Optimal Time-Interval for Time-based Location Update in Mobile Communications

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***Abstract*-In this paper, we analyze a time-based update method in location management and calculate the optimal time-interval. We obtain the probability that an MT is *j* rings away from the center cell. And using these values, an exact analysis is made for the time-based location management cost. From the result, when the time-based method is applied to location update, we can get the optimal time-interval which minimizes the location management cost.**

I. Introduction

Location management and call setup process play an important role in the PCS performance [1, 2, 3, 4]. The whereabouts of a user in mobile communication systems must first be known in order to correctly route an incoming call. A user's location information can be obtained from the registration initiated by the user and the paging issued by the system. However, the registration cost and the paging cost have the relation of inverse coupling in the use of network resources.

Researches has been made about the performance of PCS in the three models; movement-based, distance-based and time-based. However, exact numerical analyses have not been made for the case of time-based model. In this paper, we make an exact numerical analysis of a time-based model and determine the optimal time interval for location update.

II. Time-based location update method

A simple dynamic strategy in location update is a time-based method in that a mobile terminal(MT) transmits its location update messages periodically every T units of time[5, 6, 7, 8]. However, if a call arrival occurs within T interval, the system pages the MT and the MT restarts its timer. The paging mechanism of the time-based method are more complex than the movement-based and distance-based updates in that it is difficult to estimate the paging area because the MT does not update its location until the time interval T expires. Therefore, it is difficult to calculate how long distance a MT moves from the recently registered location.

Figure 1 shows the hexagonal cell structure of the mobile communication system considered in this paper. Each cell is surrounded by rings of cells. The innermost ring(ring 0) consists of only one cell and we call it center cell. Ring 0 is surrounded by ring 1 which in turn is surrounded by ring 2, and so on. When the system routes an incoming call to an MT, it first pages the center cell which is the recently registered location of the MT. If it does not succeed in finding the MT, it pages next surrounded ring. The paging goes on until it finds the MT.

From the implementation point of view, the time-based strategy is the simplest since mobile users need to send location update messages according to their local clocks. However, in this strategy, even though an MT does not move from and stays at one cell for a long time, the MT should updates its location periodically every T. It is a waste of control channels which could be used for other jobs. In this case, the value of T should be lengthened. For a highly mobile user, the time-based update method leads to an improvement over the simpler movement-based method. However, the paging cost is affected by location uncertainty. To minimize the location uncertainty, the value of T should be shortened. Therefore, it needs to analyze the optimal time-interval to save the signalling channels and to minimize the location management cost.

III. Model and analysis of location management cost for time-based update

We denote some notations and assumptions. Call arrival distribution to an MT is Poisson with rate

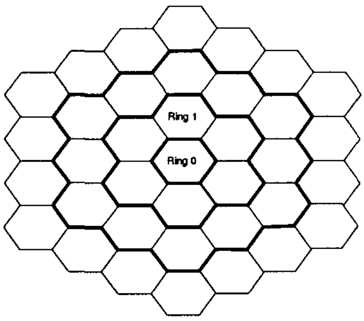

Fig. 1. Hexagonal call structure.

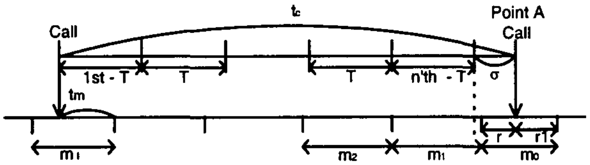

Fig. 2. The time diagram in time-based update mechanism

λ_c. Let a random variable t_c be the interval between two consecutive calls to the MT. Let m_i be the cell residence time at cell i and be the independent identically distributed random variable with the general distribution function $F_{m_i}(t)$, the density function $f_{m_i}(t)$ and the mean rate λ_m. Figure 2 shows the timing diagram of the MT between two consecutive calls. The MT visits l cells(numbered inversely in the figure) and gets a new call. The MT resides in the intermediate i'th ($0 \le i \le l$) cell for a period m_i.

Since the cell residence time for each cell should be identical, we have $f_{m_i}(t) = f_m(t)$ and $F_{m_i}(t) = F_m(t)$. The Laplace-Stieltjes Transform of $f_m(t)$ is $F^*_m(s)$ and has the relation of $F^*_m(s) = \int_{t=0}^{\infty} e^{-st} f_m(t)\,dt$. In t_c, there are n expirations of T and the remnant σ. r_1 and r are the residual life and the age of m_0, respectively. Let $\alpha(K)$ be the probability that the MT moves across K cells during the remnant σ. Let $\beta(j, K)$ be the probability that the MT is j rings away from the center cell given that K cell boundary crossings are performed.

