2003 International Symposium on Advanced Intelligent Systems
September 25-28, 2003, Jeju, Korea

Fuzzy Relational Calculus based Component Analysis Methods and their Application to Image Processing

Hajime Nobuhara and Kaoru Hirota
Department of Computational Intelligence and Systems Science,
Tokyo Institute of Technology,
4259 Nagatsuta, Midiri-ku, Yokohama, 226-8502, Japan
Tel/Fax : +81-45-924-5682/45-924-5676

{nobuhara, hirota}@hrt.dis.titech.ac.jp

Abstract— Two component analysis methods based on the fuzzy relational calculus are proposed in the setting of the ordered structure. First component analysis is based on a decomposition of fuzzy relation into fuzzy bases, using gradient method. Second one is a component analysis based on the eigen fuzzy sets of fuzzy relation. Through experiments using the test images extracted from SIDBA and View Sphere Database, the effectiveness of the proposed component analysis methods is confirmed. Furthermore, improvements of the image compression/reconstruction and image retrieval based on ordered structure are also indicated.

1 Introduction

In the image processing based on space-frequency structure, various methods of component analysis have been developed, e.g. eigen values and corresponding vectors [1, 2]. In this paper, by using the fuzzy relational calculus, two component analysis methods based on ordered structure are proposed. By using the image intensity normalization of each pixel into [0,1], an original image can be regard as a fuzzy relation. The fuzzy relational calculus can be applied to the analysis of the original image. First one is a component analysis based on a decomposition of fuzzy relation into fuzzy bases. Second one is based on the eigen fuzzy sets that correspond to the component of images. By using two proposed methods, analytical results of images extracted from SIDBA and View Sphere Database, are presented, respectively.

2 Component Analysis (I)

2.1 Problem Formulation

First component analysis based on a decomposition of fuzzy relation (original image) into fuzzy bases, is shown. An original image of size $M \times N$ (pixels) can be treated as a fuzzy relation $R \in F(\mathbf{X} \times \mathbf{Y})$, $\mathbf{X} = \{x_1, x_2, \ldots, x_M\}$, $\mathbf{Y} = \{y_1, y_2, \ldots, y_N\}$, by normalizing the intensity range of each pixel into $[0, 1]$. The original image R is approximated by the composition of two pairs of fuzzy bases, $\{A_i \in F(\mathbf{X}) | i = 1, \ldots, c\}$ and $\{B_i \in F(\mathbf{Y}) | i = 1, \ldots, c\}$ such that

$$R(x,y) \approx \tilde{R}(x,y) = \bigvee_{i=1}^{c} (A_i(x) \wedge B_i(y)), \quad (1)$$

for all $x \in \mathbf{X}$, $y \in \mathbf{Y}$, where c denotes the Schein rank of the fuzzy relation R, that is the smallest integer satisfying Eq. (1). As it can be seen from Eq. (1), the max-min composition of fuzzy bases $\mathbf{A}$ and $\mathbf{B}$ corresponds to the approximated image $\tilde{R}$, that is fuzzy bases $\mathbf{A}$ and $\mathbf{B}$ express components of the original images. Obviously c measures the approximate performance $\tilde{R}$ of R, in the sense that the possible larger the value c, the better approximation $\tilde{R}$ is. The approximation problem (Eq. (1)) which is equivalent to identification of the component of the original image, can be seen as an optimization that minimizes the cost function:

$$Q = \sum_{(x,y) \in \mathbf{X} \times \mathbf{Y}} \left(R(x,y) - \bigvee_{i=1}^{c} (A_i(x) \wedge B_i(y)) \right)^2 . \quad (2)$$

A solution of the optimization problem, based on the gradient method has been presented in [6]. The following improvement, [3] i.e., a fast solution of the optimization problem is presented as follows:

$$A_i^{(iter+1)}(x) = A_i^{(iter)}(x) - \alpha \frac{\partial Q_{iter}}{\partial A_i^{(iter)}(x)}, \quad (3)$$

$$B_i^{(iter+1)}(y) = B_i^{(iter)}(y) - \alpha \frac{\partial Q_{iter}}{\partial B_i^{(iter)}(y)}, \quad (4)$$

for all $x \in \mathbf{X}$, $y \in \mathbf{Y}$. where "iter" denotes the iteration number of the gradient method. For simplicity of notation, by setting $Q = Q_{iter}$, $A_i = A_i^{(iter)}$ and $B_i = B_i^{(iter)}$, the derivation of the cost function Q with respect to $A_l(x')$ can be written for

