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Deadlock Points of Fuzzy Flip-Flops

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***Abstract*— A concept of deadlock point of fuzzy sequential circuit is proposed. There are six cases of fuzzy sequential circuits of 1 state and 1 input variables with deadlock points. Examples of each case are shown both in a form of characteristic equation and in a graphical illustration. As fuzzy sequential circuit with 1 state and 1 input variables, D and T fuzzy flip-flops are also characterized using the proposed concept. Thus one of the four types of D fuzzy flip-flops and T fuzzy flip-flop have a deadlock point 1/2.**

I. Introduction

Since the proposal of the concept of JK fuzzy flip-flop in 1989 [1], fuzzy sequential circuits, particularly fuzzy flip-flops, have been studied from both application oriented and theoretical points of view [5] [4] [6] [7]. A fuzzy sequential circuit is a fuzzy system with fuzzy output depending on the value of its current fuzzy output(fuzzy state), in other words, a fuzzy stateful system.

As the research area of fuzzy sequential circuits is growing, some problem involved in these systems need to be exposed. Hirota and Ozawa mentioned that JK fuzzy flip-flops extended from binary irreducible logical forms (minterm and maxterm expression) are not able to control their value in [0, 1] completely (i.e. set-type and reset-type JK fuzzy flip-flop) [1]. In order to avoid the problem, they have proposed another JK fuzzy flip-flop combining set-type and reset-type under the conditions $J \leq K$ or $J \geq K$. A few years later, unified forms of JK fuzzy flip-flop have been proposed as another way to solve the problem [2]. Mori and Mukaidono also mentioned this problem and have proposed another type of unified logical form of JK fuzzy flip-flop [4]. T fuzzy memory cells also have this problem at 1/2 as the values of their state, Virant et al. [8] have added external signal to the systems in order to set and reset system's value forcibly.

In this paper, a concept of deadlock of fuzzy sequential system is proposed in order to reveal the behavior of these problems. The concept of a deadlock point of a fuzzy sequential circuit is mentioned and the behavior of fuzzy sequential systems with 1 state and 1 input variables is analyzed. And thus deadlock points of these systems are categorized into 6 cases. The proposed concept is applied to 1 state, 1 input variables fuzzy flip-flops, D and T fuzzy flip-flops. As a result, one of the four types of D fuzzy flip-flops and the T fuzzy flip-flop have a deadlock point 1/2.

II. Concept of Deadlock of Fuzzy Sequential Circuit

The minterm and maxterm expressions of fuzzy logical function of T fuzzy flip-flop are,

$$Q(t+1) = (T(t) \textcircled{\scriptsize T} Q^{\textcircled{\scriptsize D}}(t)) \textcircled{\scriptsize S} (T^{\textcircled{\scriptsize D}}(t) \textcircled{\scriptsize T} Q(t)), \quad (1)$$

$$Q(t+1) = (T(t) \textcircled{\scriptsize S} Q(t)) \textcircled{\scriptsize T} (T^{\textcircled{\scriptsize D}}(t) \textcircled{\scriptsize S} Q^{\textcircled{\scriptsize D}}(t)), \quad (2)$$

respectively, where t is time variable, $T(t)$ and $Q(t)$ is input and state variable at time t, respectively.

Using Max-Min logical operation system $(1-\cdot, \wedge, \vee)$ as fuzzy logical operation system $(\cdot^{\textcircled{\scriptsize D}}, \textcircled{\scriptsize T}, \textcircled{\scriptsize S})$ in the above equations, these two equations (1) and (2) are equivalent in the same way under binary logic.

If the state $Q(t)$ is 1/2, then the next state $Q(t+1)$ is also 1/2 regardless of value of input $T(t)$. Once the T fuzzy flip-flop transits to state 1/2, it is not able to transit to other states in the future.

For analysis of such a behavior of a fuzzy sequential circuit, a concept of deadlock points of fuzzy sequential circuits is defined. In this paper, fuzzy sequential circuits with 1 state and 1 input variables, such as

$$q(t+1) = f(q(t), x(t)), \quad (3)$$

are dealt with, where $q \in [0,1]$ and $x \in [0,1]$ are a fuzzy state variable and a fuzzy input variable, respectively. D and T fuzzy flip-flops are examples of this.