For the hexagonal cell configuration in the figure

TABLE I. Value of $\beta(j, K)$

j \ K	1	2	3	4	5	…
0	0	0.166666	0.055555	0.069444	0.046296	…
1	1	0.333333	0.416666	0.277777	0.262345	…
2		0.5	0.333333	0.379629	0.324074	…
3			0.194444	0.203703	0.243055	…
4				0.069444	0.100308	…
5					0.023919	…
…	…	…	…	…	…	…

1, we assume that each MT resides in a cell for a time period then moves to one of its neighbors with the equal probability, i.e., 1/6. This assumption is meaningful when users are moving within city area because the moving area is not large and then the movement pattern can be seen to be random. Therefore, we can get $\beta(j, K)$ from computer programming. Let the costs for performing a location update and for paging a cell be V and U, respectively. These costs account for the wireless and wireline bandwidth utilization and the computational overheads in order to process the location update and the paging. Let C_u be the expected location update cost per call arrival. The expected paging cost per call arrival is denoted by C_v.

From the figure 2, r_1 is the residual time of cell residence time m_0 and $r + r_1 = m_0$. At first, we derive the expected cost for location updates. The probability that there are n and more location update messages due to expirations of T timer during t_c is

$$P[t_c > nT] = e^{-\lambda_c \cdot nT}. \quad (1)$$

Similarly, the probability that there are n+1 and more location update messages due to expirations of T timer during t_c is

$$P[t_c > (n+1)T] = e^{-\lambda_c \cdot (n+1)T}. \quad (2)$$

Therefore, we can get the probability that there are n location update messages due to expirations of T timer during t_c.

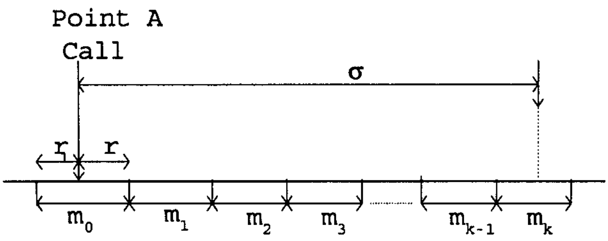

Fig. 3. The reverse time diagram

$$
\begin{aligned}
&P[(n+1)T > t_c > nT] \\
&= P[t_c > nT] - P[t_c > (n+1)T] \\
&= e^{-\lambda_c \cdot nT} - e^{-\lambda_c \cdot (n+1)T} \\
&= e^{-\lambda_c nT}(1 - e^{-\lambda_c T}) \ . \qquad (3)
\end{aligned}
$$

We let p_n be the probability that there are n update messages between two successive calls. Then we have

$$p_n = e^{-\lambda_c nT}(1 - e^{-\lambda_c T}) \qquad (4)$$

The expected cost for location updates for the period between two consecutive calls, C_u is

$$C_u = U \sum_{n=0}^{\infty} np_n = U \sum_{n=0}^{\infty} ne^{-\lambda_c nT}(1 - e^{-\lambda_c nT}) \qquad (5)$$

Now, we derive the expected cost for paging cells. Let $f_\sigma(t)$, $f_r(t)$ and $f_{r_1}(t)$ be the probability density function of σ, r and r_1, respectively. Then we have

$$
\begin{aligned}
f_\sigma(t) &= \frac{\lambda_c e^{-\lambda_c t}}{1 - e^{-\lambda_c T}} \qquad (0 \le t \le T), \\
&= 0 \qquad \textit{otherwise}. \qquad (6)
\end{aligned}
$$

From the random observer property, we can get

$$f_r(t) = f_{r_1}(t) = \lambda_m[1 - F_m(t)]. \qquad (7)$$

The Laplace-Stieltjes Transform for the $f_r(t)$ is

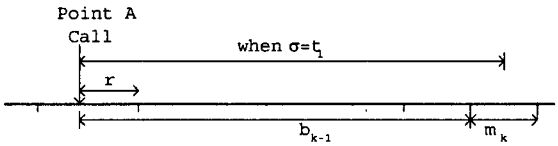

Fig. 4. The time diagram at point A

$$F^*_r(s) = \frac{\lambda_m}{s}[1 - F^*_m(s)]. \qquad (8)$$

At the point A in the figure 2, the time axis can be displayed reversly as shown in the figure 3. We consider the instant, $\sigma = t_1$. We let $\alpha(0, t_1)$ be the probability that the MT does not cross the boundary of the current cell during the remnant time $\sigma = t_1$. Then we have

$$
\begin{aligned}
\alpha(0, t_1) &= P[t_1 < r] = \int_{t_1}^{\infty} f_r(t)\, dt \\
&= \int_{t_1}^{\infty} \lambda_m[1 - F_m(t)]dt \qquad (9)
\end{aligned}
$$

From (6) and (9), we get

$$\alpha(0) = \int_0^T \alpha(0, t_1) f_\sigma(t_1) dt_1. \qquad (10)$$

We let $\alpha(k, t_1)$ be the probability that the MT crosses cell boundaries k times during the remnant time $\sigma = t_1$. Then we have

$$
\begin{aligned}
\alpha(k, t_1) = P[&r + m_1 + m_2 + \cdots + m_{k-1} < t_1 \\
&< r + m_1 + m_2 + \cdots + m_{k-1} + m_k]. \qquad (11)
\end{aligned}
$$

