한국공작기계학회 2003 추계학술대회 논문집 pp.390-395

Dynamic Analysis of a Flexible Structure in Motion

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운동 중인 유연한 구조물의 동적 해석

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Abstract

Moving flexible structures such as transfer systems in press machine, crane, working table of machine tools have vibration problems because of starting, feeding and stopping. An analysis method is suggested and experimentally studied in order to solve a vibration problem of a moving flexible structure. In this method, the concepts of substructure synthesis method and semi-static displacement including rigid body mode were used. Total deformation of a structure was assumed to be composed of quasi-static and dynamic components. Experimental results from an elementary model of a transfer feeder showed good agreements with computational results.

Key Words : Flexible structures(유연한 구조물), Vibration(진동), Dynamic analysis(동적 해석)

1. Introduction

Flexible structures like cross bar and feeder drive system in press transfer machine, crane, traveling table of machine tool and arms of industrial robots are needed to be driven at higher operating speed[1~3]. Also, flexible structures are undergoing less weighed for energy saving. In many vibration problems, we used to increase natural frequencies and damping of the system. However, the increase of stiffness results in that of weight, and the increase of weight looses energy saving. Also, there is limitation in the increase of the natural frequencies and addition of damping cannot be damping cannot be effected on the reduction of peak value of vibration displacement in case that the panel is moving[4].

Therefore, we need to exactly predict the vibration of a moving flexible structure. For this research, concepts of substructure synthesis[5~7] was used. The vibration displacements in this study are assumed to be two components[5, 8]. The one is quasi-static component generated from acceleration

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and/or deceleration of the trajectory of motion, and the other is dynamic component to represent relative vibration between exciting and excited parts. Also, a simplified model is suggested and applied to the press transfer machine. The validity would be proved from experiments.

2. Analysis method

To build a mathematical model of a structure, let mass matrix be [M], stiffness matrix [K], damping matrix [C]. Total equation of motion is

$$[M]\{\ddot{q}_t(t)\}+[C]\{\dot{q}_t(t)\}+[K]\{q_t(t)\}=\{Q_t(t)\} \quad (1)$$

where $\{q_t(t)\}$ is total displacement vector, and $\{Q_t(t)\}$ is force vector. Total displacement is divided into exciting part(drive part) and the other part[8, 9]. And the former is assumed to be $\{q_b(t)\}$, and the latter is $\{q(t)\}$ like,

$$\{q_t(t)\} = \begin{Bmatrix} q(t) \\ q_b(t) \end{Bmatrix} \quad (2).$$

Rearranging eq. (1) by eq. (2) gives two split equations of motion as

$$\begin{aligned}[][M_1]\{\ddot{q}\} + [M_2]\{\ddot{q}_b\} + [C_1]\{\dot{q}\} + [C_2]\{\dot{q}_b\} \\ + [K_1]\{q\} + [K_2]\{q_b\} = \{Q\} \quad (3)\end{aligned}$$

$$\begin{aligned}[][M_3]\{\ddot{q}\} + [M_4]\{\ddot{q}_b\} + [C_3]\{\dot{q}\} + [C_4]\{\dot{q}_b\} \\ + [K_3]\{q\} + [K_4]\{q_b\} = \{Q_b\} \quad (4)\end{aligned}$$

The displacement of the remaining part $\{q\}$ is assumed to be combination of two parts as,

$$\{q\} = \{q_d\} + \{q_s\} \quad (5)$$

where $\{q_d\}$ is dynamic component, and $\{q_s\}$ is semi-static component[5, 8]. Semi-static components means the displacements generated by motion of the exciting part, and defined as,

$$[K_1]\{q_s\} + [K_2]\{q_b\} = \{0\}$$

$$\{q_s\} = -[K_1]^{-1}[K_2]\{q_b\} = [T]\{q_b\} \quad (6)$$

Namely, $\{q_s\}$ is elastic displacement by exciting part, $\{q_b\}$, and satisfies the static compatibility equation. Equations (5) and (6) are substituted to (3), and is shown as,

$$[M_1]\{\ddot{q}_d\} + [C_1]\{\dot{q}_d\} + [K_1]\{q_d\} = \{R(t)\} \quad (7),$$

where,

$$\begin{aligned}\{R(t)\} = \{Q\} + ([M_1][K_1]^{-1}[K_2] - [M_2])\{\ddot{q}_b\} \\ + ([C_1][K_1]^{-1}[K_2] - [C_2])\{\dot{q}_b\}.\end{aligned}$$

Above equation can be directly solved from time history analysis.

