Image enhancement using the local statistics

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Abstract

A nonlinear iterative filtering based on local statistics and anisotropic diffusion is introduced. Local statistics determines the diffusion coefficient at each iteration step. Anisotropic diffusion can be seen as estimates a piecewise smooth image from the noisy input image. In the experimental section, our results are shown to suppress noise with preserving the edges. Therefore, it enhances the image and improves performance.

I. Introduction

Enhancing the images with preserving the edges are one of the major issues in image processing. Partial differential equations (PDEs) have dominated image processing research recently. After Witkin introduced a clean formalism for the scale-space filtering [1]. Perona-Malik introduced the elegant formulation of anisotropic diffusion [2]. Research in the anisotropic diffusion has been oriented toward understanding the mathematical properties of anisotropic diffusion and related variational formulations [3], developing related well-posed and stable equations [4], modifying anisotropic diffusion equations for medical applications [5]. Our method is motivated by the great recent interest in using evolutions specified by PDE as image processing procedures for tasks such as edge enhancement , segmentation and detection [6], [7].

II. Conventional Methods

A. Anisotropic Diffusion

Diffusion methods remove noise from an image by modifying the image via PDE. For simple example, consider linear diffusion equation (the heat equation) given by:

$$\frac{\partial I(x,y,t)}{\partial t} = div(\nabla I(x,y,t)) \tag{1}$$

using the original image $I(x,y,0) = I_0(x,y)$ as the initial condition, where t specifies the image scale, and where ∇I is the image gradient. Linear diffusion smoothes out noise and edges equally. This causes difficulty in tracking features across multiple scales. In order to keep sharp edges, while filtering noise and small details, Perona and Malik [2] first introduced the idea of nonlinear anisotropic diffusion by replacing the classical isotropic diffusion equation with the PDE as follows:

$$\frac{\partial I(x,y,t)}{\partial t} = div\left[g(\|\nabla I\|)\nabla I\right] \tag{2}$$

where $\|\nabla I\|$ is the gradient magnitude, and $g(\bullet)$ is chosen so as to suppress diffusion in the regions of high gradient and to encourage diffusion in low gradient regions. Note that if $g(\bullet)$ is equal to one, then (2) would turn into the linear diffusion equation (1). They suggested tow "edge-stopping" functions:

$$g(\|\nabla I\|) = \left[1 + \frac{\|\nabla I\|^2}{K^2}\right]^{-1} \quad \text{and} \quad g(\|\nabla I\|) = \exp\left[-\frac{\|\nabla I\|^2}{K^2}\right]$$

where K is a positive constant. Perona-Malik discretized their anisotropic diffusion equation as follows:

$$I_s^{t+1} = I_s^t + \frac{\lambda}{|\eta_s|}\sum_{p\in\eta_s} g(\nabla I^t_{s,p})\nabla I^t_{s,p} \tag{3}$$

where I_s^t is a discretely sampled image, s denotes the pixel position, and t denotes the iteration. The constant λ is scalar that determines the rate of diffusion, p takes one of the four neighbors and η_s is the set of four neighbors of s.

Perona-Malik approximated the image gradient in a particular direction as: $\nabla I^{t}_{s,p} = I_p^{\ t} - I_s^{\ t}, \quad p \in \eta_s$ (4)

B. Robust Anisotropic Diffusion

Black [8] showed a comparative study between Perona-Malik anisotropic diffusion based on a combination of PDE and estimator. In this image process, the goal is to find an image I that satisfies the following optimization criterion:

$$\min_{I} \sum_{s \in I} \sum_{p \in \eta_s} \rho(I_p - I_s, \sigma) \quad (5)$$

where $\rho(\bullet)$ is a robust error norm and σ is a "scale" parameter. Equation (5) can be solved by gradient descent using the calculus of variations as follow:

$$\frac{\partial I(x,y,t)}{\partial t} = div\left[\rho'\left(\|\nabla I\|\right)\frac{\nabla I}{\|\nabla I\|}\right] \quad (6)$$

where $g(x) \triangleq \rho'(x)/x$. This demonstrates that anisotropic diffusion is the gradient descent of an estimation problem with a familiar robust error norm. Moreover, Black proposed to use Tukey's biweight error norm to define a better "edge-stopping" function:

$$\rho(x,\sigma) = \begin{cases} \frac{x^2}{\sigma^2} - \frac{x^4}{\sigma^4} + \frac{x^6}{3\sigma^6} & |x| \le \sigma \\ 1/3 & otherwise \end{cases} \quad (7)$$

Using Tukey's function, the diffusion process converges faster and yields sharper edges because of the Tukey's biweight function give zero weight to edges whose magnitude is above a certain value.