We let $b_k = r + m_1 + m_2 + \cdots + m_k = r + \sum_{i=1}^{k} m_i$ as in the figure 4. We get the Laplace-Stieltjes Transform for the probability density function of $b_k(t)$ as following.

$$F^*_{b_k}(s) = F^*_r(s) \cdot (F^*_m(s))^k \qquad (k \ge 0). \qquad (12)$$

For $b_{k-1}(t)$, we can get

$$F^*_{b_{k-1}}(s) = F^*_r(s) \cdot (F^*_m(s))^{k-1} \qquad (k \ge 1). \qquad (13)$$

From the equation (11), we get $\alpha(k, t_1)$ as following.

$$\alpha(k,t_1)=\int_0^{t_1} P[m_k > t_1 - t]\cdot b_{k-1}(t)\,dt = \int_0^{t_1}\left[\int_{t_1-t}^{\infty} f_m(t_2)dt_2\right] b_{k-1}(t)\,dt. \quad (14)$$

From (6) and (14), we have $\alpha(k)$.

$$\alpha(k)=\int_0^{T}\alpha(k,t_1)f_\sigma(t_1)dt_1 . \quad (15)$$

Let π_j be the probability that the MT is located in a ring j cell when a call arrival occurs. Then we have

$$\pi_j=\sum_{K=0}^{\infty}\alpha(K)\beta(j,K). \quad (16)$$

Given that the MT is residing in ring j, let ω_j be the number of cells from ring 0 to ring j.

$$\omega_j=1+\sum_{i=1}^{j}6i = 1+3j(j+1) \quad (17)$$

The paging cost for the time-based update for the period between two consecutive calls, C_v is expressed as

$$C_v=V\sum_{j=0}^{\infty}\pi_j\omega_j . \quad (18)$$

The expected total cost for location updates and paging per call arrival in the time-based method is therefore

$$C_T=C_u+C_v . \quad (19)$$

IV. Result and discussion

Figure 5 and 6 show the values of total cost, C_T as a function of T. The values of U and V can be set to any values. However, to demonstrate the analysis of this paper, U and V are, for convenience, set to one. The distribution of cell resident time used in the results is gamma distributrion. The value of gamma used in the figure 5 is γ=1, that is exponential distribution. The value of gamma used in

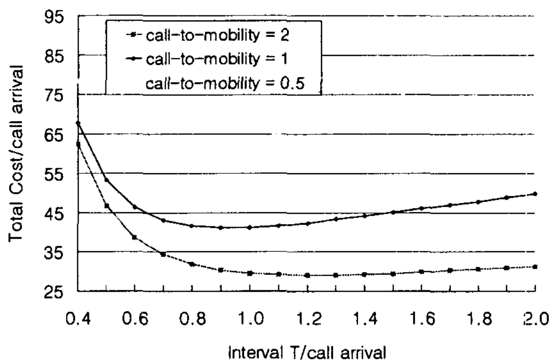

Fig. 5. Total cost versus time interval in the time-based method when γ=1.

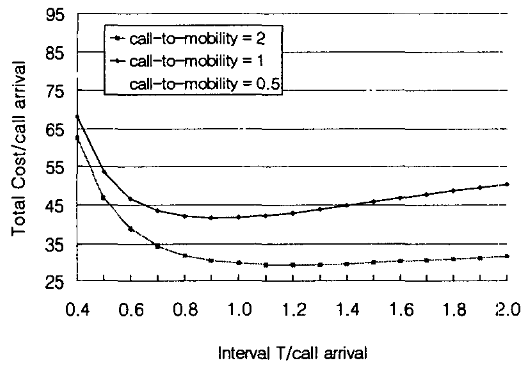

Fig. 6. Total cost versus time interval in the time-based method when γ=2.

the figure 6 is γ=2. To show the effect of call-to-mobility patterns, three call-to-mobility ratio $\frac{\lambda_c}{\lambda_m}$ = 0.5, 1.0 and 2.0, are considered. The values of C_T of the figure 5 and 6 have similar values. This is because the mean resident time of MT at a cell makes major effects than its variance on the values of C_T. In the figures, it can be seen that the value of the total cost varies widely as T changes. By selecting the appropriate value of T, the total cost per call arrival, C_T, could be minimal.

In the figures, when T is small, C_T is large because location updates occurs frequently which increases the location update cost. When T is large, location update cost is small but paging area gets large which results in high paging cost. In the figures, we also see that when mobility is large(when call-to-mobility is small), the total cost gets high. This is because more cells should be paged to find the location of MT which makes the paging cost high.

V. CONCLUSIONS

In this paper, we analyzed a time-based update method in location management and calculated the optimal time-interval. We obtained the probability that an MT is j rings away from the center cell by computer programming. And using these values, we made an exact analysis for the time-based location management cost. The location management cost varies according to the values of the time interval T. The optimal time-interval to minimize the location management cost is dependent on the call arrival rate and user mobility. Using the result in this paper, when the time-based method is applied to location update, we can determine the optimal time-interval which minimizes the location management cost.

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