3. Semi-static displacement including rigid body mode

Internal area of the system is defined as e, and boundary area as c. Among the displacements of area e, the displacement which is generated by unit displacement of the boundary area c can be given as a similar form like,

$$\{x_e\} = -[K_{ec}]^{-1}[K_{cc}]\{x_c\} \quad (6')$$

In above equation, arbitrary displacement in area e is defined as sum of only elastic deformation by unit displacement per one degree of freedom in boundary area c, where arbitrary displacement includes definitely rigid body displacement. However, it is not appropriate for rigid body displacement to be given as sum of only elastic deformation and the accuracy of the computer analysis becomes lower. The displacement in the internal area, $\{x_e\}$ which is generated by that in the boundary area is considered as sum of the rigid body and elastic displacements[5].

First, the displacements of the boundary area are divided into two parts in a whole system. Degrees of freedom equal to the number of degrees of freedom in the rigid body displacement is selected randomly among the boundary area c, and that displacement is assumed to be $\{x_{co}\}$. And the displacements of the other degrees of freedom in the boundary area are $\{x_{cc}\}$. Also, it is assumed that, and a whole system is in a rigid body motion for that degree of freedom if the state of a whole system is free, and unit displacement is given to an arbitrary degree of freedom in the $\{x_{co}\}$. This is the rigid body mode of the system. By applying this way to the total degrees of freedom of $\{x_{co}\}$, total rigid body modes can be obtained. Assuming that [o] is rigid body mode of the system, the rigid body displacement of can be partitioned as,

$$\{x\}_o = \begin{Bmatrix} x_e \\ x_{co} \\ x_{cc} \end{Bmatrix}_o = [\Phi_o]\{x_{co}\} = \begin{bmatrix} [\Phi_{oe}] \\ [I] \\ [\Phi_{oc}] \end{bmatrix}\{x_{co}\} \quad (8),$$

where $[\Phi_{oe}]$ and $[\Phi_{oc}]$ are components corresponding to area, e and c_c, respectively. Area c_c represents degree of freedom except the number of degree of freedom equal to that of rigid body displacement from the boundary area c.

The displacement in area c_c can be considered as the sum of two part; the rigid body displacement as shown in the lower part of eq. (8), and the relative displacements, $\{x_{cc}\}$ when the displacement of all degrees of freedom in area c_o is set to zero ($\{x_{co}\} = \{0\}$). Then,

$$\{x_{cc}\} = [\Phi_{oc}]\{x_{co}\} + \{x_{cc}'\} \quad (9).$$

Also, as the displacement to be generated semi-statically by the displacement of boundary area, c among the displacements in area e is the sum of the rigid body displacement as shown in the upper part of eq. (8), and the relative displacement generated by the displacement of $\{x_{cc}\}$ in area, c_c, the displacement in area e can be written as,

$$\{x_e\} = [\,[\Phi_{oe}]\ [T']\,]\begin{Bmatrix} x_{co} \\ x_{cc}' \end{Bmatrix} \quad (10),$$

where $[T']$ is the matrix removing the column equivalent to the number of the degree of freedom of $\{x_{co}\}$ in the transformation matrix $[T]$ of equation (6), and represents the relation between the displacement of area e and that of area c_c.

Combining equations (9) and (10),

$$\begin{aligned}\{x_e\} &= [\Phi_{oe}]\{x_{co}\} + [T'](\{x_{cc}\} - [\Phi_{oc}]\{x_{co}\}) \\ &= ([\Phi_{oe}] - [T'][\Phi_{oc}])\{x_{co}\} + [T']\{x_{cc}\} \\ &= [T'']\{x_c\} \qquad (11)\end{aligned}$$

where, $[T''] = [\,[\Phi_{oe}] - [T'][\Phi_{oc}] \,\vdots\, [T']\,]$.

