2003 International Symposium on Advanced Intelligent Systems
September 25-28, 2003, Jeju, Korea

Situation-Dependent Fuzzy Rating

Atsushi Hayashi

Graduate School of Systems and Information Engineering, University of Tsukuba

1-1-1 Tennodai, Tsukuba, Ibaraki 305-8573, JAPAN

email: atsushi@fhuman.esys.tsukuba.ac.jp

Takehisa Onisawa

Institute of Engineering Mechanics and Systems, University of Tsukuba

1-1-1 Tennodai, Tsukuba, Ibaraki 305-8573, JAPAN

email: onisawa@fhuman.esys.tsukuba.ac.jp

Abstract - **Fuzzy set expressing category in fuzzy rating, which is a kind of psychological scaling, is dependent on situations. This paper assumes that a mapping exists between fuzzy sets expressing categories in some situation and those expressing same categories in another situation. Fuzzy sets expressing categories in some situation are obtained by fuzzy sets expressing categories in another situation and the mapping between them. The usefulness of the present method is confirmed by the experiments comparing fuzzy sets obtained by the presented method with those identified directly by fuzzy rating. The normalized distance is used to compare both fuzzy sets and the experimental results show that the normalized distances between both fuzzy sets are enough small and that the presented method is useful for psychological scaling.**

I. INTRODUCTION

Linguistic expressions such as "he is very tall" or "it is a little cold" are often used to categorize the physical quantity rather than expressions such as "he is 180cm tall" or "it is 10 degrees Celsius". The former linguistic expressions are considered as expressions of the category of psychological scale [1]. Fuzzy rating [2], which is a kind of psychological scaling, is useful when the category of psychological scale is expressed by linguistic expressions, e.g., "very tall", since fuzziness is included in the meaning of the linguistic expression of the category. Therefore, the meanings of categories are expressed well by fuzzy sets in fuzzy rating.

By the way, psychological scale is dependent on situations. For example, the meaning of linguistic expression "very tall" as the rating of high school students' height is not the same as that of linguistic expression "very tall" as the rating of NBA players' height. A fuzzy set expressing some category in some situation is different from that expressing the same category in another situations. However, psychological scale in some situation is usually analogous to psychological scales in other situations in the sense that some relation exists between these psychological scales. Therefore, in fuzzy rating, fuzzy sets expressing categories in some situation are also analogous to those expressing same categories in another situations and the former are obtained by the latter using the analogy. Although there have been many studies on situation-dependent fuzzy sets [3][4][5], they obtain fuzzy sets directly without considering the relation between situations.

Our previous study [6] defines situations as the union of support sets of fuzzy sets and describes how a fuzzy set expressing some category in some situation is obtained by a fuzzy set expressing the same category in another situation and a mapping between situations.

However, the union of support sets of fuzzy sets should be considered as the domain evaluated by fuzzy rating rather than the situation. Therefore, this paper defines the domain of physical quantities evaluated by fuzzy rating in some situation and assumes that a mapping exists between the domains. This paper also discusses how fuzzy sets expressing categories in some situation are obtained using those expressing the same categories in another situation and the mapping between situations. The usefulness of the discussion is confirmed by the experiments comparing fuzzy sets obtained by the presented method with those obtained directly by fuzzy rating.

Basic concepts about situation-dependent fuzzy rating are mentioned in chapter 2. Chapter 3 explains the experiments and discusses the experimental results. Conclusions are mentioned in the final chapter.

II. BASIC CONCEPTS

A. Definition and assumption

Psychological scaling is a method that connects physical stimulus and psychological quantities. However, psychological quantities are often categorized and expressed by ordinal scale, e.g. numerical scale, graphic scale [7]. Fuzzy rating is a kind of psychological scaling and in this paper, physical quantities, e.g. "180cm", are categorized by fuzzy sets, e.g. "tall", " a little short".

Definition 1: Evaluation Interval

Let a set R_S be a domain of physical quantities evaluated by fuzzy rating in situation S . R_S is called "Evaluation Interval" here. The evaluation interval is a closed interval, and elements of the evaluation interval are physical quantities.

