Dynamic Instability of Rocket-Propelled Flying Bodies

Yoshihiko Sugiyama*

Abstract

This paper deals withdynamic instability of slender rocket-propelled flying bodies, such as launch vehicle and advances missiles subjected to aerodynamic loads and an end rocket thrust. A flying body is simplified into a uniform free-free beam subjected to an end follower thrust. Two types of aerodynamic loads are assumed in the stability analysis.Firstly, it is assumed that two concentrated aerodynamic loads act on the flying body at its nose and tail. Secondly, to take account of effect of unsteady flow due to motion of a flexible flying body, aerodynamic load is estimated by the slender body approximation. Extended Hamilton's principle is applied to the considered beam for deriving the equation of motion. Application of FEM yields standardeigen-value problem. Dynamic stability of the beam is determined by the sign of the real part of the complex eigen-values. If aerodynamic loads are concentrated loads that act on the flying body at its nose and tail, the flutter thrust decreases by about 10% in comparison with the flutter thrust of free-free beam subjected only to an end follower thrust. If aerodynamic loads are distributed along the longitudinal axis of the flying body, the flutter thrust decreases by about 70% in comparison with the flutter thrustof free-free beam under an end follower thrust. It is found that the flutter thrust is reduced considerably if the aerodynamic loads are taken into account in addition to an end rocket thrust in the stability analysis of slender rocket-propelled flying bodies.

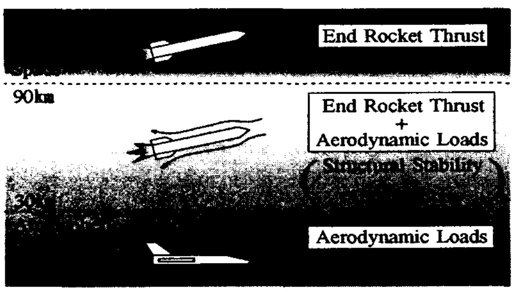
Key Words: Dynamic stability, Divergence, Flutter, Rocket thrust, Aeroelasticity

1. introduction

The light-weightedness is a paramount design key for cost-effective aerospace structures.1 However the flexibility is parasitical on light-weightedness, and thus dynamic instability is latent in flexible aerospace structures. Therefore dynamic instability of flexible flying body has been one of the interesting topics in the field of aerospace structures engineering.

*Department of Aerospace Engineering,
Osaka Prefecture University, Sakai-shi, Japan
E-mail address sugiyama@aero.osakafu-u.ac.jp

Aerodynamic loads can be written in the formsTwo kinds of force can act on the flying body; that is aerodynamic loads and an end rocket thrust. In the atmosphericregion, aerodynamic loads are the main loading to the aircraft. At an altitude higher than about 90km, as there is no air, only an end rocket thrust acts on the flying body. However at an altitude lower than about 90km, aerodynamic loads act on the flying rocket-propelled body in addition to an end rocket thrust. Figure 1. shows a relation between the altitudes and loads that act on the bodies. So far aeroelasticity of aircraft structures has been studied by many scientists. The state of art of the branch of applied mechanics has been compiled in book form by Bisplinghoff and Ashley2, and Dowell, et al.3 Lately dynamic instability of space transfer vehicle and advanced missiles have been one of the main interests of the aerospace structures engineers. Stability of free-free beams subjected to a follower tangential thrust has been studied by several researchers.4 It has been found that the free-free beams under an end follower thrust may lose its stability by bending flutter. The body bending flutter of an actual slender rocket-propelled missile was reported in the Journalof Royal Aeronautical Society in 1956.5 As to the effect of aerodynamic loads on a slender flying body, there have been published some papers on body divergence due to the aerodynamic loads. Ikeda6 studied the dynamic stability of rocket body through rigid-bar mechanical models having two degrees of freedom. Tomita7 discussed body divergence of small test rockets. His paper concluded that the failure of launched test rockets was caused possibly by body divergence. However Kobayashi8 conducted experiments of a model slender spaceplane to conclude that there is a possibility of body flutterfor spaceplanes with a particular planform. Under the circumstances on the topic, the intended aim of the present paper is to discuss the effect of aerodynamic loads on dynamic instability of the free-free beams subjected to an end rocket thrust.