If we use transformation matrix $[T'']$ in equation (11) instead of $[T] = -[K_e]^{-1}[K_{ec}]$, the semi-static displacement where rigid body mode is included can be obtained effectively, and the accuracy of the simulation also can be improved.

4. Experiments and Discussions

In order to verify the effectiveness of the analytical method, experimental set-ups with two kinds of the vibration systems with flexible structures were used, and the simulation results were compared with the experimental results.

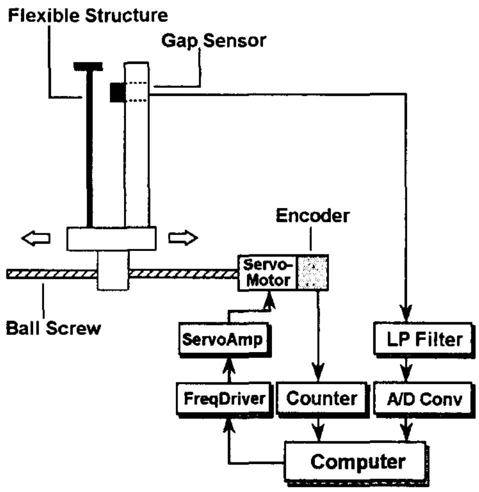

Fig. 1. Experimental set-up

A schematic diagram of experimental set-up is shown in Fig. 1. This set-up has the motion generator driven by servomotor, a flexible structure which is a vibration model, and motion measurement rig. The thickness of a flexible part is 4.5 *mm*, width is 100.0 *mm*, and length is 500.0 *mm*. The displacement of a flexible structure is measured from gap sensor. Given driving shapes are cycloid and trapezoid, and the driving time is 1 or 2 seconds.

First, input of the acceleration as a cycloid shape at experimental set-up is shown in Fig. 2. In case that the gain of the servomotor is large, the big noise is generated around the servo- motor. Therefore some tests are performed at low level of the gain. As the results of the tests, distortions of the displacements are shown around 0.5 second, and 1.0 second. By applying the input as in Fig. 2, the signal of the relative displacement is given as shown in Fig. 3. As being predicted from the previous theoretical analysis, the effects of the acceleration component of input displacement on the system are shown around 0.0~0.3 seconds, and 0.7~1.0 seconds. In order to compare the results between the simulation and the test by using input I, the simulation was executed by using the computer program coded and the results

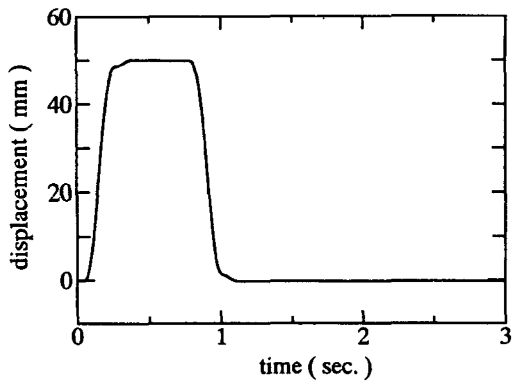

Fig. 2. Input A at experimental set-up

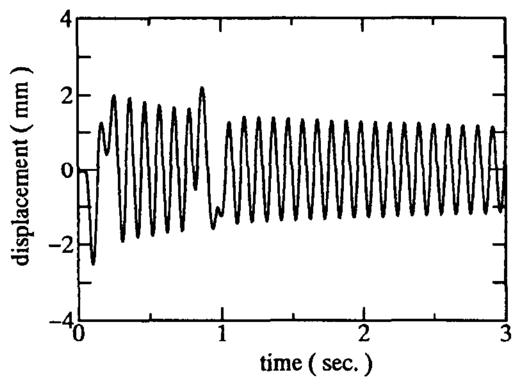

Fig. 3. Relative displacement obtained from tests by input A

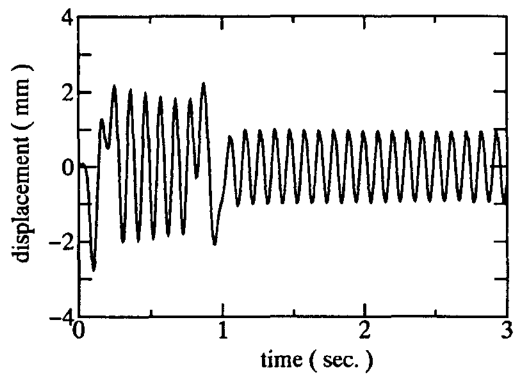

Fig. 4. Relative displacement obtained from simulations by input A

are shown in Fig. 4.