Definition 2: Category

Let the evaluation interval be categorized by c_i $(i=1,2,\cdots,n)$. Fuzzy Sets $\tilde{C}_i^S$ expressing categories c_i $(i=1,2,\cdots,n)$ in situation S are defined by formula (1).

$$\mu_{\tilde{C}_i^S} : R_S \to [0,1]\ (i=1,2,\cdots,n) \quad (1)$$

Assumption: Mapping between evaluation intervals

Let R_S be the evaluation interval in situation S, and R_T be another evaluation interval in situation T. It is assumed that a mapping f between evaluation intervals exists.

Definition 3: Fuzzy sets in some situation and those in another situation

Let $\tilde{C}_i^S$ and $\tilde{C}_i^T$ be fuzzy sets expressing categories c_i $(i=1,2,\cdots,n)$ in situation S and in situation T, respectively, where $\tilde{C}_i^S$ and $\tilde{C}_i^T$ are fuzzy set on evaluation intervals R_S in situation S and R_T in situation T, respectively. Let a mapping f be a mapping between R_S and R_T. Fuzzy sets $\tilde{C}_i^T = f(\tilde{C}_i^S)$ $(i=1,2,\cdots,n)$ are defined by formula (2) using the extension principle [8], where $f(\tilde{C}_i^S)$ is a set mapped from $\tilde{C}_i^S$.

$$\mu_{f(\tilde{C}_i^S)}(t) = \begin{cases} \bigvee_{s\in f^{-1}(t)} \mu_{\tilde{C}_i^S}(s) , & t \in f(R_S) \\ 0 , & t \notin f(R_S) \end{cases} \qquad (2)$$

$(i=1,2,\cdots,n)$

where $f^{-1}(t)=\{s \mid t=f(s), t\in f(R_S)\}$, and $f(R_S)$ is a set mapped from R_S.

Figure 1 explains the evaluation interval, categories and a mapping.

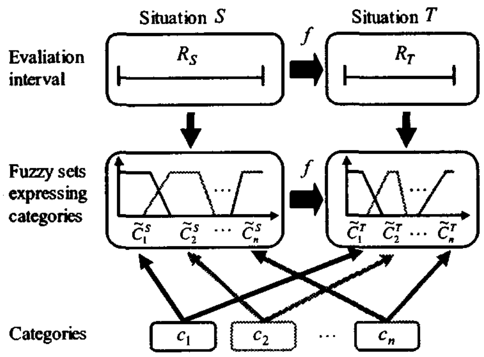

Figure 1 Evaluation interval, categories and mapping

B. Method to obtain fuzzy sets

Using above definitions and assumption, fuzzy sets in some situation are obtained by fuzzy sets in another situation and a mapping between evaluation intervals. For example, let R_S be the evaluation interval in the situation "We are talking about NBA players", and R_T be the evaluation interval in the situation "We are talking about junior high school students". R_S and R_T are the domain of height considered as NBA player's height and the domain of height considered as junior high school students' height, respectively.

Let categories c_i $(i=1,2,\cdots,n)$ categorize evaluation intervals and let fuzzy sets $\tilde{C}_i^S$ $(i=1,2,\cdots,n)$ express categories c_i in situation S. For example, when category c_1 is category "very tall", fuzzy set $\tilde{C}_1^S$ expresses "very tall" in situation S. Fuzzy sets $\tilde{C}_i^T$ $(i=1,2,\cdots,n)$ expressing categories c_i in situation T are obtained by fuzzy sets $\tilde{C}_i^S$ $(i=1,2,\cdots,n)$ and a mapping f between R_S and R_T.

III. EXPERIMENTS

Subjects experiments are performed in order to confirm that fuzzy sets obtained by presented method are useful as psychological scale. The experiments are done by 10 subjects, who are graduate or undergraduate students in the University of Tsukuba. Eight of them are male and others are female.

In the experiments, Fuzzy sets obtained by the presented methods are compared with those obtained directly by fuzzy rating.

A. Fuzzy rating

As shown in Table 1, six situations are presented to subjects for fuzzy rating. Fuzzy ratings about "Temperature" are performed in situation 1, 2, 3 and those about "Height" are performed in situation 4, 5, 6.

