

Adaptive PID Controller for Nonlinear Systems using Fuzzy Model

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Abstract – This paper presents an adaptive PID control scheme for nonlinear system. TSK(Takagi-Sugeno-Kang) fuzzy model is used to estimate the error of control input, and the parameter of PID controller are adapted using the error. The parameters of TSK fuzzy model are also adapted to plant. The proposed algorithm allows designing adaptive PID controller which is adapted to the uncertainty of nonlinear plant and the change of parameters. The usefulness of the proposed algorithm is also certificated by the several simulations.

I. INTRODUCTION

With the development of industry, designing and expression at the control systems with high performance are also necessary, and its robustness and economic problem are also appeared important. It is specially important to know accurately the characteristics of system in development of control system. But it is very difficult to modeling and control correctly the system by the large scale and complexity of system which include some unreliabilities such as nonlinear, the change of parameter and disturbance from the outside. In this case, a general linear controller with fixed gain which designed based on linear model from one behavior point, couldn't correspond rapidly to the change of environment, and it is difficult to get satisfied result. Therefore in order to design a robust and stable control system it needs to approach adaptive control system, which can reduce the dependence on mathematical model and acquire good control result in kinetics change. In recently, many adaptive control methods are proposed, and continuous researches progress [1-4].

This paper proposes a new algorithm of adaptive PID controller for nonlinear systems using fuzzy model, and combine it with the technique of PID controller and fuzzy model. Model of system was estimated by using TSK fuzzy model which has a low number of fuzzy rule, a linear input-output equation with a constant term, and can also present a large class of nonlinear system with good accuracy [5-6]. The parameters of PID controller and fuzzy model are adapted to real time by using online learning algorithm[7]. The effectiveness of adaptive controller and advantage of linear controller are verified by the proposed computer simulation for nonlinear system.

This paper is composed of four important sections. Chapter 2 explains the adaptive PID control system, chapter 3 shows the results and discussions of simulation and chapter 4 shows conclusions.

II. ADAPTIVE PID CONTROL SYSTEM

The adaptive PID control system proposed in this paper first request TSK fuzzy model for nonlinear system, second estimate error of control input by using TSK fuzzy model, then adapt the parameter of PID controller to real time from that error. The parameter of TSK fuzzy model are also adapted to real time by the error from the comparison of actual output of plant and fuzzy model. Fig. 1 shows the adaptive PID control system structure.

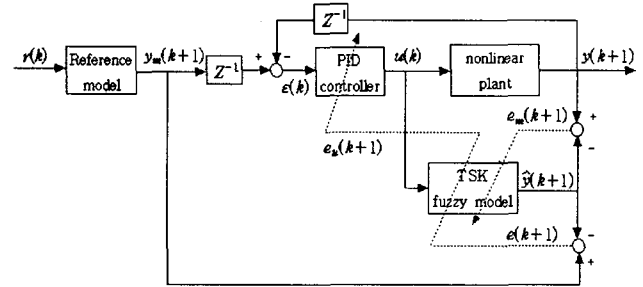


Fig. 1. The structure of adaptive PID control system

A. TSK fuzzy model

A TSK(Takagi-Sugeno-Kang) fuzzy model consists of TSK fuzzy rules as follows:

$$M^i : \text{if } z_1 \text{ is } F_1^i, \dots, z_l \text{ is } F_l^i \\ \text{then } y^i = a_0^i + a_1^i + \dots + a_n^i x_n \quad (1)$$

The output \hat{y} of fuzzy model is inferred as follows:

$$\hat{y} = \frac{\sum_{i=1}^r w^i(z) y^i}{\sum_{i=1}^r w^i(z)} \quad (2)$$

where r is the number of the fuzzy rules and $w^i(z)$ is the fitness of rules, and estimated as follows:

$$w^i(z) = \prod_{j=1}^m F_j^i(z^j) \quad (3)$$

where $F_j^i(z_j)$ is membership value of z_j in fuzzy set F_j^i .

$\sum_{i=1}^r w^i(z) = 1$ is presumed form now in this paper.

B. Online learning algorithm

In this paper online learning algorithm (Recursive Least Square algorithm) is expressed in equations (4) and (5).

$$\begin{aligned}\hat{\theta}(n+1) &= \hat{\theta}(n) + P(n+1)z(n+1) \times \\ &\quad [y(n+1) - z^T(n+1)\hat{\theta}(n)]\end{aligned}\quad (4)$$

$$P(n+1) = P(n) - \frac{P(n)z(n+1)z^T(n+1)P(n)}{1 + z^T(n+1)P(n)z(n+1)} \quad (5)$$

where $\hat{\theta}(0) = 0$, $P(0) = \alpha I$ (α is large value), $\hat{\theta}(n)$ is the estimated parameter, $y(n+1)$ is the desired value, $z^T(n+1)\hat{\theta}(n)$ is the actual output.

