

Minimum Entropy Deconvolution을 이용한 지하수 상대 재충진양의 시계열 추정법

김태희, 이강근*

한국지질자원연구원

* 서울대학교 지구환경과학부

<요약문>

There are so many methods to estimate the groundwater recharge. These methods can be categorized into four groups. First group is related to the water balance analysis, second group is concerned with baseflow/springflow recession, and third group is interested in some types of tracers; environmental tracers and/or temperature profile. The limitation of these types of methods is that the estimated results of recharge are presented in the form of an average over some time period. Fourth group has a little different approach. They use the time series data of hydraulic head and specific yield evaluated from field test, and the results of estimation are described in the sequential form. But their approach has a serious problem. The estimated results in fourth type of methods are generally underestimated because they cannot consider the discharge phase of water table fluctuation coupled with the recharge phase. Ketchum et al. (2000) proposed calibrated method, considering recharge- and discharge-coupled water table fluctuation. But the discharge is considered just as the areal average with discharge rate. On the other hand, there are many methods to estimate the source wavelet with observed data set in geophysics/signal processing and geophysical methods are rarely applied to the estimation of groundwater recharge. The purpose of this study is the evaluation of the applicability of one of the geophysical methods in the estimation of sequential recharge rate. The applied geophysical method is called minimum entropy deconvolution (MED). For this purpose, numerical modeling with linearized Boussinesq equation was applied. Using the synthesized hydraulic head through the numerical modeling, the relative sequence of recharge is calculated inversely. Estimated results are very concordant with the applied recharge sequence. Cross-correlations between applied recharge sequence and the estimated results are above 0.985 in all study cases. Through the numerical test, the availability of MED in the estimation of the recharge sequence to groundwater was investigated

key word : relative recharge, sequence, minimum entropy deconvolution, numerical simulation

1. Introduction

The estimation of groundwater recharge is one of various classical topics in hydrogeology. And there are a plenty of methods in the estimation of groundwater recharge. Scanlon et. al. (2002) reviewed various kind of techniques for quantifying groundwater recharge. In this paper, they subdivided techniques for estimation recharge into four types; water budget, techniques based on surface-water studies, techniques based on unsaturated studies, physical techniques. (Bradbury and Rushton, 1998; Finch, 1998; Bekesi and McConchie, 1999; Rutledge and Daniel, 1994; Avery et. al., 1999 Ketchum et. al., 2000; Solomon et. al., 1993; Leduc et. al., 1997; Bromley et. al., 1997 Taniguchi, 1993; Taniguchi et. al., 1999a; Taniguchi et. al., 1999b). The applications included in referred three groups are all concerned with the estimation of average annual recharge rate.

All of referred groups concentrate their all efforts to the average recharge rate. However, the average recharge rate is not enough to explain the natural phenomena, especially, the seasonal variation of the water supply ability from groundwater. Unfortunately, it is a very hard work to find out some studies on sequential approaches to estimate the recharge rate to groundwater. Forth group is related to the sequential estimation of recharge rate to groundwater. Sometimes, the methods related to forth group are called 'saturated-volume fluctuation (SVF) analysis method'. Ketchum et. al. (2000) proposed the storage accumulation method calibrated with the spring discharge records. Healy and Cook (2002) tried to extend the discussion on SVF to the estimation of specific yield in fractured media. But in these studies, the local discharge, described with the recession of hydraulic head, did not considered.

On the other hand, there are rare applications of the geophysical methods to the estimation of the groundwater recharge. Even though predictive deconvolution technique is the one of the most prevalent methods for the estimation of source wavelet in geophysics and signal processing, it was rarely applied to the estimation of recharge to groundwater.

In general, the classical predictive deconvolution technique is based on the minimum phase condition. However, minimum phase condition can hardly be satisfied in nature. On the contrary, the input signal in natural system can be considered as a random signal. To apply the classical predictive deconvolution, some appropriate sequence satisfying the minimum phase condition must be picked up. To avoid the strong restriction of minimum phase condition, Wiggins (1978) proposed another deconvolution technique, called minimum entropy deconvolution (MED). Wiggins insisted that MED process seeks the smallest number of large spikes that is consistent with data. In other words, MED finds out the simplest signals from the observed data. Wiggins represented the simplicity with varimax norm. However, solution process with varimax norm is non-linear. To acquire the desired digital filters for the estimation of source wavelet, some iteration are needed in MED proposed by Wiggins. For the linearization of MED problem, Carbrelli (1984) suggested another criterion, the D norm, instead of varimax norm. Carbrelli (1984) showed that the behavior of the D norm is consistent with varimax norm. Using the D norm as a criterion, the solution process in MED is linear, and iteration is not needed.