2. NOMENCLATURE

α : angle of attack at nose and tail, respectively

$C_{L\alpha}$: lift slope coefficient

EI : flexural rigidity

L : length of beam

L_N, L_T : concentrated aerodynamic loads at nose and tail

$L(x)$: distributed aerodynamic loads

P : end rocket thrust

Q : non-dimensional thrust

S : reference area of beam

T : kinetic energy

U : potential energy

V : flight velocity

$\bar{V}$: non-dimensional flight velocity

W : work

m : mass of beam per unit length

t : time

x, y : coordinates

ν^2 : mass ratio

ρ : fluid density

τ : non-dimensional time

ξ, η : non-dimensional coordinates

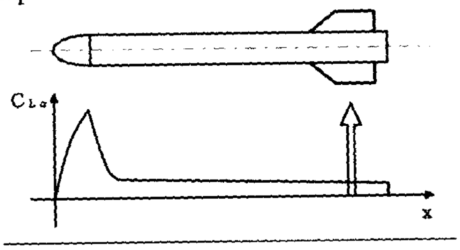
$1/2\rho V^2$: dynamic pressure

3. ANALYSIS

3.1 Mathematical model

General image of aerodynamic loads applied on slender flying body is sketched in Figure 2. It is assumed in the present paper that the flying body is accommodated with tail fines.

Figure 3 shows a mathematical model of a slender rocket-propelled body subjected to an end follower thrust and aerodynamic loads. The flying body is simplified into a uniform free-free beam. It is assumed that the end follower thrust is constant and tangent to the beam at its end. The aerodynamic loads are assumed as two concentrated loads that act on the rocket body at its nose and tail, since the aerodynamic loads between the nose and tail are very small compared with those at its noseand tail.



3.2 Concentrated aerodynamic loads

$$L_N = \alpha_N C_{L\alpha N} \frac{1}{2} \rho v^2 S, \qquad (1)$$

$$L_T = \alpha_T C_{L\alpha T} \frac{1}{2} \rho v^2 S. \qquad (2)$$

The angles of attack α_N, α_T can be written in the forms

$$\alpha_N = -\left[\frac{\partial y}{\partial x} + \frac{\partial y / \partial t}{V}\right]_{x=0}, \qquad (3)$$

$$\alpha_T = -\left[\frac{\partial y}{\partial x} + \frac{\partial y / \partial t}{V}\right]_{x=L}, \qquad (4)$$

where y is the displacement of the slender body.

3.3 Distributed aerodynamic loads

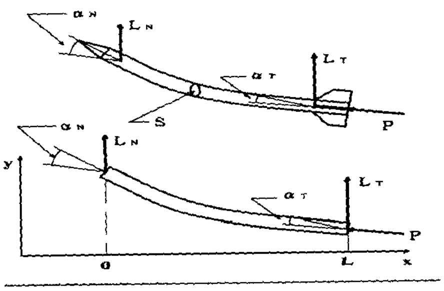
To take account of the effect of unsteady flow due to lateral motion of a flexible flying body, aerodynamic loads are estimated by the slender body theory2,3

$$L(x) = -\rho S\left[V^2 \frac{\partial y^2}{\partial x^2} + 2V \frac{\partial y^2}{\partial x \partial t} + \frac{\partial y^2}{\partial t^2}\right], \qquad (5)$$

$$L_T = \alpha_T C_{L\alpha T} \frac{1}{2} \rho V^2 S. \qquad (6)$$

The attack angle α_T can be written in the form

$$\alpha_T = -\left[\frac{\partial y}{\partial x} + \frac{\partial y / \partial t}{V}\right]_{x=L}. \qquad (7)$$



3.4 Formulation through FEM

Extended Hamilton's principle for the nonconservative system can be written in the form

$$\delta \int_{t_1}^{t_2} (T - U + W_c) dt + \int_{t_1}^{t_2} (\delta W_{L(x)} + \delta W_{LT} + \delta W_{nc}) dt = 0, \qquad (8)$$

where T is the total kinetic energy of the system, U the elastic potential energy of the beam, W_c the work done by the conservative component of the end follower force.

$\delta W_{L(x)}$ and δW_{LT} are the virtual work done by aerodynamic force, while δW_{nc} the virtual work done by the nonconservative component of the end follower force. The energy and work components are given as follows

$$T = \frac{1}{2} \int_0^L m \left(\frac{dy}{dt}\right)^2 dx,$$

$$U = \frac{1}{2} \int_0^L EI \left(\frac{d^2 y}{dx^2}\right)^2 dx,$$

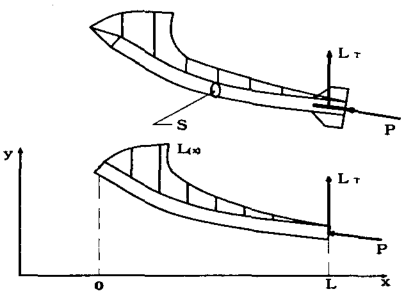
$$W_c = \frac{1}{2} \int_0^L P \frac{x}{L} \left(\frac{dy}{dx} \right)^2 dx,$$

$$\delta W_{L(x)} = -\rho S \int \left\{ \left(V^2 \frac{\partial y^2}{\partial x^2} + 2V \frac{\partial y^2}{\partial x \partial t} + \frac{\partial y^2}{\partial t^2} \right) \delta y \right\} dx,$$