Definition Deadlock points of fuzzy sequential circuit

In Eq. (3), $q(t)$ is called a deadlock point of f if and only if $q(t+1) = f(q(t), x(t)) = q(t)$ for all values of input variables x.

□

If a fuzzy sequential circuit has a deadlock point q, then once the system takes the value q, the system cannot change the state value to other than q.

Because the domain of a fuzzy logic variable is [0,1], the number of fuzzy states is infinite in general. If Max-Min logical operation system $(1-\cdot, \wedge, \vee)$ is used as fuzzy logical operation system $(\cdot^{\textcircled{\scriptsize D}}, \textcircled{\scriptsize T}, \textcircled{\scriptsize S})$, for any fuzzy logical functions f, g, $f \equiv g$ under Max-Min fuzzy logic $([0,1], 1-\cdot, \wedge, \vee)$ if and only if $f \equiv g$ under B-ternary logic $(\{0, 1/2, 1\}, 1-$

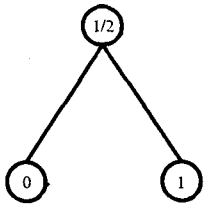

Fig. 1. Partially ordered relation of ambiguity

$\cdot, \wedge, \vee$) [3]. It is sufficient to check the value $\{0, 1/2, 1\}$ of the variables to determine the characteristic of a fuzzy logical function.

Definition Partially ordered relation of ambiguity [3].

Let $a, b \in [0,1]$, $a \overset{a}{\succeq} b$ if and only if $1/2 \geq a \geq b$ or $b \geq a \geq 1/2$.

Let $\boldsymbol{x}, \boldsymbol{y} \in [0,1]^n$ be fuzzy vectors of dimension n. Then $\{x_1, x_2, \cdots, x_n\}$, $\{y_1, y_2, \cdots, y_n\}$, $\boldsymbol{x} \overset{a}{\succeq} \boldsymbol{y}$ if and only if $x_i \overset{a}{\succeq} y_i$ for every $i \in \{0, \cdots, n\}$.

Let $f(\boldsymbol{x}), g(\boldsymbol{x})$ be fuzzy logical functions of n variables. Then $f \overset{a}{\succeq} g$ if and only if $f(\boldsymbol{x}) \overset{a}{\succeq} g(\boldsymbol{x})$ for all $\boldsymbol{x} \in [0,1]^n$

□

It is clear that 1/2 is the most ambiguous, and 0 and 1 are the least ambiguous. Figure 1 shows the Hasse diagram of the partially ordered relation of ambiguity.

Proposition (Monotonicity of Ambiguity) [3] Let $\boldsymbol{x}$, $\boldsymbol{y}$ be n-dimensional ternary vector $\{0, 1/2, 1\}^n$, and let $f : \{0, 1/2, 1\}^n \to \{0, 1/2, 1\}$ be a B-ternary logical function. Then

$$\boldsymbol{x} \overset{a}{\succeq} \boldsymbol{y} \Longrightarrow f(\boldsymbol{x}) \overset{a}{\succeq} f(\boldsymbol{y}).$$

From this proposition, B-ternary logical functions hold ambiguity.

III. Six Cases of Deadlock Points

In the previous section, it is mentioned that it is sufficient to show the deadlock points only under the value $\{0, 1/2, 1\}$ for variables of any fuzzy logical function in the case of Max-Min logical operation system. Hereafter, fuzzy systems with fuzzy negation, t-norm, and s-norm restricted to Max-Min logical operation system $(1 - \cdot, \wedge, \vee)$ are considered. Thus it is possible to consider 8 cases of deadlock points for any fuzzy logical function with 1 state variable.

1) deadlock = {0}
2) deadlock = {1}
3) deadlock = {1/2}
4) deadlock = {1/2,1}
5) deadlock = {0,1/2}
6) deadlock = {0,1/2,1}
7) deadlock = {0,1}
8) deadlock = $\emptyset$