C. Design of adaptive PID controller

The required control input $u(k)$ computed by using PID control law is

$$\begin{aligned}u(k) &= u(k-1) + K_1(k)h_1(k) + \\ &\quad K_2(k)h_2(k) + K_3(k)h_3(k)\end{aligned}\quad (6)$$

where $K_1(k)$, $K_2(k)$, $K_3(k)$ are the parameters of PID controller,

$h_1(k) = \varepsilon(k) - \varepsilon(k-1)$, $h_2(k) = \varepsilon(k)$, $h_3(k) = \varepsilon(k) - 2\varepsilon(k-1) + \varepsilon(k-2)$. $\varepsilon(k) = y_m(k) - y(k)$ is the feedback error. $y_m(k)$ is output of reference model, $y(k)$ is actual output of nonlinear plant.

The controller parameter $K_1(k)$, $K_2(k)$, $K_3(k)$ are adapted to system. This method can express the nonlinear system as TSK fuzzy model, and adjust the parameters of controller by the estimated error of control input from the model.

The estimated error $e_u(k+1)$ of control input by using TSK fuzzy model is shown in equation (7).

$$e_u(k+1) = e(k+1) / \frac{\partial \hat{y}(k+1)}{\partial u(k)} \quad (7)$$

where $e_u(k+1) = u^*(k+1) - u(k+1)$, $e(k+1) = y_m(k+1) - y(k+1)$. $u^*(k+1)$ is an ideal control input which equal the output of system $y(k+1)$ to the output of reference model $y_m(k+1)$. $u(k+1)$ is computed by controller.

From the equation (2) and the presumption $\sum_{i=1}^r w^i(z) = 1$,

$$\frac{\partial \hat{y}(k+1)}{\partial u(k)} = \sum_{i=1}^r w^i(z) b^i \quad (8)$$

where b^i is parameter of input $u(k)$ in fuzzy model.

In equation (6), control input $u(k)$ is expressed by linear equation of parameters $K_1(k)$, $K_2(k)$, $K_3(k)$. The error $e_u(k+1)$ estimated by using TSK fuzzy model, and equations (4) and (5) adapt the parameters $K_1(k)$, $K_2(k)$, $K_3(k)$ of control input $u(k)$ to the real time.

D. Adaption of parameter of TSK fuzzy model

Because the error of control input was estimated by using TSK fuzzy model, and then the parameters of PID controller are adapted, so TSK fuzzy model must express exactly the system for good adapt the parameters of controller. Also the TSK fuzzy model must be adapted to the changes of system parameters. Furthermore, it is necessary to adapt the parameters of TSK fuzzy model to online.

Because TSK fuzzy model is adapted to the change of system parameter, so consequence parameters of fuzzy model are adapted to plant.

The output (2) of TSK fuzzy model can be expressed as follows:

$$\begin{aligned}\hat{y} &= \sum_{i=1}^r w^i(z) y^i / \sum_{i=1}^r w^i(z) \\ &= a_0^1 g_0^1 + a_1^1 g_1^1 + a_2^1 g_2^1 + \dots + a_n^1 g_n^1 + \\ &\quad a_0^2 g_0^2 + a_1^2 g_1^2 + a_2^2 g_2^2 + \dots + a_n^2 g_n^2 + \\ &\quad \vdots \\ &\quad a_0^r g_0^r + a_1^r g_1^r + a_2^r g_2^r + \dots + a_n^r g_n^r\end{aligned}\quad (9)$$

where, $g_j^i = w^i(z) x_j / \sum_{i=1}^r w^i(z)$ (only, $x_0 = 1$).

In equation (9), inference equation \hat{y} from the output of model is the formation of linear from the consequent parameters a_0^1 , a_1^1 , \dots , a_n^r . Therefore the error from the comparison of actual output of plant and fuzzy model, and algorithm equations (4) and (5) can adapt the parameters a_0^1 , a_1^1 , \dots , a_n^r to real time.