Table 1 Situations used in fuzzy rating

Situation 1	You are staying outdoors in summer.
Situation 2	You are staying outdoors in winter.
Situation 3	You are talking about climate through a year.
Situation 4	You are talking about first graders in junior high school.
Situation 5	You are talking about sixth graders in junior high school.
Situation 6	You are talking about junior high school students.

Five categories about "Temperature" and "Height" are chosen by the subjects as shown in Tables 2 and 3, respectively.

Let categories about "Temperature" be expressed as $t_i\,(i=1,2,\cdots,5)$ and categories about "Height" be expressed as $h_i\,(i=1,2,\cdots,5)$. Although various categories are chosen by subjects, a subject uses the same categories for all situations in fuzzy rating about "Temperature" or "Height" through the experiments. For example, Subject 3 uses categories "very cold", "a little cold", "average", "a little hot" and "very hot" through fuzzy rating about "Temperature".

Table 2 Categories related to "Temperature" chosen in fuzzy rating

Subjects	t_1	t_2	t_3	t_4	t_5
1	Extremely cold	A little cold	Good temperature	A little hot	Extremely hot
2	Cold	A little cold	Average	A little hot	Hot
3	Very cold	A little cold	Average	A little hot	Very hot
4	Extremely cold	A little cold	Average	A little hot	Extremely hot
5	Very cold	A little cold	Neutral	A little hot	Very hot
6	Very cold	A little cold	Neutral	A little hot	Very hot
7	Very cold	A little cold	Average	A little hot	Very hot
8	Very cold	A little cold	Average	A little hot	Very hot
9	Very cold	A little cold	Neutral	A little hot	Very hot
10	Very cold	A little cold	Average	A little hot	Very hot

Table 3 Categories related to "Height" chosen in fuzzy rating

Subjects	h_1	h_2	h_3	h_4	h_5
1	Very short	Slightly short	Average	Slightly tall	Very tall
2	Short	A little short	Average	A little short	Tall
3	Extremely short	Slightly short	Average	Slightly tall	Extremely short
4	Extremely short	A little short	Average	A little short	Extremely short
5	Very short	Slightly short	Normal	Slightly tall	Very tall
6	Very short	A little short	Neutral	A little short	Very tall
7	Very short	A little short	Average	A little short	Very tall
8	Very short	A little short	Average	A little short	Very tall
9	Very short	A little short	Average	A little short	Very tall
10	Very short	A little short	Average	A little short	Very tall

Fuzzy sets expressing categories in situations are identified by the BASE method [9]. In the BASE method assuming that the membership function of a fuzzy set is a trapezoidal one, subjects evaluate elements in the evaluation interval with a three-scale point whether elements belong to a fuzzy set completely or not, or elements belong to a fuzzy set to some extent. This method identifies a support set and a 1-level set of a fuzzy set indirectly using subjects evaluation data.

B. Identification of mapping between evaluation intervals

The following four functions are considered as mappings since these functions are simple.

$$f(x) = a_1 x + a_0 \tag{3}$$

$$f(x) = b_2 x^2 + b_1 x + b_0 \tag{4}$$

$$f(x) = c_2 \log(x + c_1) + c_0 \tag{5}$$

$$f(x) = d_2 e^{d_1 x} + d_0 \tag{6}$$

Parameter values (a_i, b_j, c_j, d_j, i = 0,1, j = 0,1,2) are identified by the least-mean-square method [10] using 1-level sets and support sets of fuzzy sets identified by the BASE method. Fuzzy sets used for the identification of parameter values of mappings are those expressing categories t_1, t_4 and t_5 about "Temperature" and those expressing categories h_1, h_4 and h_5 about "Height".

C. Comparison of fuzzy sets

Fuzzy sets expressing categories about "Temperature" or "Height" in some situations, which are obtained by the presented method, are compared with those obtained by fuzzy rating, where the comparison is based on the normalized distance [11].