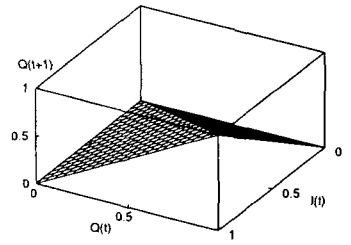

Fig. 2. Characteristics of Case 1

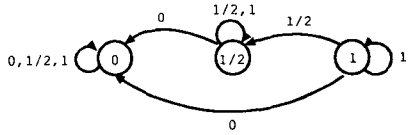

Fig. 3. State transition of Case 1

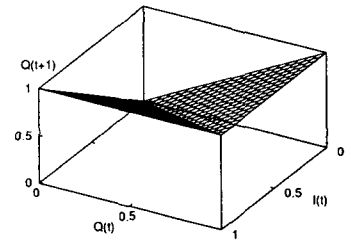

Fig. 4. Characteristics of Case 2

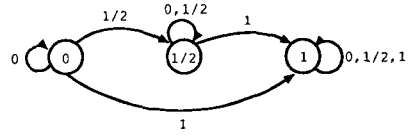

Fig. 5. State transition of Case 2

The last case shows that the logical function does not have any deadlock point, and case 8 cannot occur actually because of the monotonicity of the order of ambiguity. For each of case 1 through 6, the examples of fuzzy logical function with 1 input and 1 state variables are shown next:

$$\begin{aligned}
&\text{Case 1)} \quad q(t+1) = q \wedge x \\
&\text{Case 2)} \quad q(t+1) = q \vee x \\
&\text{Case 3)} \quad q(t+1) = q \wedge (x \vee (1-x)) \vee (1-q) \wedge x \wedge (1-x) \\
&\text{Case 4)} \quad q(t+1) = q \vee x \wedge (1-x) \\
&\text{Case 5)} \quad q(t+1) = q \wedge (x \vee (1-x)) \\
&\text{Case 6)} \quad q(t+1) = q
\end{aligned}$$

The characteristics and state transition of these logical functions are graphically illustrated in Fig. 2 through Fig. 13.

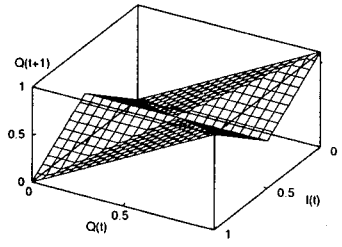

Fig. 6. Characteristics of Case 3

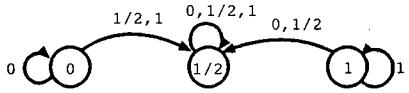

Fig. 7. State transition of Case 3

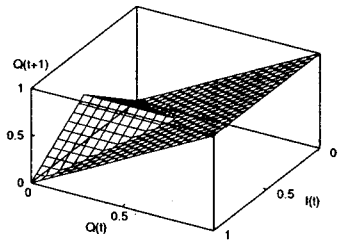

Fig. 8. Characteristics of Case 4

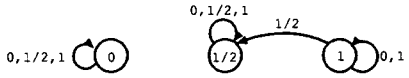

Fig. 9. State transition of Case 4

IV. Deadlock Points of Fuzzy Flip-Flops

Deadlock points of D and T flip-flops are investigated in this section.

A. D fuzzy flip-flop

Under binary logic, the characteristic function of D flip-flop is simplified as

$$Q(t+1) = D(t).$$

The truth table of D flip-flop is extended to that under B-ternary logic(Fig. 14).

When $(D(t), Q(t)) = (1/2, 0), (1/2, 1/2)$, or $(1/2, 1)$, $Q(t+1)$ is uniquely determined as 1/2 because there are both 0 and 1 in the neighbor values of $Q(t+1)$. Thus, for those cases, $Q(t+1)$ must be a more ambiguous value than both 0 and 1, that is 1/2.

When $(D(t), Q(t)) = (0, 1/2)$, $Q(t+1)$ can be one of 0 or 1/2, and when $(D(t), Q(t)) = (1, 1/2)$, $Q(t+1)$ can take one of 1/2 or 1 as values.

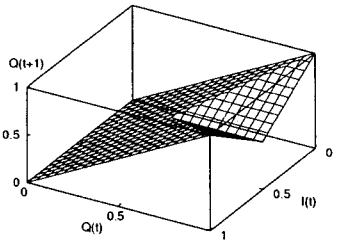

Fig. 10. Characteristics of Case 5

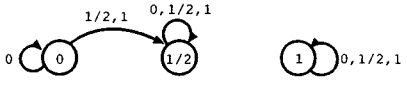

Fig. 11. State transition of Case 5

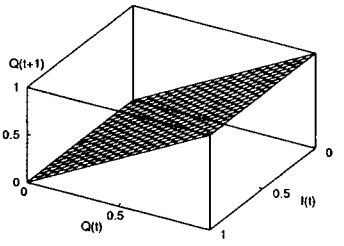

Fig. 12. Characteristics of Case 6

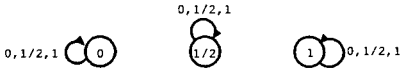

Fig. 13. State transition of Case 6

Q(t) / D(t)	0	1/2	1
0	0	1/2 / 0	0
1/2	1/2	1/2	1/2
1	1	1 / 1/2	1

Fig. 14. Truth table of D fuzzy flip-flop

Therefore in the extension of D flip-flop to Max-Min logic, there are four distinct types of characteristics, i.e.,