III. SIMULATION AND RESULTS

A nonlinear plant is described by the following different equation:

$$y(k+1) = \frac{y(k)y(k-1)y(k-2)u(k-1)(y(k-2)-1)+u(k)}{(1+y(k-2)^2+y(k-1)^2)} \quad (10)$$

The fuzzy model (11) represents exactly equation (10) in the region of $-1 \leq u(k) \leq 1$.

M^1 : if $y(k-1)$ is A_1 and $y(k-2)$ is B_1

$$\text{then } y(k+1)^1 = 0.008 + 0.974y(k) + 0.333y(k-1) - 0.279y(k-2) + 0.494u(k) - 0.468u(k-1)$$

M^2 : if $y(k-1)$ is A_1 and $y(k-2)$ is B_2

$$\text{then } y(k+1)^2 = 0.021 - 0.030y(k) + 0.058y(k-1) + 0.127y(k-2) + 0.419u(k) + 0.029u(k-1)$$

M^3 : if $y(k-1)$ is A_2 and $y(k-2)$ is C_1

$$\text{then } y(k+1)^3 = 0.004 - 0.173y(k) + 0.211y(k-1) + 0.009y(k-2) + 0.619u(k) + 0.111u(k-1)$$

M^4 : if $y(k-1)$ is A_2 and $y(k-2)$ is C_2

$$\text{then } y(k+1)^4 = 0.002 - 0.012y(k) + 0.005y(k-1) + 0.007y(k-2) + 0.977u(k) + 0.007u(k-1)$$

M^5 : if $y(k-1)$ is A_2 and $y(k-2)$ is C_3

$$\text{then } y(k+1)^5 = 0.003 - 0.099y(k) + 0.011y(k-1) - 0.020y(k-2) + 0.587u(k) + 0.058u(k-1)$$

M^6 : if $y(k-1)$ is A_3 and $y(k-2)$ is D_1

$$\text{then } y(k+1)^6 = -0.003 - 0.062y(k) + 0.010y(k-1) - 0.192y(k-2) + 0.380u(k) + 0.079u(k-1)$$

M^7 : if $y(k-1)$ is A_3 and $y(k-2)$ is D_2

$$\text{then } y(k+1)^7 = -0.112 + 0.131y(k) - 0.001y(k-1) + 0.144y(k-2) + 0.382u(k) - 0.138u(k-1)$$

(11)

Fuzzy sets of the fuzzy model are shown in Fig. 2.

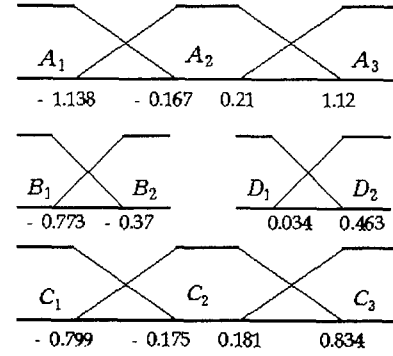


Fig. 2. Fuzzy sets of the fuzzy model (11)

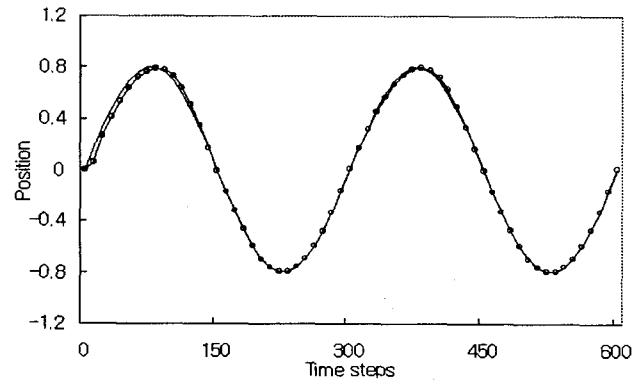


Fig. 3. Reference model and actual system output (— the reference model, \circ — the system)

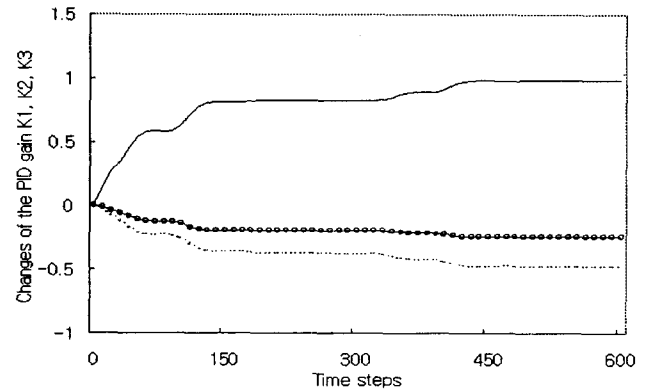


Fig. 4. Changes of the PID controller parameter (..... K_1 , — K_2 , \circ — K_3)

Fig. 3 shows the response of the adaptive control when the desired output of reference model is $y_m(k+1) = r(k) = 0.8 \sin(2\pi k / 300)$. The output of active system is tracking well to the output of reference model.

