2003년도 대한전자공학회 하계종합학술대회 제26권 제1호

Lifting Scheme을 이용한 이미지 잡음 제거

박영석, 곽훈성
전북대학교 컴퓨터공학과
E-mail : eenhasu@mail.chonbuk.ac.kr

Image De-noising Using Lifting Scheme

Young-Seok Park, Hoon-Sung Kwak
Dept. of Computer Engineering, Chonbuk University
E-mail : eenhasu@mail.chonbuk.ac.kr
hskwak@moak.chonbuk.ac.kr

ABSTRACT

In this paper, we describe an approach for image denoising using the lifting construction, with the spatial adaptive wavelet transform. The adaptive lifting scheme is implemented in spatial domain to be adjusted thresholds to reduce noise. In this approach we represent adaptive characteristics of biorthogonal wavelets for choosing predictors effectively. Predict filter is changed from sample to sample according to local signal features with their vanishing moments. We in this approach have implemented and applied to image denoising by finding a relevant minimax threshold. Experimental results show that the adaptive method of denoising process is compared with existing ones, such as non-adaptive wavelet, CRF(13, 7) and SWE(13, 7) wavelets used by JPEG2000.

I. INTRODUCTION

Digital image denoising is used to produce good estimates of the original image from noisy observation in signal and image processing areas. Denoising algorithms based on wavelet thresholding have led a very adaptive representation [1]. The processing of wavelet denoising is carried on the transformed domain for each subband. The resultant wavelet coefficients are used as threshold estimation. However, they give drawbacks of these wavelet transforms that perform real-to-real transforms, requiring more memory resources, and also fixing over the entire signal for deriving the scaling and wavelet coefficients. Thus, a new technique, the Lifting scheme (LS), has been proposed by Sweldens [2], that is based on the representation of the wavelet filters through their polyphase matrix, leading to a scheme which can factor any wavelet transform into lifting steps [3], thus constructing prediction/update operations. Lifting scheme also offers several advantageous properties with respect to the classical one, having the properties of faster in-place calculation, easily invertible, non-linearity, and adaptive design.

Denoising adaptiveness is affected by adjusting thresholding spatially, based on the reason that detailed regions having such as edges and textures tolerate some noise but not blurring [4]. In wavelet transform region, the signal energy is compacted into a few transform coefficients and noise contributes to the high frequency and insignificant coefficients. Based on these neighboring correlated properties taken from a spatial image and its wavelet transform, we can exploit the localized property of the coefficients to make it more suitable for locally adaptive image processing. Thus the absolute value of the wavelet coefficients is used as a measure that a coefficients close to zero contains little information and is relatively strongly influenced by noise.

In this paper, we describe the degree of noise depression exploiting lifting based construction, with the spatial adaptive wavelet transform. In this case update filter is especially first changed from sample to sample according to local signal features [6]. This allows not only to adapt the predictor to the signal but also to change the wavelet basis functions at each point and scale. We in this approach have implemented and applied to image denoising by finding a relevant threshold to be fixed with taking advantage of non-garrote shrinkage [7]. In the rest of the paper, basic lifting scheme and adaptive transform are presented in the next section and followed by experimental results. Finally, concluding remarks are presented.

II. LIFTING TRANSFORM

2-1. Lifting Scheme

The basic idea, LS, is a relationship among certain biorthogonal wavelets sharing the scaling function, which also exploits the correlation structure present in real signals [3]. Discrete wavelet transform(DWT) also can be viewed as prediction-error decomposition. Thus the scaling coefficients at a given scale *j* are 'predictors' for the next scale *j-1*, while the wavelet coefficients are 'prediction errors' between the scaling coefficients and the high resolution data that they are attempting predict. A lifting scheme comprised of following three steps. Let x[n] be a set of digital signal:

1) Splitting step:
Dividing the data into two distinct data sets. A simple way is to split the original data x[n] into the even indexed subset $x_e[n]$ and the odd indexed subset $x_o[n]$. This subdividing is sometimes called as the *lazy wavelet transform*.

$$x_e[n] = x[2n] \quad (1)$$
$$x_o[n] = x[2n+1] \quad (2)$$

2) Predict step:
Performing to obtain the wavelet coefficients *d*[n] which are the prediction errors, and predicting $x_o[n]$ from $x_e[n]$ using the predictor operator *P*. By subtracting this prediction from the odd samples, we reduce redundancy.