$$\begin{aligned}
\text{(D1)} \quad Q(t+1) &= D \wedge Q \vee D(1-Q), \\
\text{(D2)} \quad Q(t+1) &= D \wedge Q \vee D(1-Q) \\
&\quad \vee (1-D) \wedge Q \wedge (1-Q), \\
\text{(D3)} \quad Q(t+1) &= D, \\
\text{(D4)} \quad Q(t+1) &= D \vee (1-D) \wedge Q \wedge (1-Q).
\end{aligned}$$

(D3) is the same as the irreducible form under binary logic. The characteristics of each equation are shown in Fig. 15, Fig. 16, Fig. 17, and Fig. 18, respectively.

The characteristics of (D1), (D3), and (D4) have no deadlock points, whereas (D2) has the deadlock points 1/2.

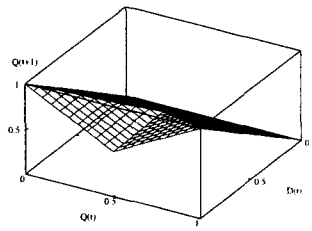

Fig. 15. Characteristics of (D1)

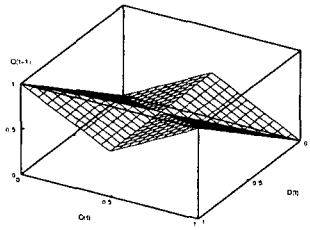

Fig. 16. Characteristics of (D2)

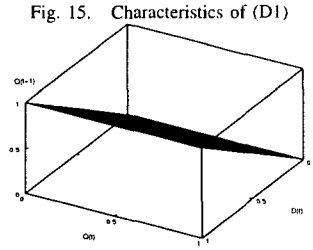

Fig. 17. Characteristics of (D3)

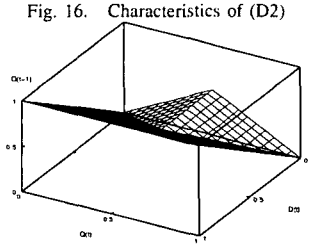

Fig. 18. Characteristics of (D4)

T(t) \ Q(t)	0	1/2	1
0	0	1/2	1
1/2	1/2	1/2	1/2
1	1	1/2	0

Fig. 19. Truth table of T fuzzy flip-flop

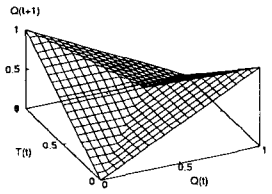

Fig. 20. Characteristics of T fuzzy flip-flop

B. T fuzzy flip-flop

The fuzzy extension of T fuzzy flip-flop under Max-Min logic is uniquely determined. The truth table of T flip-flop under B-ternary logic is shown in Fig. 19. The characteristics of T fuzzy flip-flop is shown in Fig. 20. T fuzzy flip-flop has deadlock points 1/2. Moreover there are no T fuzzy flip-flop cannot be used without a deadlock point. This fact means that T fuzzy flip-flop cannot be used without external set or reset signal likewise Virant's T fuzzy memory cell.

V. Conclusions

The concept of deadlock point of fuzzy sequential circuit is proposed. In the case of 1 state and 1 input variables of fuzzy sequential circuit, it is shown that there are six cases of deadlock points. This concept is applied for conventional fuzzy flip-flop with 1 state and 1 input variables, D and T fuzzy flip-flops. One of D fuzzy flip-flop and T fuzzy flip-flop have a deadlock point 1/2. This means if these circuits are used in fuzzy systems, the deadlock should be avoided using other design techniques, e.g., adding external set or reset signal.

This result gives the foundation of constructing design method of complex fuzzy systems and automatic design of fuzzy systems.

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