C. Stabilized Inverse Diffusion Equation (SIDE)

The crucial parameter that controls the amount of blurring in the Perona-Malik equation is K. For example, if K is very large, then the results of Perona-Malik filtering will be quite similar to the linear averaging. Smaller K will lead to more edge preservation. Pollak [7] introduces a limiting case by setting $K=0$, to obtain a discontinuous $\psi(\bullet)$. This resulting equation is called a stabilized inverse diffusion equation (SIDE):

$$I_n^{i+1} = I_n^i + \Delta t\left[\psi\left(I_{n+1}^i - I_n^i\right) - \psi\left(I_n^i - I_{n-1}^i\right)\right] \quad (8)$$

Notice that the right-hand side of $\psi(\bullet)$ function has a discontinuity at a point I if and only if $I_i = I_{i+1}$ for some i. Therefore, If $I_i(t_0) = I_{i+1}(t_0)$ for some time instant t_0 and some index i, then $I_i(t) = I_{i+1}(t)$ for all future $t > t_0$, that is, as soon as the values of two neighboring samples become equal to each other, they stay equal for the remainder of the evolution. The SIDE successively merges neighboring pixels together, resulting in larger and larger flat regions. For a very simple case [9], SIDE can solve the following estimation problem:

$$u_n = x_n + w_n, \qquad for \ \ n = 1,\cdots,N \quad (9)$$

$$\text{Subject to } \ TV(x) = \sum_n |x_{n+1} - x_n| \le \nu$$

Above constrained estimation can be solved by evolving the SIDE with $\psi(\bullet) = sign(\bullet)$ and with the initial signal $u(0)$.

SIDE provides good estimates of edge locations. However, it is not necessarily very accurate in estimating the intensity values within each region.

III. Proposed Method

We propose a new approach directly motivated local statistics and anisotropic diffusion. Our proposed method is designed to eliminate the noise with preserving edges in noisy image. Our filter produces the enhanced data according to:

$$I_s^{\ t+1} = (1 - a_{i,j}) \bullet I_s^{\ t} + a_{i,j} \bullet \overline{I}_s^{\ t} \quad (10)$$

where $\overline{I}_s^{\ t}$ is the mean value of the sampled image and $a_{i,j}$ is coefficient that is determined by the local statistics, and is calculated by:

$$a_{i,j} = \begin{cases} b + \frac{\sigma_n^{\ 2}}{\mathrm{var}(I_{i,j}^t)}, & b + \frac{\sigma_n^{\ 2}}{\mathrm{var}(I_{i,j}^t)} \le 1 \\ 1 \quad , & otherwise \end{cases} \quad (11)$$

The parameter b plays an important role in controlling the performance of the filter. The larger b is, the better the filtering effect will be, but the more it will blur sharp edges. In the extreme cases, when $b=0$, the resulting coefficient $a_{i,j}$

is the same as the Lee filter coefficient, when $b=1$, the filter reduces to a simple arithmetic mean filter. Fig.1 shows these facts conceptually. Experiments show that choosing $b \in [0.2, 0.3]$ can obtain a good result

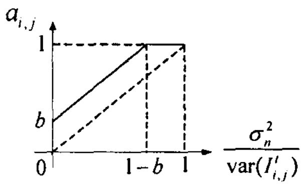


Fig.1 local coefficients: $a_{i,j}$

Local variance $\mathrm{var}(I_s^t)$ and local mean $\overline{I}_s^t$ are given by:

$$\mathrm{var}(I_s^t) = (1/|\eta_s|)\sum_{p\in\eta_s}(I_p^t - \overline{I}_s^t)^2 \quad \text{and}$$

$$\overline{I}_s^t = \frac{1}{N}\sum_{i=1}^{N} I_i^t = I_s^t + \frac{1}{|\eta_s|}\sum_{p\in\eta_s}(I_p^t - I_s^t) = I_s^t + \frac{1}{|\eta_s|}\nabla^2 I_s^t$$

The local statistics $\overline{I}_s^t$, $\mathrm{var}(I_s^t)$ plays an essential role in controlling the filter. At homogeneous regions $\left(\mathrm{var}(I_s^t) \approx \sigma_n^2\right)$, then $I_s^{t+1} \approx \overline{I}_s^t$, leading to smooth image. At edge regions $\left(\mathrm{var}(I_s^t) \gg \sigma_n^2\right)$, then $I_s^{t+1} \approx I_s^t$, preserving the image edges. We can express (10) in the following form:

$$I_{i,j}^{t+1} = I_{i,j}^t + a_{i,j}\bullet(\overline{I}_{i,j}^t - I_{i,j}^t) = I_{i,j}^t + a_{i,j}\bullet\frac{1}{|\eta_s|}\nabla^2 I_{i,j}^t$$

$$= I_{i,j}^t + \frac{1}{|\eta_s|}\left[a_{i+1,j}\bullet\left(I_{i+1,j}^t - I_{i,j}^t\right) + \cdots + a_{i,j-1}\bullet\left(I_{i,j-1}^t - I_{i,j}^t\right)\right]$$

$$= I_{i,j}^t + \frac{1}{|\eta_s|}div\left(a_{i,j}\bullet\nabla I_{i,j}^t\right) \qquad (12)$$