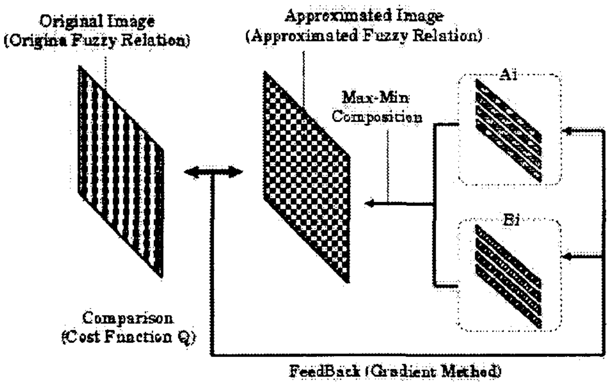

Figure 1: An overview of the component analysis (I)

$l = 1, 2, \ldots, c$, as

$$\frac{\partial Q}{\partial A_l(x')} = -2\sum_{y \in \mathbf{Y}} \left(R(x', y) - \bigvee_{i=1}^{c} \left(A_i(x') \wedge B_i(y) \right) \right) \cdot \phi(A_l(x'), \tilde{R}(x', y)), \quad (5)$$

and

$$\frac{\partial Q}{\partial B_l(y')} = -2\sum_{x \in \mathbf{X}} \left(R(x, y') - \bigvee_{i=1}^{c} \left(A_i(x') \wedge B_i(y') \right) \right) \cdot \phi(B_l(y'), \tilde{R}(x, y')), \quad (6)$$

where

$$\phi(a, b) = \begin{cases} 1, & if \quad a = b, \\ 0, & otherwise. \end{cases} \quad (7)$$

An overview of the first component analysis method is shown in Fig. 1.

2.2 Experiments of Image Component Analysis

A result of application of the component analysis (I) to an image extracted from Standard Image DataBAse (SIDBA), is presented. Figure 2 (left) shows an original image 'cameraman'. Under the condition that the learning rate α is set at 0.005, the approximated image $\tilde{R}$ is obtained. The approximated image $\tilde{R}$ with the Schein rank being 25, 50, and 100 are shown in Figs. 2 (right) - 3. The value of the cost function Q with respect to *iter* is shown in Fig. 7. The fuzzy bases **A** and **B** with the Schein rank being 25, 50, and 100 are shown in Figs. 4 - 6. As it can be seen from Figs. 2 - 7, if the Schein rank is higher, the approximation of $\tilde{R}$ is better. In other words, the Schein rank is higher, the component of the original image included in the fuzzy bases **A** and **B** is larger.

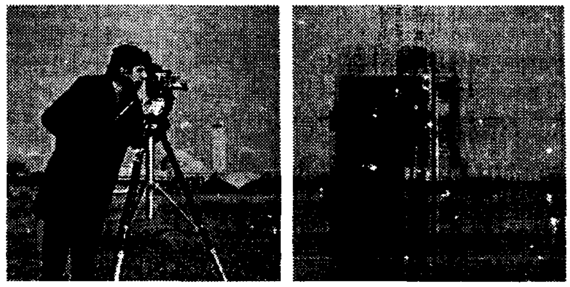

Figure 2: Original image (left)and approximated image $\tilde{R}$, Schein rank = 25, (right)

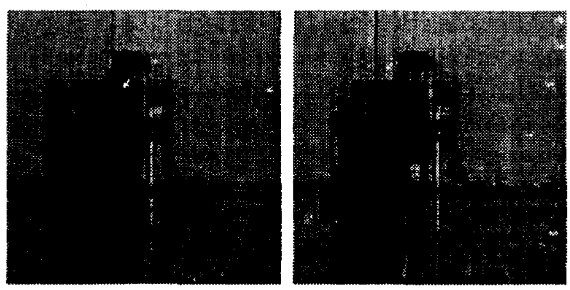

Figure 3: Approximated image $\tilde{R}$, Schein rank = 50 (left), and Schein rank = 100 (right)