$$\delta W_{LT} = \alpha_T C_{L\alpha T} \frac{1}{2} \rho V^2 S \delta y_{x=L}$$

$$= -C_{L\alpha T} \frac{1}{2} \rho S \left(V^2 \frac{\partial y}{\partial x} + V \frac{\partial y}{\partial t} \right)_{x=L} \delta y_{x=L},$$

$$\delta W_{nc} = -P \left(\frac{dy}{dx} \right)_{x=L} \delta y_{x=L}. \tag{9}$$



For simplicity the following dimensionless quantities are introduced;

$$\xi = \frac{x}{L}, \quad \eta = \frac{y}{L}, \quad \tau = \frac{t}{L^2} \sqrt{\frac{EI}{m}},$$

$$Q = \frac{PL^2}{EI}, \quad \overline{V} = VL \sqrt{\frac{m}{EI}}, \quad v^2 = \frac{\rho S}{m}. \tag{10}$$

4. RESULTS AND DISCUSSIONS

4.1 Effect of an end follower thrust

Here we consider the standard case of the free-free beam subjected only to an end follower thrust. The horizontal axis means a real part of the complex eigen-value λ, while the vertical axis an imaginary part. Increasing the non-dimensional thrust parameter Q above 109.7 results in eigenvaluewith positive real part. That is why this value is the critical force for flutter-type instability, i.e. Q_{cr}=109.7. This value was found first by Feodosiev4 in 1965.

4.2 Combined effect of an end follower thrust and two concentrated aerodynamic loads

Now let us consider the slender rocket-propelled flying body subjected to an end follower thrust and two concentrated aerodynamic loads.

It is proved that the critical flutter thrust decreases by about 10% in comparison with the flutter thrust for the free-free beam subjected only to an end follower thrust.

It is noted that the reduction of the flutter thrust is attributed to aerodynamic damping shown in Eqs (3) and (4), as it has been known that velocity-dependent forces may have a destabilizing effect on flutter of elastic systems subjected to nonconservative forces.

4.3 Combined effect of an end follower thrust and distributed aerodynamic loads

Here we consider the slender flying body subjected to an end follower thrust and distributed aerodynamic loads.

Figure 5 shows the relation between the critical thrust force Q_{cr} and flight velocity $\overline{V}$. It turns out that the critical flutter thrust decreases greatly by about 70%.

The reduction of the flutter thrust in this case may be attributable to the velocity-dependence components shown in expressions (5) and (6).

Figure 6 shows that the unstable eigen-value crosses the imaginary-axis at Q_{cr}=29.0. The flutter at Q_{cr}=29.0 is likely of the 'mild'type, since the unstable eigen-value branch runs parallel to the imaginary axis and thus the growth rate of the amplitude, that is the positive real part, is very small. Figure 7 shows the corresponding bending flutter mode.

5. CONCLUDING REMARKS

It is interesting to note that flutter of free-free beam under an end follower thrust has been studied in the framework of structural stability, while dynamic stability of free-free flying bodies subjected to aerodynamic loads has been investigated in the fields of aeroelasticity. What is a new point in present paper is to combine the concept of structuralstability and aeroelasticity which have been developed independently each other.

It has been turned outby the present study that the flutter thrust of a free-free beam under an end rocket thrust may be reduced considerable by aerodynamic loads. If aerodynamic loads areassumed as two concentrated loads that act on the flying body at its nose and tail, the flutter thrust is reduced by about 10% in comparison with the reference flutter thrust. If aerodynamic loads are distributed along the longitudinal axis of the slender flying body, the flutter thrust is reduced byabout 70%. The reduction of the flutter thrust may be attributable to the effect of velocity-dependence components involved in the aerodynamic loads.

In conclusions, it is confirmed that not only an end follower thrust but also aerodynamic loads should be taken into account in the discussion of the dynamic instability of slender rocket-propelled flying bodies.

REFERENCES

1. T. P. Sarafim, Spacecraft Structures and Mechanisms from Consept to Launch, Kluwer Academic Publishers, Dordrech, The Netherlands (1995).
2. R. L. Bisplinghoff, and H. Ashley, Principles of Aeroelasticity, John Wiley and Sons, INC, New York (1962).
3. E. H. Dowell, et al., A Modern Course in Aeroelasticity, Kluwer Academic Publishers, Dordrech, The Netherlands, Chap.3 (1978).
4. V. I. Feodosiev, Prikladnaya Matematika Mekhanika (PMM), 29-2, 391-392 (1965).
5. D. J. Farrar, Journal of the Royal Aeronautical Society, 60, 712-726 (1956).
6. K. Ikeda, Journal of the Japan Society of Aeronautical Engineering, 7-70, 265 -269 (1959).
7. N. Tomita, Trans. JSME, Series C, 60-580, 59-66 (1994).
8. S. Kobayashi, Proceedings of the JSASS/JSME Structures Conference, 37, 85-88 (1995).