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Fuzzy Group Decision Making for Multiple Decision Maker-Multiple Objective Programming Problems

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***Abstract*— In this paper, we propose a fuzzy group decision making method for multiple decision maker-multiple objective programming problems to obtain the agreeable solution. In the proposed method, considering the vague nature of human subjective judgement, it is assumed that each of multiple decision makers has a fuzzy goal for each of his/her own objective functions. After eliciting the membership functions from the decision makers for their fuzzy goals, total M-Pareto optimal solution concept is defined in membership spaces in order to deal with multiple decision maker-multiple objective programming problems. For generating a candidate of the agreeable solution which is total M-Pareto optimal, the extended weighted minimax problem is formulated and solved for some weighting vector which is specified by the decision makers in their subjective manner. Given the total M-Pareto optimal solution, each of the decision makers must either be satisfied with the current values of the membership functions, or update his/her weighting vector. However, in general, it seems to be very difficult to find the agreeable solution with which all of the decision makers are satisfied perfectly because of the conflicts between their membership functions. In the proposed method, each of the decision makers is requested to estimate the degree of satisfaction for the candidate of the agreeable solution. Using the estimated values of satisfaction of each of the decision makers, the core concept is defined, which is a set of undominated candidates. The interactive algorithm is developed to obtain the agreeable solution which satisfies core conditions.**

I. Introduction

In order to deal with the real-world decision problems, many kinds of methods for multiple objective programming problems [6], [7] have been proposed to obtain a compromise or satisfactory solution, where a *single* decision maker is involved and he/she has his/her own multiple, noncommensurable and conflicting objective functions. However, in most real-world complicated decision situations, not a single decision maker but *multiple* decision makers are often involved. They may have their own multiple objective functions and each of them may have his/her own inherent point of view for his/her objective functions. In order to handle such multiple decision maker-multiple objective decision problems (MDMOP), many types of group decision making methods have been developed [2], [3], [4]
to find out the agreeable solution through the interaction between the decision makers.

In this paper, we propose interactive algorithms to obtain the agreeable solution of multiple decision makers in MDMOP on the basis of multiple objective programming techniques [6], [7]. In the proposed algorithm, considering the vague nature of human subjective judgement in MDMOP, it is assumed that each of multiple decision makers has a fuzzy goal [6], [9] for each of his/her own objective functions. It is also assumed that each of the decision makers may have the different decision power to reach the agreeable solution.

After eliciting the membership function for each of the objective functions from each of the decision makers in his/her subjective manner, he/she derives his/her own satisfactory solution from among his/her partial M-Pareto optimal solution set which is defined in his/her own membership space. Each of the satisfactory solutions of multiple decision makers is adopted as the ideal point for each of the decision makers in MDMOP. However, such satisfactory solutions can not be usually obtained simultaneously. In order to obtain the agreeable solution, the extended weighted minimax problem is formulated, where their own satisfactory solutions are used as ideal points, and the decision makers set their decision powers corresponding to the power relationships in the decision situation. For generating a candidate of the agreeable solution which is total M-Pareto optimal, the extended weighted minimax problem is solved for some weighting vector which is specified by the decision makers in their subjective manner. Given the total M-Pareto optimal solution, each of the decision makers must either be satisfied with the current values of the membership functions, or update his/her weighting vector. However, in general, it seems to be very difficult to find the agreeable solution with which all of the decision makers are satisfied perfectly because of the conflicts between their membership functions. In the proposed method, each of the decision makers is requested to estimate the degree of satisfaction for the candidate of the agreeable solution. Using the estimated values of satisfaction of each of the decision makers, the core concept is defined, which is a set of undominated candidates. The interactive algorithm is developed to obtain the agreeable solution which satisfies core conditions.

II. Fuzzy Multiple Decision Maker-Multiple Objective Programming Problems

We focus on the following multiple decision maker-multiple objective programming problem (MDMOP) where each of multiple decision makers ($DM_i, i = 1, \cdots, p$) has his/her own multiple objective functions $f_{ij}(\boldsymbol{x}), j = 1, \cdots, k_i$ which conflict each other, and all of the objective functions are defined in the common feasible region X.