Figure 4: Fuzzy bases **A** and **B**, Schein rank = 25

Figure 5: Fuzzy bases **A** and **B**, Schein rank = 50

Figure 6: Fuzzy bases **A** and **B**, Schein rank = 100

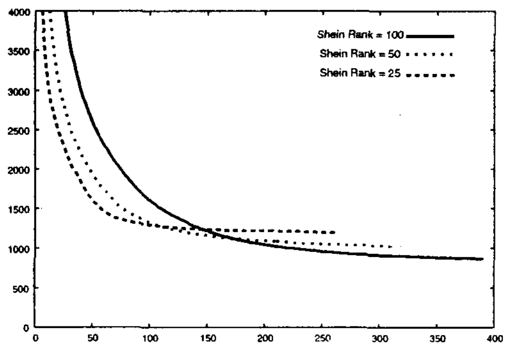

Figure 7: The value of the cost function Q with respect to *iter*

2.3 Extension Proposals

A lossy image compression method based on fuzzy relational structure (fuzzy relational equations) has been proposed in [4]. The component analysis method (I) can be a candidate of the design of appropriate coders in the compression scheme, in order to achieve an efficient compression. Furthermore, the fuzzy relation R can be regard as the concept of keywords in natural language. Therefore, the proposed component analysis method (I) can be also used as a visualization scheme in order to perform an efficient data mining in text processing.

3 Component Analysis Method (II)

3.1 Problem Formulation

Second component analysis based on eigen fuzzy sets [7] is shown. In this paper, the eigen fuzzy sets of max-min and min-max composition type are considered.

Let R be a fuzzy relation between a finite set $\mathbf{X}$ and A be a fuzzy subset of $\mathbf{X}$, i.e., $R \in F(\mathbf{X} \times \mathbf{X})$ and $A \in F(\mathbf{X})$.

[Max-min Composition Type]

An eigen fuzzy set associated with R, is a fuzzy set A such that

$$A = A \circ R, \tag{8}$$

where

$$A(x') = \max_{x \in \mathbf{X}}\{\min(A(x), R(x', x))\} \qquad \forall x' \in \mathbf{X}. \tag{9}$$

[Min-max Composition Type]

An eigen fuzzy set associated with R, is a fuzzy set A such that

$$A = A \bullet R, \tag{10}$$

where

$$A(x') = \min_{x \in \mathbf{X}}\{\max(A(x), R(x', x))\} \qquad \forall x' \in \mathbf{X}. \tag{11}$$

An overview of the eigen fuzzy equations (Eqs. (8) - (11)) is shown in Fig. 8. The eigen fuzzy set A corresponds to a component of the fuzzy relation R in terms of the ordered structure. The greatest eigen fuzzy set (GEFS) for max-min composition type, and the smallest eigen fuzzy set (SEFS) for min-max composition one, are presented using numerical example as follows:

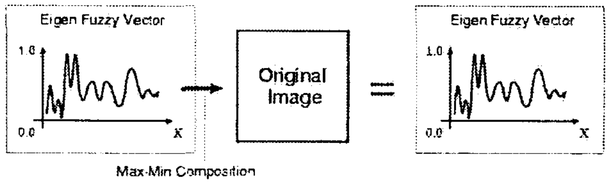

Figure 8: An overview of the component analysis (II)

[The greatest eigen fuzzy set]

Let R be given by

$$\begin{pmatrix} 0.3 & 0.5 & 0.9 \\ 0.6 & 0.8 & 0.1 \\ 0.1 & 0.2 & 0.4 \end{pmatrix}. \tag{12}$$

Step G-1: Find the greatest element in their corresponding columns (See Eq. (13)), and construct a candidate fuzzy set A_1 by using them (See Eq. (14)),

$$\begin{pmatrix} 0.3 & 0.5 & \boxed{0.9} \\ \boxed{0.6} & \boxed{0.8} & 0.0 \\ 0.1 & 0.2 & 0.4 \end{pmatrix}, \tag{13}$$

$$A_1 = [0.6, 0.8, 0.9]. \tag{14}$$

Step G-2: Calculate the composition of R and $A_n, (n = 1, \ldots)$ until the convergence of the fuzzy set A_n.

The composition of R and $A_n, (n = 1, \ldots)$ is shown as follows:

$$A_1 \circ R = [0.9, 0.8, 0.4] = A_2, \tag{15}$$

$$A_2 \circ R = [0.5, 0.8, 0.4] = A_3, \tag{16}$$

$$A_3 \circ R = [0.5, 0.8, 0.4] = A_4 = A_3. \tag{17}$$

Step G-3: When the fuzzy set A_n is converged, GEFS can be obtained as the fuzzy set A_n.

The SEFS of min-max composition type is also obtained by the dual process of the above GEFS algorithm [5].