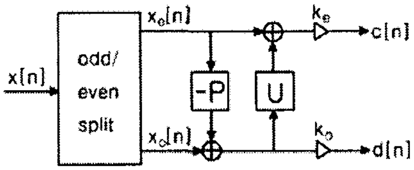
$$d[n] = x_o[n] - P(x_e[n]) \quad (3)$$

3) Update step:
Using $x_e[n]$ and *d*[n] to obtain the scaling coefficients *c*[n] which represent predictors, i.e., a coarse approximation to the original signal x[n]. An update operator *U* is used to calculate *c*[n] as following.

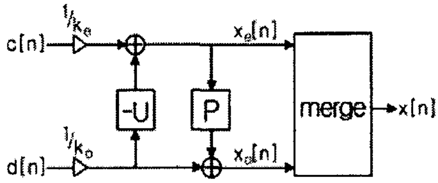
$$c[n] = x_e[n] + U(d[n]) \quad (4)$$

Figure 1 shows the principle construction of lifting scheme. In Figure 1b, the inverse transform can be easily constructed from forward transform by inverting the operation and signs in (3) and (4), and leading to the following synthesis equations (5) and (6).

$$x_e[n] = c[n] - U(d[n]) \quad (5)$$
$$x_o[n] = d[n] + P(x_e[n]) \quad (6)$$



a) forward transform



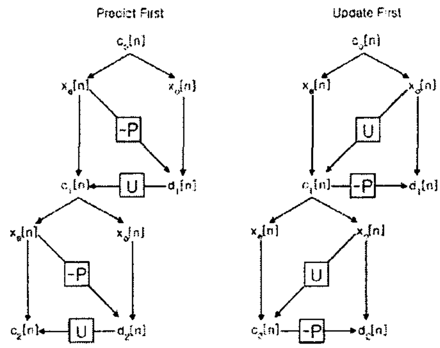
b) inverse transform

Figure.1 Lifting scheme

Filters *P*(.) and *U*(.) can be chosen arbitrary. Since the signal is reconstructed exactly in reversed order, the transform is always invertible.

2-2. Update-first wavelet transform

When the order of predict and update filters are reversed, updating-first lifting scheme is obtained as Figure 2. When predicting-first, the prediction *P*(.) is performed prior to construction of the approximation coefficients and iteration to the next. By updating-first, the prediction operator is outside the loop so that the approximation coefficients can be iterated to the lowest scale, quantized, and reconstructed prior to the prediction. A stability and performance is important to transforms. Quantized predictors are stably kept from augmenting propagation of errors throughout the entire pyramid as Figure 2.



a) predict first b) update first

Figure.2 Two-iteration trees

We use update-first structure to introduce adaptivity into the wavelet transform. Thus, prediction is only depended on low-pass coefficients that are computed as in the classical wavelet transform.

III. ADAPTIVE IMAGE DENOISING

3-1. Adaptive transform

In this paper we use space adaptive transform that tends to adapt the predictor to the signal and change the wavelet basis functions at each point and scale [6]. In a space adapted transform we employ the update-predict sequence framework and choose a predictor from a suite of predictors to minimize each d[n] value. In a $\tilde{N}$=1 point update, this adaptive algorithm chooses one point prediction of N∈{-5, -3, -1, 0, 1, 3, 5} symmetric points, and minimizing the value d[n] as shown in (7). This (N, $\tilde{N}$) pair is chosen for each n point. The transform is able to lock-on to the dominant signal structure at each point, and avoid discontinuities and other high-order polynomial phenomena that would decease the quality of prediction [5].

$$d[n] = \min \| P_N(x_e[n]) \| \quad (7)$$

where N is each prediction point that is chosen with N($\tilde{N}$) vanishing moments of the family of biorthogonal Cohen-Daubechies-Feauveau(CDF) wavelets for the primal(dual) wavelet [11]. Note that N=1, $\tilde{N}$=1is the Haar wavelet.