The normalized distance between fuzzy sets $\tilde{A}$ and $\tilde{B}$ is defined by formula (7). Figure 2 explains the normalized distance.

$$d(\tilde{A},\tilde{B}) = \frac{1}{2(\gamma_2 - \gamma_1)} \int_0^1 (|a_1(\alpha) - b_1(\alpha)| + |a_2(\alpha) - b_2(\alpha)|) d\alpha \tag{7}$$

where $\tilde{A}_\alpha = [a_1(\alpha), a_2(\alpha)]$ and $\tilde{B}_\alpha = [b_1(\alpha), b_2(\alpha)]$ are α-cut sets of $\tilde{A}$ and $\tilde{B}$, respectively

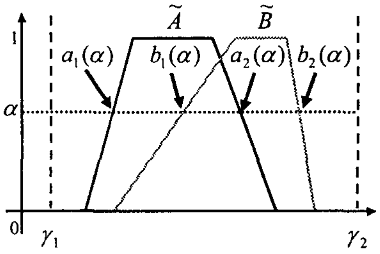

Figure 2 Normalized Distance

The normalized distance ranges from 0 to 1 and if the normalized distance is 0, the fuzzy set $\tilde{A}$ is equivalent to the fuzzy set $\tilde{B}$. In the experiments, γ_1 and γ_2 are the minimum and the maximum of the evaluation interval, respectively.

Fuzzy sets expressing categories t_2 and t_3 about "Temperature", and those expressing categories h_2 and h_3 about "Height" are used for the comparison, where these fuzzy sets are not used for the identification of the parameter values of mappings.

D. Results and remarks

Normalized distances between fuzzy sets expressing categories t_1, t_4, t_5, h_1, h_4 and h_5 obtained by fuzzy rating and those obtained by the presented method are shown in Table 4, where only fuzzy sets expressing categories t_1, t_4, t_5, h_1, h_4 and h_5 are used for the identification of parameter values of mappings.

Table 4 Averages of normalized distances for each mapping

(a) Attribute "Temperature"

Mapping	t_1	t_4	t_5
$a_1x + a_0$	0.049	0.034	0.032
$b_2x^2 + b_1x + b_0$	0.024	0.027	0.024
$c_2 \log(x + c_1) + c_0$	0.153	0.076	0.105
$d_2e^{d_1x} + d_0$	0.101	0.152	0.298

(b) Attribute "Height"

Mapping	h_1	h_4	h_5
$a_1x + a_0$	0.033	0.041	0.041
$b_2x^2 + b_1x + b_0$	0.030	0.032	0.027
$c_2 \log(x + c_1) + c_0$	0.037	0.043	0.043
$d_2e^{d_1x} + d_0$	0.033	0.047	0.049

It is found that the averages of normalized distances by the function (4) are the shortest among all functions. Therefore, the function (4) with identified parameter values is used as mappings between evaluation intervals.

Table 5 shows the averages of normalized distances in each situation between fuzzy sets expressing categories t_2, t_3, h_2 and h_3 obtained by fuzzy rating and those obtained by the presented methods.

Table 6 shows the temperature differences or height differences, which are obtained by formula (8).

Table 5 Averages of normalized distances for each situation

(a) Attribute "Temperature"

Situation	t_2	t_3
1	0.057	0.056
2	0.051	0.043
3	0.060	0.053

(b) Attribute "Height"

Situation	h_2	h_3
4	0.047	0.058
5	0.039	0.053
6	0.047	0.049

Table 6 Temperature differences or height differences correspond to normalized distances in Table 5

(a) Attribute "Temperature" ($^{\circ}C$)

Situation	t_2	t_3
1	1.2	1.2
2	1.3	1.1
3	2.5	2.3

(b) Attribute "Height" (cm)

Situation	h_2	h_3
4	2.1	2.6
5	1.6	2.2
6	3.6	3.7

$$D = (\Gamma_2 - \Gamma_1) \cdot \Delta \quad (8)$$

where Γ_1 and Γ_2 are the average values of the minimum and the maximum of the evaluation intervals among subjects, respectively, and Δ is the average values of normalized distances shown in Table 5.

In order to confirm that fuzzy sets obtained by the presented method are useful for psychological scale, temperature differences and height differences shown in Table 6 are compared with the width of fuzzy sets obtained by fuzzy rating, where the widths of a fuzzy set are defined by Figure 3. If the former is less than the latter, fuzzy sets obtained by the presented method are allowable as psychological scale.