[MDMOP]

$$\begin{aligned} DM_1 : \quad & \min \quad (f_{11}(\boldsymbol{x}), \cdots, f_{1k_1}(\boldsymbol{x})) \quad (1) \\ & \cdots\cdots\cdots\cdots\cdots \\ DM_p : \quad & \min \quad (f_{p1}(\boldsymbol{x}), \cdots, f_{pk_p}(\boldsymbol{x})) \end{aligned}$$

$$\text{subject to} \quad \boldsymbol{x} \in X = \{\boldsymbol{x} \in R^n \mid g_j(\boldsymbol{x}) \leq 0, \ j = 1, \cdots, m\}$$

By considering the vague nature of human subjective judgement, it is quite natural to assume that all of the decision makers may have fuzzy goals [6], [9] for their own objective functions. Through the interaction with the decision makers, these fuzzy goals can be quantified by eliciting the corresponding membership functions which are denoted by $\mu_{ij}(f_{ij}(\boldsymbol{x})), i = 1, \cdots, p, j = 1, \cdots, k_i$ respectively. Then, the corresponding fuzzy multiple decision maker-multiple objective programming problem (FMDMOP) can be expressed as the following form.

[FMDMOP]

$$\begin{aligned} DM_1 : \quad & \max_{\boldsymbol{x} \in X} \quad (\mu_{11}(f_{11}(\boldsymbol{x})), \cdots, \mu_{1k_1}(f_{1k_1}(\boldsymbol{x}))) \quad (2) \\ & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ DM_p : \quad & \max_{\boldsymbol{x} \in X} \quad (\mu_{p1}(f_{p1}(\boldsymbol{x})), \cdots, \mu_{pk_p}(f_{pk_p}(\boldsymbol{x}))) \end{aligned}$$

In order to deal with FMDMOP, we first introduce the following extended Pareto optimal concept which is defined in membership space for all of the decision makers.

[Definition 1]
In FMDMOP, $\boldsymbol{x}* \in X$ is said to be a total M-Pareto optimal solution for all of the decision makers, if and only if there does not exist another $\boldsymbol{x} \in X$ such that $\mu_{ij}(f_{ij}(\boldsymbol{x})) \geq \mu_{ij}(f_{ij}(\boldsymbol{x}*))$, $i = 1, \cdots, p, j = 1, \cdots, k_i$, with strict inequality holding for at least one i and one j.

Now, consider the decision situation in FMDMOP where only one decision maker (DM_i) is involved. Then, FMDMOP can be reduced to the following usual multiple objective programming problem for single decision maker DM_i.

[$FMOP_i$]

$$\max_{\boldsymbol{x} \in X} \quad (\mu_{i1}(f_{i1}(\boldsymbol{x})), \cdots, \mu_{ik_i}(f_{ik_i}(\boldsymbol{x}))) \qquad (3)$$

In $FMOP_i$, the partial M-Pareto optimal concept for the decision maker DM_i can be defined in membership space as follows.

[Definition 2]
In $FMOP_i$, $\boldsymbol{x}* \in X$ is said to be a partial M-Pareto optimal solution for the decision maker DM_i, if and only if there does not exist another $\boldsymbol{x} \in X$ such that $\mu_{ij}(f_{ij}(\boldsymbol{x})) \geq \mu_{ij}(f_{ij}(\boldsymbol{x}*))$, $j = 1, \cdots, k_i$, with strict inequality holding for at least one j.

In the following, denote the partial M-Pareto optimal solution set for the decision maker DM_i as $X_i^P, i = 1, \cdots, p$, and the total M-Pareto optimal solution set for all of the decision makers as X_0^P. In $FMOP_i$, the decision maker DM_i can obtain his/her own satisfactory solution from among X_i^P by applying multiple objective decision making methods [6], [7]. If all of the decision makers $DM_i, i = 1, \cdots, p$ obtain their own satisfactory solution $\boldsymbol{x}_i^s$ from among X_i^P,

$$(\mu_{i1}(f_{i1}(\boldsymbol{x}_i^s)), \cdots, \mu_{ik_i}(f_{ik_i}(\boldsymbol{x}_i^s))), i = 1, \cdots, p \qquad (4)$$

can be regarded as an ideal point in membership space for all of the decision makers, which reflects their own preferences. Unfortunately, such an ideal point is not usually attainable simultaneously because the membership functions of the decision makers conflict each other. In the next section, using the extended weighted minimax method, we propose interactive algorithms to obtain the agreeable solution in FMDMOP, which is, in a sense, close to the satisfactory solutions for each of the decision makers $\boldsymbol{x}_i^s, i = 1, \cdots, p$.