3-2. Threshold selection for denoising

Denoising algorithms based on wavelet thresholding replace small wavelet coefficients by zero, and keeping or shrinking the coefficients with absolute threshold value without greatly affecting the reconstruction. The transform of the noisy signal w[n] at the receiver end is expressed as follows:

$$w[n] = s[n] + e[n], \quad n = 0, 1, \ldots, L-1 \quad (8)$$

s[n] is a wavelet coefficient and e[n] is a wavelet coefficient of the noise, generally assumed as independent and identically distributed Gaussian with zero mean and variance σ^2, i.e., $n_{ij} \sim N(0,\sigma^2)$. w[n] is a wavelet coefficient of the noisy signal, and representing the property of additive white noise in the wavelet domain.

Thresholding procedure of noisy image is handled in three steps: 1)transform data into the wavelet domain, 2)shrink the wavelet coefficients according to thresholding, 3)perform the inverse transform to reconstruct the shrunken coefficients. Choosing the threshold to denoise is that the problem depends on at each level and for each subband separately. When the noise is spread over all coefficients within each level, the noise-free image can be represented by a limited number of large wavelet coefficients. From these, we replace the small coefficients by zero, because they are dominated by noise and carry only a small amount of information. At the j-th resolution level, all wavelet coefficients with absolute value below a certain threshold are classified as noisy, and replacing them by zero [1][8]. Typically the hard thresholding procedure keeps coefficients with an absolute value above the threshold, while soft thresolding shrinkes coefficients with higher absolute value above the threshold, and replacing coefficients with an absolute value below the threshold by zero. To remedy the drawbacks of the hard and soft shrinkage, we apply non-garrote shrinkage $\delta_\lambda^G(x)$ [7] that is defined as follows:

$$\delta_\lambda^G(x) = x(1 - (\lambda/x)^2) \quad (9)$$

$$\text{where } |x| \le \lambda \text{ then } 0, \quad |x| > \lambda \text{ then } x - \lambda^2/x$$

The threshold value is calculated adaptive to each subband components to be $\lambda = \beta\, \sigma^2/\sigma_x$, where σ^2 is the noise variance that is estimated from the subband HH1, using the formular, $\sigma^2=[median(|Y_{ij}|)/0.6745]^2$, where Y_{ij} belongs to the coefficients of the diagonal direction subband HH1 [12], and σ_x is the standard deviation of the subband. Where, the scale parameter β is applied to the threshold as the global factor, $\beta = 2^{-J/2}$, J is the number of decomposition levels.

IV. EXPERIMENTAL RESULTS

The experiments used natural images of size 256x256, lena retaining the degree of variation overall sharper, and showing good qualities at the decomposition level 1 and 4. *i.i.d.* Gaussian noise generated using *randn* function at different noise levels σ =20. For our adaptive transform, we chose a filter from the (1, N) symmetric branch of the CDF family [11], where N∈{-5, -3, -1, 0, 1, 3, 5} symmetric points, and minimizing the value d[n]. For providing smoothness and preserving edge, it was possible to cope with any sequences by finding locally adaptive value. PSNR results for noise reduction are shown in Table 1 and Figure 3, and showing that lifting denoising with adaptive method gives better results than with nonadaptive ones. Figure 4 shows the empirical images to be denoised with σ =20.

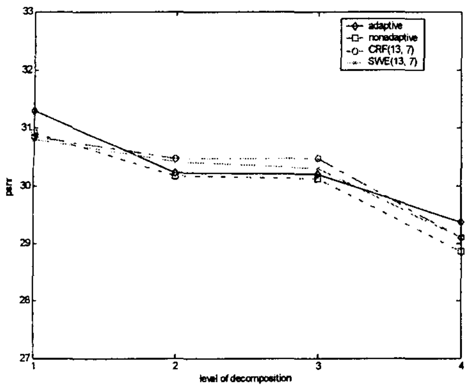


Figure 3. PSNR vs. levels of decomposition for lena

V. CONCLUSIONS

This paper has described an adaptiveness and nonlinearity of image denoising for the prediction lifting problem. Using the non-garrote shrinkage, we have also represented a denoising algorithm more effective at the decomposition level 1 and 4. It informs us on reducing the propagation error by only depending on the scaling coefficients as using the update-first operation, and also adapting to selecting threshold from optimality criterion in the strong intensity. Future research will be focused on working out algorithms for more adaptive selection in the update filter itself.