$a_{i,j}$ assigns different weights to the four directional differences. Using $|\eta_s| = 4$ and the following equalities:

$$\mathrm{var}(I_s^t) = \frac{1}{|\eta_s|}\sum_{p\in\eta_s}(I_p^t - \overline{I}_{i,j}^t)^2 = \overline{(I_{i,j}^t)^2} - (\overline{I}_{i,j}^t)^2$$

$$\overline{(I_{i,j}^t)^2} = (I_{i,j}^t)^2 + \frac{1}{|\eta_s|}\nabla^2(I_{i,j}^t)^2$$

$$\nabla^2(I_{i,j}^t)^2 = 2\left|\nabla I_{i,j}^t\right|^2 + 2I_{i,j}^t\nabla^2 I_{i,j}^t$$

Finally, We can obtain the following form of $a_{i,j}$:

$$a_{i,j} = b + \frac{\sigma_n^2}{\frac{1}{2}\left|\nabla I_{i,j}^t\right|^2 - \frac{1}{16}(\nabla^2 I)^2} \qquad (13)$$

$a_{i,j}$ combines a gradient magnitude operator and a Laplacian operator to act like an edge detector for noisy image.

IV. Experimental Results

To evaluate the performances, PSNR between the filtered output and the original image is used as a quantitative measurement. Consider the Lenna image shown in Fig.2(a) and its noisy version is shown in Fig 2(b). Also Fig.2(c)-(g) show the each filtered results after 300 iterations.

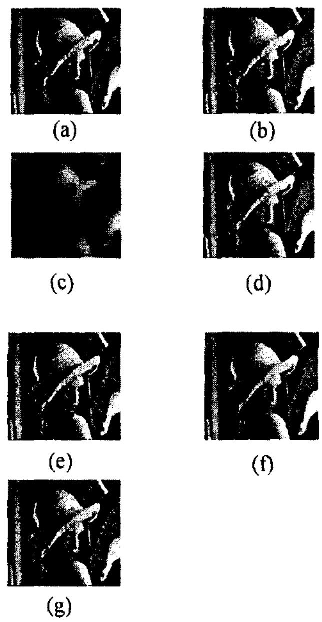
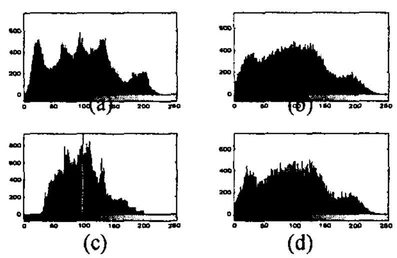


Fig.2 filtered results after 300 iterations. (a) original image (b) noisy image (c) linear (d) PM (e) Tukey (f) SIDE (g) proposed



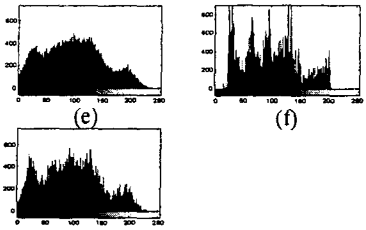


Fig.3 histograms on the image Fig.2. (a) original image (b) noisy image (c) linear (d) PM (e) Tukey (f) SIDE (g) proposed

To see how much filtered output is different from the original image, we also plot the each filtered output histograms in Fig.3 (a)-(g). Finally, the resulting PSNR computations at the various iteration numbers are listed:

PDE \ iteration	10	20	50	100	200	300	500
Noisy	26.92	26.92	26.92	26.92	26.92	26.92	26.92
linear	24.45	22.94	21.10	19.79	18.57	17.90	17.10
PM	26.94	26.97	27.05	27.17	27.41	27.63	28.05
Tukey	26.92	26.92	26.92	26.92	26.92	26.92	26.92
SIDE	27.77	28.56	30.36	30.74	28.72	27.25	25.44
proposed	27.02	27.12	27.41	27.84	28.60	29.21	29.89

V. Conclusions

In this paper, we have developed a nonlinear anisotropic diffusion technique based on local statistics for removing noise in images. Our proposed method remains more high values rather than other methods. This means that edges or high values are properly reserved against blurring. Tukey's function exhibits not much difference even after the higher iteration. This result comes from the reason that tukey's function gives zero diffusion coefficients above a certain value. If we increase scale parameter σ in (7), then it will improve the PSNR. However, it is negligible compared to others methods. SIDE both increases the PSNR and blurs the image. We can expect these properties from the influence function $\psi(\cdot)$ and at more iteration above 300, SIDE suffers from the serious blurring defects. Hence, reduce the image quality. On the other hand, our proposed method responds highly homogeneous regions by smoothing, and shuts down its smoothing in edges regions. In addition, the window sizes can affect filter performances.

Consequently, our proposed new method has effective performance in reducing the noise simultaneously with good edge preservation.

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