3.2 Experiments of Image Component Analysis

A result of application of component analysis method (II) to images extracted from 'View Sphere Database' [8] is presented. The size of the original image is 256×256 pixels, therefore, $\mathbf{X} = \{x_1, x_2, \ldots, x_{256}\}$. By using image intensity normalization from $\{0, \ldots, 255\}$ into $[0, 1]$, the original images can be considered as fuzzy relations

$R \in F(\mathbf{X} \times \mathbf{X})$. An example of the original images, the GEFS, and the SEFS are shown in Fig. 9. Table 1 shows a comparison of each eigen fuzzy sets in terms of iteration number of the proposed algorithm, where the iteration number denotes the number of times of composition to calculate the transitive closure of fuzzy relation.

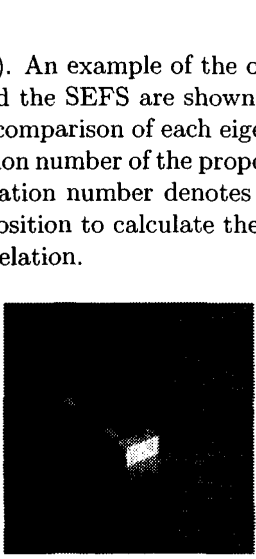

Figure 9: Original image (File-32-36)

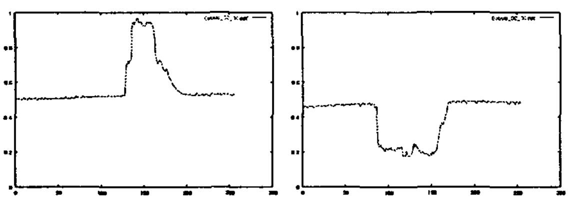

Figure 10: The GEFS (left) and the SEFS (right) of (File-32-36)

Table 1: Comparison of Iteration Number of the Algorithm

	Average	Max	Min
GEFS of Max-Min	5.52	14	1
SEFS of Min-Max	5.93	11	3

3.3 Extension Proposals

The proposed component analysis method (II), that is, eigen fuzzy sets of the original images can be used for are useful candidate of image feature, in order to perform an image retrieval.

4 Conclusions

In terms of ordered structure, two component analysis methods have been proposed by using fuzzy relational calculus. First component analysis is based on a decomposition of fuzzy relation (original image) into fuzzy bases. The decomposition is formulated as an optimization problem and it can be solved by using a gradient method. [3] Through the experiment using the test images of SIDBA, the results of the proposed component analysis method (I) is shown. Second method corresponds to the eigen fuzzy set of the fuzzy relation under the condition that the composition type is max-min and min-max, respectively. The proposed algorithm to obtain the eigen fuzzy sets of the image is based on the transitive closure of fuzzy relation. By using the image of 'View Sphere Database', the greatest and smallest eigen fuzzy sets are shown, and the iteration number of the algorithm (II) is also shown. The proposed analysis methods can be applied to various fields, e.g., an image compression based on fuzzy relational equations, an image retrieval, and a text mining. These developments should be a future study.

Acknowledgements

Support from is 'The Telecommunications Advancement Foundation' gratefully acknowledged.

References

[1] M. S. Bartlett, J. R. Movellan, and T. J. Sejinowski, "Face recognition by independent component analysis", *IEEE Transaction on Neural Networks*, Vol. 13, No. 6, 1450–1464, 2002.

[2] A. Leonardis, H. Bischof, and J. Maver, "Multiple Eigenspaces", *Pattern Recognition*, Vol. 35, No. 11, 2613–2627, 2002.

[3] H. Nobuhara, K. Hirota, W. Pedrycz, and S. Sessa, "Two Iterative Methods of Decomposition of a Fuzzy Relation for Image Compression/Decomposition Processing", Soft Computing, to appear.

[4] Nobuhara, H., Pedrycz, W. and Hirota, K.: Fast Solving Method of Fuzzy Relational Equation and its Application to Image Compression/Reconstruction. IEEE Transaction on Fuzzy Systems, Vol. 8, No. 3, (2000) 325–334

[5] Nobuhara, H. and Hirota, K.: Eigen Fuzzy Sets of Various Composition and their Application to Image Analysis, The 7th World Multiconference on Systems, Cybernetics and Informatics, Orland, Florida, USA,(2003) (to appear)

[6] W. Pedrycz, K. Hirota, and S. Sessa, "A Decomposition of Fuzzy Relations", *IEEE Transaction on Systems, Man, and Cybernetics, Part B*, Vol. 31, No. 4, 657-663, 2001.

[7] Sanchez, E.: Resolution of Eigen Fuzzy Sets Equations. Fuzzy Sets and Systems, Vol. 1, No. 1, (1978) 69–74

[8] http://www-prima.inrialpes.fr/