III. Interactive Algorithms for FMDMOP

After obtaining the satisfactory solutions $\boldsymbol{x}_i^s, i = 1, \cdots, p$ for $FMOP_i$, the decision makers have to find out their agreeable solution which reflects not only the preferences of the decision makers for their membership functions but also the power relationships between decision makers. In this paper, we assume that all of the decision makers find out their agreeable solution from among the total M-Pareto optimal solution set X_0^P. Then, the candidate of the agreeable solution is obtained by solving the following extended weighted minimax problem.

[Extended Weighted Minimax Problem]

$$\min_{\boldsymbol{x} \in X} \quad x_{p+1} + \rho \sum_{i=1}^{p} \sum_{j=1}^{k_i} \{\mu_{ij}(f_{ij}(\boldsymbol{x}_i^s)) - \mu_{ij}(f_{ij}(\boldsymbol{x}))\} \qquad (5)$$

$$\text{subject to} \quad w_{ij}\{\mu_{ij}(f_{ij}(\boldsymbol{x}_i^s)) - \mu_{ij}(f_{ij}(\boldsymbol{x}))\} \leq x_{p+1}, \quad (6)$$
$$i = 1, \cdots, p, j = 1, \cdots, k_i$$

where $w_{ij} \geq 0, i = 1, \cdots, p, j = 1, \cdots, k_i$ are weighting parameters specified by the decision makers $DM_i, i = 1, \cdots, p$, and ρ is a sufficiently small positive number.

In order to cope with the degrees of influence between the decision makers in the decision making processes, we introduce the concept of the decision powers which are defined by the weighting parameters w_{ij} as follows:

$$C_i = \sum_{j=1,\cdots,k_i} w_{ij}, \ i = 1, \cdots, p, \qquad (7)$$

where $C_i, i = 1, \cdots, p$ are decision powers of the decision maker DM_i in FMDMOP. It is easily understood that if only one decision power C_i is positive and all of the others is zero, then it is clear that FMDMOP is essentially reduced to $FMOP_i$. If all of the decision makers are equal in FMDMOP, the decision powers should be set as follows.

$$C_1 = C_2 = \cdots\cdots = C_p > 0 \qquad (8)$$

The relationships between the optimal solutions of the extended weighted minimax problem and the total M-Pareto optimal solutions can be characterized by the following theorem.

TABLE I
LINGUISTIC ASSESSMENTS

linguistic assessment	degree of satisfaction
perfectly satisfactory level	1
satisfactory level	$c(0.5 \le c \le 1)$
middle level	0.5
unsatisfactory level	$d(0.0 \le d \le 0.5)$
definitely unsatisfactory level	0.0

[Theorem 1]
If $x* \in X$ is an optimal solution of the extended weighted minimax problem for some weighting parameters $w_{ij} \ge 0, i = 1, \cdots, p, j = 1, \cdots, k_i$, then $x* \in X$ is a total M-Pareto optimal solution.

Now given the total M-Pareto optimal solution $x* \in X_0^P$ corresponding to some weighting parameters, if they are able to compromise at $x*$, then $x*$ can be regarded as one of the agreeable solutions. However, at least one of the decision makers can not compromise at $x*$, they have to find out the agreeable solution from among the total M-Pareto optimal solution set through the negotiation between them. In order to help the decision makers update their weighting parameters of the extended weighted minimax problem according to their own preferences, the trade-off information between membership functions seems to be very useful. Such trade-off rates between membership functions are easily obtained explicitly under some appropriate conditions [1].