REFERENCES

[1] D. Donoho, I. Johnstone, Ideal spatial adaptation via wavelet shrinkage, Biometrica, Vol.81, pp.424-455, Dec. 1994

[2] W. Sweldens, The lifting scheme: A construction of second generation wavelets, Siam J. Math. Anal, Vol. 29, Nr.2, pp.247-269, 1998

[3] I. Daubechies, W. Sweldens, Factoring wavelet transforms into lifting steps, J. Fourier Anal. Appl., Vol.4, Nr3, pp.247-269, 1998

[4] S. G. Chang, M. Vetterli, Spatial adaptive wavelet thresholding for image denoising, IEEE Conf. on Image Processing, Oct. 1997

[5] R. L. Claypoole, G. Davis, W. Sweldens, R. Baraniuk, Nonlinear wavelet transforms for image coding, In Proc. 31st Asiloma Conf., (Pacific Grove, CA), 1997

[6] R. L. Claypoole, R. Baraniuk, R. Nowak, Adaptive wavelet transforms via lifting, IEEE Conf. on Acoustics, Speech and Signal Processing, Phoenix, 1999

[7] H. Y. Gao, A. G. Bruce, WaveShrink with Firm Shrinkage, Statistica Sinica, 7, 855-874, 1997

[8] M. Jansen, G. Uytterhoeven, A. Bultheel, Image de-nosing by integer wavelet transforms and generalized cross validation, Report TW264, K.U.Leuven, Aug. 1997

[9] D. Donoho, I. Johnstone, Threshold selection for wavelet shrinkage of noisy data, IEEE, 1994

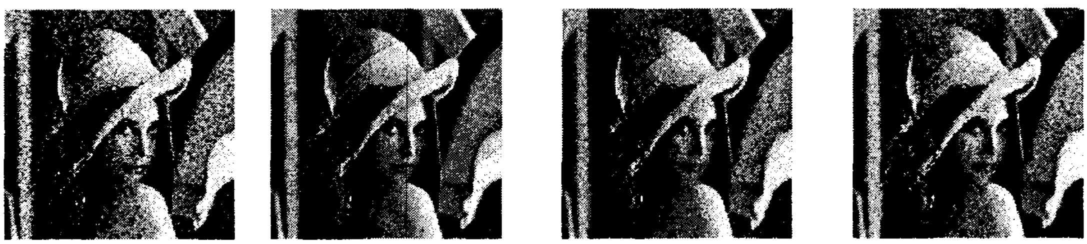
[10] H.G. Gao, Wavelet shrinkage denoising using the non-negative garrote, Journal of computational and graphical statistics, vol.7, no.4, pp.469-488, 1998

[11] A. Cohen, I. Daubechies, J. Fearvear, Bi-orthogonal bases of compactly supported wavelets, Comm. Pure Appl.,Math., vol 45, pp. 485-560, 1992

[12] S. G. Chang, M. Vetterli, Adaptive wavelet thresholding for image denoising and compression, Image Processing, IEEE Transaction on, Vol 9, No. 9, Sep. 2000, Page(s): 1532-1546

Table 1. PSNR results for lena image, adaptive and non-adaptive wavelets at σ =20

Decomposition level	1	2	3	4
Adaptive	31.30	30.22	30.20	29.37
Nonadaptive	30.91	30.17	30.12	28.86
CRF(13, 7)	30.85	30.46	30.46	29.10
SWE(13, 7)	30.80	30.42	30.30	29.10



a) noisy b) adaptive c) CRF(13, 7) d) SWE(13, 7)

Figure 4. Comparison of denoised lena image at σ =20