_1$, $\tilde{S}_2$ and $\tilde{S}_3$ be fuzzy spheres with circularity functions $\mu_{\tilde{S}_1}(s,t)$, $\mu_{\tilde{S}_2}(s,t)$, and $\mu_{\tilde{S}_3}(s,t)$, respectively. Then

(1) $\tilde{S}_1 \cap \tilde{S}_2 \leqslant \tilde{S}_1 \cup \tilde{S}_2$

(2) $(\tilde{S}_1 \cup \tilde{S}_2) \cup \tilde{S}_3 \simeq \tilde{S}_1 \cup (\tilde{S}_2 \cup \tilde{S}_3)$

(3) $(\tilde{S}_1 \cap \tilde{S}_2) \cap \tilde{S}_3 \simeq \tilde{S}_1 \cap (\tilde{S}_2 \cap \tilde{S}_3)$

(4) $\mathfrak{S}_1 \cup (\mathfrak{S}_2 \cap \mathfrak{S}_3) \simeq (\mathfrak{S}_1 \cup \mathfrak{S}_2) \cap (\mathfrak{S}_1 \cup \mathfrak{S}_3)$

(5) $\mathfrak{S}_1 \cap (\mathfrak{S}_2 \cup \mathfrak{S}_3) \simeq (\mathfrak{S}_1 \cap \mathfrak{S}_2) \cup (\mathfrak{S}_1 \cap \mathfrak{S}_3)$

(6) $\mathfrak{S}_1 \leqslant \mathfrak{S}_2 \Rightarrow \mathfrak{S}_2^c \leqslant \mathfrak{S}_1^c$

(7) $\mathfrak{S}_1^c \cup \mathfrak{S}_2^c \simeq (\mathfrak{S}_1 \cap \mathfrak{S}_2)^c$

(8) $\mathfrak{S}_1^c \cap \mathfrak{S}_2^c \simeq (\mathfrak{S}_1 \cup \mathfrak{S}_2)^c$

Corollary 2.5. [8] Let $\mathfrak{S}$ be a fuzzy spheres with circularity function $\mu_{\tilde{\mathfrak{S}}}(s,t)$, and let $\mathfrak{S}_n (n = 1, 2, \cdots)$ be a sequence of fuzzy spheres with circularity functions $\mu_{\tilde{\mathfrak{S}}_n}(s,t)\ (n = 1, 2, \cdots)$, respectively. Then

(1) $\bigcap_{n=1}^{\infty} \mathfrak{S}_n \leqslant \bigcup_{n=1}^{\infty} \mathfrak{S}_n$

(2) $\mathfrak{S} \cup (\bigcap_{n=1}^{\infty} \mathfrak{S}_n) \simeq \bigcap_{n=1}^{\infty} (\mathfrak{S} \cup \mathfrak{S}_n)$

(3) $\mathfrak{S} \cap (\bigcup_{n=1}^{\infty} \mathfrak{S}_n) \simeq \bigcup_{n=1}^{\infty} (\mathfrak{S} \cap \mathfrak{S}_n)$

(4) $\bigcup_{n=1}^{\infty} \mathfrak{S}_n^{\,c} \simeq (\bigcap_{n=1}^{\infty} \mathfrak{S}_n)^c$

(5) $\bigcap_{n=1}^{\infty} \mathfrak{S}_n^{\,c} \simeq (\bigcup_{n=1}^{\infty} \mathfrak{S}_n)^c$

III. Convergence of Fuzzy Spheres

For each $n = 1, 2, \cdots$, let

$$\mathfrak{S}_n = \{ ((x_n(s,t), y_n(s,t), z_n(s,t)), \mu_{\tilde{\mathfrak{S}}_n}(s,t)) \mid (s,t) \in R \}$$

be a fuzzy sphere with the circularity functions $\mu_{\tilde{\mathfrak{S}}_n}(s,t)$. Also, let

$$\mathfrak{S} = \{ ((x(s,t), y(s,t), z(s,t)), \mu_{\tilde{\mathfrak{S}}}(s,t)) \mid (s,t) \in R \}$$

be a fuzzy sphere with the circularity function $\mu_{\tilde{\mathfrak{S}}}(s,t)$.

Definition 3.1. The sequence $\{\mathfrak{S}_n\}$ of fuzzy spheres is said to converge in the sense of circularity to the fuzzy sphere $\mathfrak{S}$ if $\mu_{\tilde{\mathfrak{S}}_n}(s,t)$ converges to $\mu_{\tilde{\mathfrak{S}}}(s,t)$ for all $(s,t) \in R$.

Theorem 3.2. Let $\mathfrak{S}_1 \leqslant \mathfrak{S}_2 \leqslant \cdots \leqslant \mathfrak{S}_n \leqslant \cdots$ be a non-decreasing sequence of fuzzy spheres. Then $\{\mathfrak{S}_n\}$ converges in the sense of circularity to the fuzzy sphere $\bigcup_{n=1}^{\infty} \mathfrak{S}_n$.

Lemma 3.3. Let R is a two-dimensional rectangular region, and $\{f_n\}, \{g_n\}$ are sequences of real-valued functions defined on R. Let f, g are real-valued functions such that $f_n \to f$ and $g_n \to g$ on R. Then $f_n \vee g_n$ converges to $f \vee g$ on R.

Theorem 3.4. Let $\{\mathfrak{S}_n\}$ and $\{\mathcal{T}_n\}$ converge in the sense of circularity to the fuzzy sphere $\mathfrak{S}$, $\mathcal{T}$ respectively. Then $\{\mathfrak{S}_n \cup \mathcal{T}_n\}$ converges in the sense of circularity to the fuzzy sphere $\mathfrak{S} \cup \mathcal{T}$.

Definition 3.5. The scalar multiple of $\mathfrak{S}$, denote by $\alpha\mathfrak{S}$, is defined to be the fuzzy sphere

$$\alpha\mathfrak{S} = \{ (x(s,t), y(s,t), z(s,t), \mu_{\alpha\tilde{\mathfrak{S}}}(s,t) \mid (s,t) \in R \}$$

with the circularity function $\mu_{\alpha\tilde{\mathfrak{S}}}(s,t) = (\alpha, \mu_{\tilde{\mathfrak{S}}})(s,t) = \alpha \cdot \mu_{\tilde{\mathfrak{S}}}(s,t)$ and $0 \le \alpha \le 1$.

Theorem 3.6. Let $\{\mathfrak{S}_n\}$ be a sequence of fuzzy spheres, and $\{\mathfrak{S}_n\}$ converges in the sense of circularity to the fuzzy sphere $\mathfrak{S}$. Then $\{\alpha\mathfrak{S}_n\}$ converges in the sense of circularity to the fuzzy sphere $\alpha\mathfrak{S}$.

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