[Theorem 2]
Let us assume that the extended weighted minimax problem has a unique optimal solution $x* \in X$ satisfying the conditions [1] that, $x*$ is a regular point, the second-order sufficiency conditions are satisfied at $x*$, and there are no degenerate constraints at $x*$. Also assume that the constraints (6) for the indices (i_1, j_1) and (i_2, j_2) are active, *i.e.*,

$$w_{ij}\{\mu_{ij}(f_{ij}(x_i^s)) - \mu_{ij}(f_{ij}(x*))\} = x_{p+1}^*. \tag{9}$$

Then it holds [8] that

$$\frac{\partial \mu_{i_1 j_1}(f_{i_1 j_1}(x*))}{\partial \mu_{i_2 j_2}(f_{i_2 j_2}(x*))} = -\frac{w_{i_2 j_2}\{\lambda_{i_2 j_2}^* + \rho\}}{w_{i_1 j_1}\{\lambda_{i_1 j_1}^* + \rho\}}, \tag{10}$$

where $\lambda_{i_1 j_1}^*, \lambda_{i_2 j_2}^* > 0$ are the Lagrangian multipliers corresponding to the constraints (6).

In general, it seems to be possible for each of decision makers to reply the degree of satisfaction for the given candidate of the agreeable solution instead of replying yes or no. From such a point of view, in this paper, let us assume that each of decision makers can specify the degree of satisfaction for the given candidate of the agreeable solution in their subjective manner according to Table 1.

If the decision maker can not specify the degree of satisfaction for the given candidate of the agreeable solution $x*$ according to Table 1, he/she may set his/her degree of satisfaction for $x* \in X_0^P$ on the basis of his/her own satisfactory solution $x_i^s \in X_i^P, i = 1, \cdots, p$ as follows.

$$a_i(x*) = \frac{\sum_{j=1}^{k_i} \mu_{ij}(f_{ij}(x*))}{\sum_{j=1}^{k_i} \mu_{ij}(f_{ij}(x_i^s))}, i = 1, \cdots, p \tag{11}$$

For the degrees of satisfaction $a_i(x*), i = 1, \cdots, p$ which are specified by the decision makers in their subjective manner according to Table 1, we define the agreeable solution concept in FMDMOP as follows, which depends on the majority parameter $r(0 \le r \le p)$ and the threshold level $\alpha(0.5 \le \alpha \le 1)$.

[Definition 3]
$x* \in X_0^P$ is said to be an agreeable solution in FMDMOP, if and only if the corresponding degrees of satisfaction $a_i(x*) \ge \alpha$ for at least r decision makers, where r is the majority parameter and $\alpha(0.5 \le \alpha \le 1)$ is the threshold level of the degree of satisfaction.

Now, we can construct an interactive algorithm based on the extended weighted minimax problem to obtain an agreeable solution from among the total M-Pareto optimal solution set.

[Interactive algorithm 1]
[Step 1]
Each of the decision makers (DM_i) obtains his/her own satisfactory solution $x_i^s \in X_i^P, i = 1, \cdots, p$ for the usual multiple objective programming problem $FMOP_i$ by applying multiple objective programming techniques.
[Step 2]
Set the decision powers $C_i, i = 1, \cdots, p$ which reflect the power relationships between the decision makers.
[Step 3]
Set the initial weighting parameters $w_{ij} = C_i/k_i, i = 1, \cdots, p, j = 1, \cdots, k_i$.
[Step 4]
Solve the extended weighted minimax problem and obtain the corresponding total M-Pareto optimal solution $x* \in X_0^P$ and the trade-off information between the membership functions.
[Step 5]
Each of decision makers $DM_i, i = 1, \cdots, p$ estimates the degree of satisfaction $a_i(x*)$ in his/her subjective manner. If the degrees of satisfaction $a_i(x*)$ for at least r decision makers are greater than or equal to the threshold level α, then stop. $x*$ is one of the agreeable solution in FMDMOP. Otherwise, go to Step 6.
[Step 6]
Considering the current values of the membership functions and trade-off information between membership functions, the decision makers who do not accept their current membership function values update their own weighting parameters, and go to Step 4.

By applying Interactive Algorithm 1, the decision makers may not be able to reach the agreeable solution for the case where the membership functions of the decision makers seriously conflict each other. In order to deal with such decision situations, we introduce, instead of the agreeable solution, the concept of the core [2] which is defined as a set of undominated alternatives. In the following, let us denote the candidates of the agreeable solution, which are generated by applying Interactive Algorithm 1, as $S = \{s_1, s_2, \cdots, s_q\}$. Then, the relative estimated values between the candidates $s_j, j = 1, \cdots, q$ can be defined as follows.

$$\begin{aligned} r^i_{j_1 j_2} &= \frac{a_i(s_{j_1})}{a_i(s_{j_1}) + a_i(s_{j_2})}, \\ i &= 1, \cdots, p, j_1, j_2 = 1, \cdots, q, j_1 \neq j_2 \quad (12) \end{aligned}$$

where the relative estimated value $r^i_{j_1 j_2}$ can be interpreted as the degree that the decision maker DM_i prefers s_{j_1} to s_{j_2}. Using the relative estimated values $r^i_{j_1 j_2}$, the core C [2] can be defined as follows.