Fig. 4 shows the changes of the adaptive PID controller parameters during tracking sine wave in the case of Fig. 3.

The result of simulation for compared performance of controller when $r(k)$ is a rectangle wave, is shown in Fig. 5. The reference

model used in here is a third order system shown as follows:

$$y_m(k+1) = 2.4y_m(k) - 1.92y_m(k-1) + 0.512y_m(k-2) + 0.008u(k) \quad (12)$$

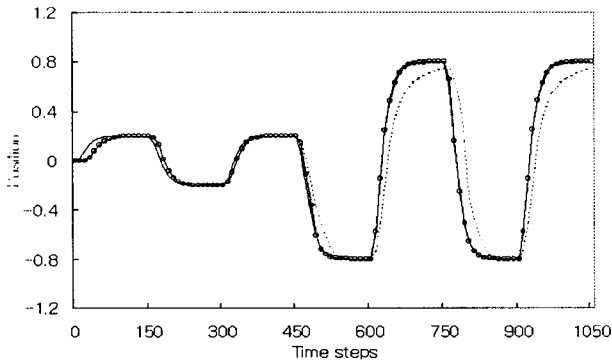


Fig. 5. Comparison of controller performance (—— Responses of the reference model, —○— Responses of the adaptive controller, Responses of the linear controller)

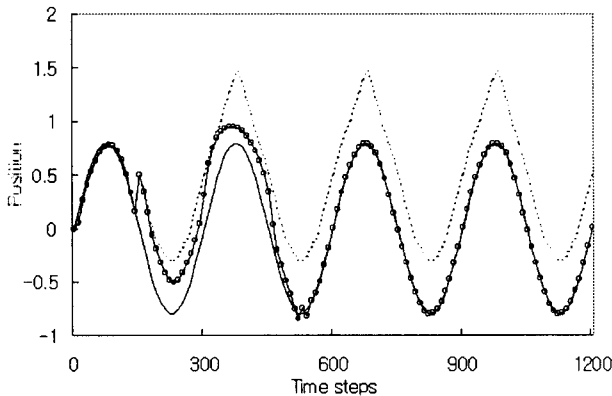


Fig. 6. Comparison of control performance (—— Responses of the reference model, —○— Responses of the adaptive fuzzy model, Responses of the nonadaptive fuzzy model)

In order to detect the adaptive effect of PID controller parameters, the results when continuous adaption of controller parameter act and interruption act to the adaptive parameter, are compared. The response from the continuous adaptive control is shown in Fig. 5. The dashed line is the result when the parameters of controller is fixed and no adaptive control acted after 300 step. When the adaption of controller parameters is interrupted, the result doesn't track well to the output of the reference model because of the linear system.

Fig. 6 shows the results of simulation for the adaptive usefulness of fuzzy model parameters. In this simulation the output of reference model is a sine wave $y_m(k+1) = r(k) = 0.8\sin(2\pi k/300)$ and a disturbance value of $w = 0.5$ is added to the plant at the time $k = 150$. In Fig. 6, —○— is the response when both of the

parameters of controller and fuzzy model were adapted, dashed line is response when the parameter of controller is adapted and the parameter of fuzzy model is fixed to value of equation (11). When the parameter of fuzzy model is adapted, the fuzzy model is adapted well and the output of system tracks well to the output of reference model though there is changes in the system. But it doesn't follow well to the output of reference model because the error of control input couldn't be estimated exactly if there is no adaption of the parameters of fuzzy model.

IV. CONCLUSIONS

This paper presented a new algorithm of designing adaptive PID controller using fuzzy model for improvement performance of nonlinear control system. The error of control input, which is estimated by using TSK fuzzy model, and adapt the parameters of PID controller to the real time. The actual output of the plant and the output of model were compared, and the parameters of TSK fuzzy model were also adapted to the real time.

The adaptive PID controller, proposed and designed by using simulation for nonlinear system, shows better effectiveness and advantage with fast convergence ability and robustness than the linear controller with fixed gain.

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