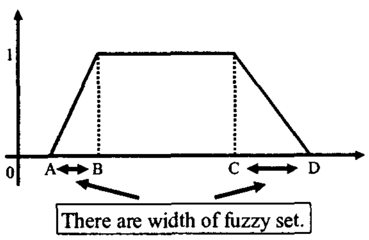

Figure 3 Widths of a fuzzy set

Table 7 shows the average values of widths of fuzzy sets expressing categories t_2, t_3, h_2 and h_3 in each situation among all subjects, where the fuzzy sets are obtained by fuzzy rating.

Comparing Table 6 with Table 7, it is found that the differences shown in Table 6 are less than the widths of fuzzy sets A-B and C-D in each situation and that the differences shown in Table 6 are allowable. Therefore, fuzzy sets obtained by the presented method can express categories in each situation well.

Table 7 Averages of widths of fuzzy sets obtained by fuzzy rating

(a) Category t_2 ($^{\circ}C$)

Situation	A-B	C-D
1	2.0	1.9
2	2.2	2.5
3	3.6	3.2

(b) Category t_3 ($^{\circ}C$)

Situation	A-B	C-D
1	1.9	2.1
2	2.1	2.6
3	3.5	3.4

(c) Category h_2 (cm)

Situation	A-B	C-D
4	3.6	4.4
5	3.6	4.8
6	6.6	7.3

(d) Category h_3 (cm)

Situation	A-B	C-D
4	4.9	4.7
5	3.7	3.6
6	8.6	7.4

IV. CONCLUSIONS

This paper discusses a situation-dependent fuzzy rating. It is assumed that a mapping between the evaluation intervals exists. The method is proposed that fuzzy sets in some situation are obtained by fuzzy sets in another situation and a mapping between evaluation intervals.

The usefulness of the presented method is confirmed by some experiments that compare fuzzy sets obtained by the present method with those obtained by fuzzy rating. The normalized distance between fuzzy sets is used in the comparison. Experimental results show that the difference is allowable. Therefore, situation-dependent fuzzy sets expressing categories are obtained by the presented method.

In a future, the applications of the present method to fuzzy relation and fuzzy reasoning are considered.

REFERENCES

[1] Shizuhiko Nishisato. *Applied psychological scaling*, Seishinshobo, Tokyo, 1975 (in Japanese).

[2] Ayumi Yoshikawa. Fuzzy Rating and Membership Function Identification Methods, *Journal of Japan Society for Fuzzy Theory and Systems*, Vol.10, No.2, pp.184-192 (1998) (in Japanese)

[3] Tomohiro Takagi, Toru Yamaguchi, Michio Sugeno. Conceptual fuzzy sets and its realization using associative memories, *7th Fuzzy system symposium*, pp.359-362, 1991. (in Japanese)

[4] Haruhiko Takeuchi. A Technique on Adjusting the System's Knowledge of Linguistic Hedges to Human Naïve Thinking, *Joint 9th IFSA World Congress and 20th NAFIPS International Conference*, pp.118-123, 2001.

[5] Alf.C.Zimmer. A model for the interpretation of verbal predictions, *International Journal of Man-Machine Studies*, Vol.20, pp.121-134, 1984.

[6] Atsushi Hayashi, Takehisa Onisawa. A study on situation-dependent fuzzy sets, *Proceeding of 1st International Conference on Soft Computing and Intelligent Systems*, 25B1-1, 2002.

[7] Yoshihisa Tanaka, *Organon of psycholozy 16 composition of scale*, University of Tokyo Press, Tokyo, 1973 (in Japanese).

[8] Japan Society for Fuzzy Theory and Systems. *Fuzzy and soft computing handbook*, Kyoritsu shuppan Co., Ltd, Tokyo 2000. (in Japanese)

[9] Ayumi Yoshikawa, Daisuke Fujimoto, Takeshi Nishimura. Objective and subjective comparison between two methods for identifying membership functions: computerized fuzzy graphic rating scale and boundary asymptotic estimation method, *11th Fuzzy system symposium*, pp.405-408, 1995. (in Japanese)

[10] Michiaki Kawaguchi. *Introduction to multivariate analysis I*, Morikita shuppan Co., Ltd, Tokyo, 1973. (in Japanese)

[11] Arnold Kaufmann, Madan M.Gupta. *Introduction to fuzzy arithmetic*, Van nostrand reinhold company, New York, 1985.