The comparison of those results shows a good agreement, and especially, good in time interval (0.0 ~ 1.0 sec.) which is displacement excitation.

The second input function is the trapezoid as shown in Fig. 5, and this function is being practically used in the transfer feeder of a press machine. The distortions by using the lower level of gain are shown a little bit like Fig. 2.

By applying the above input to the system, the result of the relative displacement is shown in Fig. 6, and this is obtained from non-contacting sensor. As in Fig. 3, the same effects of the acceleration component of input displacement are shown. In order to verify the results, the simulation was performed, and the results are given in Fig. 7. The results between the tests and the simulations are agreed well.

Through the simulations and the tests by two kinds of input functions, it is found that a little bit difference between each simulation and test is shown at the interval of free vibration after forced vibration is finished. In case of using input I, the magnitude by test is generally larger than that of simulation, but the opposite phenomenon is shown in case of using input II. However, totally, the results between the test and the simulation in each case have a good agreement.

From comparing the test with the simulation we can see that the shape and magnitude of the vibration have good agreement.

Through the simulations and the tests, analysis method proposed was proved its validity. Therefore, it is possible to perform the vibration analysis of a moving flexible structure effectively.

5. Conclusions

It is not easy to simulate the relative displacements of a moving flexible structure accurately, because the total moving displacements are relatively very large and the relative displacement between

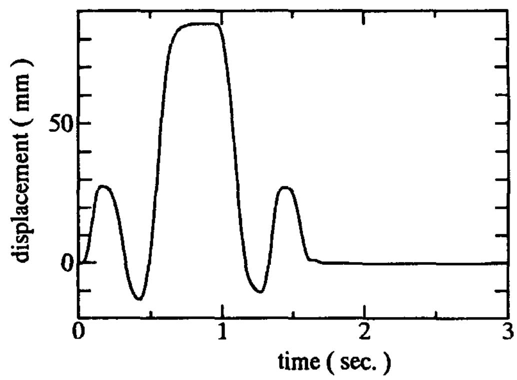

Fig. 5. Input B at experimental set-up.

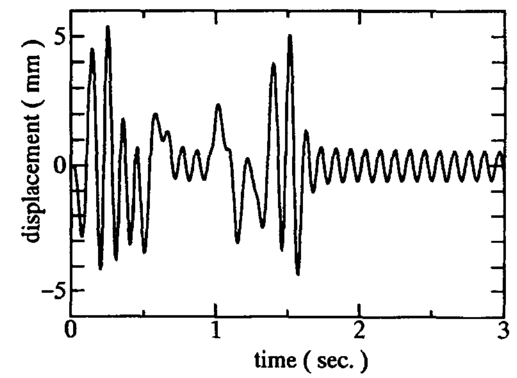

Fig. 6. Relative displacement obtained from tests by input B.

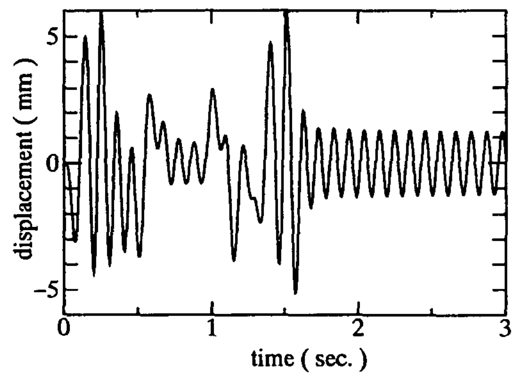

Fig. 7. Relative displacement obtained from simulations by input B.

interior points in moving structure are too small to be distinguished.

We suggested an analytical method to simulate the vibration behavior in motion precisely by using the concepts of substructure synthesis method. We could accurate solutions by introducing the concept considering a rigid body mode in the displacement of the internal area by that of the boundary area. Although it is so difficult to measure the relative displacements in experiments generating the vibration with large and small amplitudes, simultaneously, the results between simulation and test in each case have a good agreement.

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