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AM-14

Singular Representation and Finite Element Methods

Let Ω be a bounded, open, and polygonal domain in R^2 with re-entrant corners. We consider the following Partial Differential Equations:

$$\begin{cases} (I-\nabla\nabla\cdot+\nabla^{\perp}\nabla\times)\mathbf{u} &= \mathbf{f} \text{ in } \Omega, \\ \mathbf{n}\cdot\mathbf{u} &= 0 \text{ on } \Gamma_N, \\ \nabla\times\mathbf{u} &= 0 \text{ on } \Gamma_N, \\ \boldsymbol{\tau}\cdot\mathbf{u} &= 0 \text{ on } \Gamma_D, \\ \nabla\cdot\mathbf{u} &= 0 \text{ on } \Gamma_D, \end{cases} \qquad (1)$$

where the symbol $\nabla\cdot$ and ∇ stand for the divergence and gradient operators, respectively; $\mathbf{f}\in L^2(\Omega)^2$ is a given vector function; $\partial\Omega=\Gamma_D\cup\Gamma_N$ is the partition of the boundary of Ω; $\mathbf{n}$ is the outward unit vector normal to the boundary and $\boldsymbol{\tau}$ represents the unit vector tangent to the boundary oriented counterclockwise. For simplicity, assume that both Γ_D and Γ_N are nonempty. Denote the curl operator in R^2 by

$$\nabla\times=(-\partial_2,\partial_1)$$

and its formal adjoint by

$$\nabla^{\perp}=\begin{pmatrix}\partial_2\\ -\partial_1\end{pmatrix}.$$

Consider a weak formulation(WF); Find $\mathbf{u}\in V$ such that

$$a(\mathbf{u},\mathbf{v}):=(\mathbf{u},\mathbf{v})+(\nabla\cdot\mathbf{u},\nabla\cdot\mathbf{v})+(\nabla\times\mathbf{u},\nabla\times\mathbf{v})=(\mathbf{f},\mathbf{v}),\ \forall\ \mathbf{v}\in V. \qquad (2)$$

We assume there is only one singular corner.

There are many methods to deal with the domain singularities. We introduce them shortly and we suggest a new Finite Element Methods by using Singular representation for the solution.