회전 크레인의 안정성 해석

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회전크레인의 비선형 운동방정식

$$\ddot{\theta}_1 = \frac{K_1}{\tau_1} v_1 - \frac{1}{\tau_1} \dot{\theta}_1 \quad (8) \qquad \ddot{\beta} = \frac{r_2 K_2}{R' \tau_2} v_2 - \frac{R' \dot{\beta} + \dot{\beta}^2 R'}{\tau_2} \quad (9)$$

$$\ddot{L} = \frac{r_3 K_3}{\tau_3} v_3 - \frac{1}{\tau_3} \dot{L} \quad (10) \qquad \ddot{X} = -\frac{g(X-x)}{L} \quad (14)$$

$$\ddot{Y} = -\frac{g(Y-y)}{L} \quad (15)$$

회전크레인의 리아푸노브 함수

$$V(x) = \frac{1}{2}(a x_2^2 + b x_4^2 + c x_6^2) \quad (26)$$

$$\dot{V}(x) = a x_2 \left(\frac{K_1}{\tau_1} u_1 - \frac{1}{\tau_1} x_2\right) + b x_4 \left(\frac{r_2 K_2}{R' \tau_2} u_2 - \frac{R' x_4 + x_4^2 R'}{\tau_2}\right) + c x_6 \left(\frac{r_3 K_3}{\tau_3} u_3 - \frac{1}{\tau_3} x_6\right) \quad (27)$$

회전크레인 구동모터의 구속조건

$$\text{크레인 모터} : x_2 \left(\frac{K_1}{\tau_1} u_1 - \frac{1}{\tau_1} x_2\right) = x_2 \dot{x}_2 \le 0$$

$$\text{붐 모터} : x_4 \left(\frac{r_2 K_2}{R' \tau_2} u_2 - \frac{R' x_4 + x_4^2 R'}{\tau_2} R\right) = x_4 \dot{x}_4 \le 0 \quad (28)$$

$$\text{호이스트 모터} : x_6 \left(\frac{r_3 K_3}{\tau_3} u_3 - \frac{1}{\tau_3} x_6\right) = x_6 \dot{x}_6 \le 0$$

회전크레인 구동모터의 구속방정식

$$u_1 \le \frac{x_2}{r_1 K_1} \qquad u_2 \le \frac{R'(R' x_4 + x_4^2 R')}{r_2 K_2} \qquad u_3 \ge \frac{x_6}{r_3 K_3} \quad (29)$$

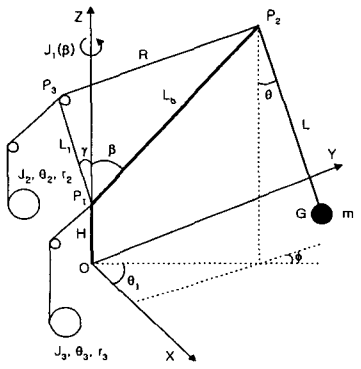
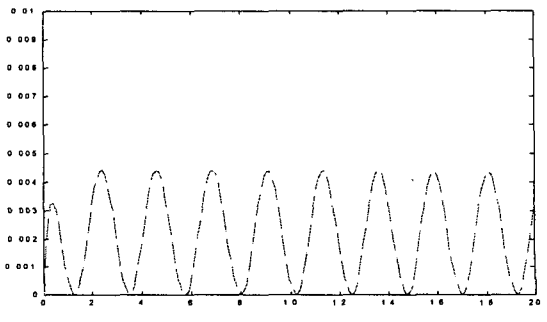


Fig. 1 Schematic diagram ofrotary crane



(b) Swing angle of object

Fig. 4 Response of rotary crane in the constraint input