In this study, MED with D norm is applied to the estimation of the sequence of relative recharge rate. And the applicability of MED to evaluation of recharge sequence will be investigated with the numerical modeling.

2. Results of the Application of MED to Synthesized Data Sets

MED with D norm was applied to estimate the sequence of relative recharge with synthesized data sets. In actual application of MED, it is the most sensitive problem how the length of applied filter can be determined. The statistical concepts of MED come from factor analysis in multivariate statistics. In factor analysis, the number of factors is determined by eigen-value analysis. In custom, the number of eigen-value above 1 is applied as the number of factors in factor analysis. In this study, to determine the length of filter, householder transform was applied to evaluate the number of eigen-value above 1 from the constructed autocovariance matrix, firstly. In most cases using eigen-value analysis, the filter length is above 20 in tested model. With very simple sequence of assumed recharge, this method is applicable to estimate the sequence of recharge. However, in the case that synthesized sequences from numerical modeling were applied, the results of estimations showed some time shift and the dissipation. In other words, the greatest simplicity was not acquired with eigen-value analysis. Then trial and error method was used in this study to determine the length of filter. In most case, it was revealed that 2 is appropriate for the length of filter to estimate recharge sequence in groundwater phenomena in applied models. All applied length of filter in this study is 2. The physical meaning of the length of filter, 2, will be discussed later. The results of estimations are provided in Table 1.

To evaluate the applicability of this method, cross-correlations between applied recharge sequence and estimated results were calculated because the results of the methods proposed in this study is presented in the relative values. The physical meaning of cross-correlation between two time series is the similarity of the signal shape. As cross-correlation is closer to 1, two compared sequences are more similar. When cross-correlation is 1, two sequences are identical in shape. In this study, all cross-correlations in study case are higher than 0.985.

3. Discussion

In this study, the full procedure using MED to estimate the sequence of relative recharge with the sequence of hydraulic head was discussed with both cases of numerical test and the application to the real situations. In numerical tests, the applicability of MED to the estimation of the relative recharge sequences was investigated. For the investigation, linearized Boussinesq equation were employed; confined case and unconfined case. In addition, four type of conceptual model approximating the real aquifer were applied. The results of these numerical tests confirmed the applicability of MED. All values of cross correlation in numerical tests are above 0.993, and the resulted wavelets of recharge are almost same with the original sequence of recharge. It is noticeable that the length of filter in every case is 2. Length 2 has an important mathematical and physical meaning that system response is the exponential decay. In all results through MED, small negative spikes follow the estimated recharge (positive spike). It is due to the length of filter, 2. The number of filter length, 2, means that the system response shows the exponential behavior. It is matter of course that the system response is not exponential. These results can be yielded from the discretization of time. The apparent response with discrete time domain can be described with the exponential decay. The assumption of

the exponential decay makes another error into the results of estimation with MED, the negative spikes following the estimated impulses. For simple comparison, the other sequence of hydraulic head was synthesized through the time domain convolution with a response function that decays exponentially along with time. The same sequence of recharge was assumed to work on the assumed exponential system. Using synthesized head sequence, the sequence of relative recharge was estimated with MED. Cross-correlation is over 0.994. When the negative spikes are removed from the estimations, cross-correlations in study case grow greatly higher, up to 0.999, and these results indicate that two sequences the applied sequence of recharge in numerical modeling and the estimated sequences of relative recharge with MED are almost identical in their shape.

However, MED has an important limitation that the result of estimation is provided in form of relative sequence, not absolute values. Therefore, for the exact estimation of recharge sequence, the reference value of recharge should be recorded, at least one time. With the real aspects, it is very difficult work to measure the recharge rate to the groundwater and long-term monitoring of recharge sequence is very expensive. From this viewpoint, MED can be a powerful and convenient method in the estimation of the recharge sequence to groundwater. In addition, it is proved through the numerical test that MED with a reference value can be a very effective and precise method for the estimation of recharge sequence.

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Table 1. Correlation between applied recharge and estimated results in study case

Node	218	221	224
Correlation	9.92E-01	9.92E-01	9.92E-01
hydraulic conductivity(m/day) around the node	4.50E+00	1.87E+00	7.78E-01
	2.37E+00	1.34E+00	2.42E+00
	3.24E-01	1.63E+00	1.35E+01
	2.84E+01	2.20E+00	3.39E+00