[Definition 4]
The core C is defined as a set of undominated options, *i.e.*

$$\begin{aligned} C &= \{s_{j_2} \in S \mid \neg \exists s_{j_1} \text{ such that } r^i_{j_1 j_2} > 0.5 \\ & \quad \text{for at least } r \text{ decision makers}\}. \quad (13) \end{aligned}$$

By extending the above core concept, the fuzzy β-core [5] is defined as follows.

[Definition 5]
The fuzzy β-core C_β is defined as a set of undominated options which depends on the parameter $\beta (0.5 \leq \beta \leq 1)$, *i.e.*

$$\begin{aligned} C_\beta &= \{s_{j_2} \in S \mid \neg \exists s_{j_1} \text{ such that } r^i_{j_1 j_2} > \beta \\ & \quad \text{for at least } r \text{ decision makers}\}. \quad (14) \end{aligned}$$

For the parameters $0.5 \leq \beta_1 \leq \beta_2 \leq 1$, the following relation holds.

$$C \subset C_{\beta_1} \subset C_{\beta_2}.$$

In order to deal with the case where the decision makers can not find out their agreeable solution in Interactive Algorithm 1, we can construct the interactive algorithm to obtain the fuzzy β-core C_β.

[Interactive algorithm 2]
[Step 1]
Each of the decision maker (DM_i) obtains his/her own satisfactory solution $x^s_i \in X^P_i, i = 1, \cdots, p$ for the usual multiple objective programming problem FMOP_i by applying multiple objective programming techniques.
[Step 2]
Set the decision powers $C_i, i = 1, \cdots, p$ which reflect the power relationships between the decision makers.
[Step 3]
Set the iteration parameter q which is the maximum value of the interaction between the decision makers.
[Step 4]
Set the initial weighting parameters $w_{ij} = C_i / k_i, i = 1, \cdots, p, j = 1, \cdots, k_i$, the initial iteration number $t = 0$ and the initial set of the candidates of the agreeable solution $S = \phi$.
[Step 5]
Solve the extended weighted minimax problem and obtain the corresponding total M-Pareto optimal solution $x* \in X^P_0$ and the trade-off information between the membership functions. Update the iteration number $t \leftarrow t + 1$ and the set of the candidates $S \leftarrow S \cup \{x*\}$.
[Step 6]
Each of decision makers $\text{DM}_i, i = 1, \cdots, p$ estimates the degree of satisfaction $a_i(x*)$ in his/her subjective manner. If the degrees of satisfaction $a_i(x*)$ for at least r decision makers are greater than or equal to the threshold level α, then stop. $x*$ is one of the agreeable solution in FMDMOP. Otherwise, go to Step 7.
[Step 7]
The iteration number $t \geq q$, go to Step 8. Otherwise, considering the current values of the membership functions and trade-off information between membership functions, the decision makers who do not accept their current membership function values update their own weighting parameters, and go to Step 5.
[Step 8]
Denote each of the elements of the set S as $s_j, j = 1, \cdots, q$. For the degrees of satisfaction $a_i(s_j), i = 1, \cdots, p, j = 1, \cdots, q$, compute the relative estimated values $r^i_{j_1 j_2}, i = 1, \cdots, p, j_1, j_2 = 1, \cdots, q, j_1 \neq j_2$ and obtain the fuzzy β-core C_β.

IV. Conclusions

In this paper, we propose interactive algorithms to obtain the agreeable solution or the fuzzy *beta*-core of multiple decision makers in FMDMOP on the basis of multiple objective programming techniques. However, the decision makers may be burdened by the subjective estimation for the candidate of the agreeable solution. For the sake of relieving such a burden of the decision makers, some kinds of group's aggregated preference intensity concepts [3] should be investigated.

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