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범주 $IRel_R(H)$의 부분범주

Some Subcategories of The Category $\mathbf{IRel}_{\mathbf{R}}$ (H)

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Abstract

We introduce the subcategories $\mathbf{IRel}_{\mathbf{PR}}$ (H), $\mathbf{IRel}_{\mathbf{PO}}$ (H) and $\mathbf{IRel}_{\mathbf{E}}$ (H) of $\mathbf{IRel}_{\mathbf{R}}$ (H) and study their structures in a viewpoint of the topological universe introduced by L.D.Nel. In particular, the category $\mathbf{IRel}_{\mathbf{R}}$ (H)(resp. $\mathbf{IRel}_{\mathbf{P}}$ (H) and $\mathbf{IRel}_{\mathbf{E}}$ (H)) is a topological universe over **Set**. Moreover, we show that $\mathbf{IRel}_{\mathbf{E}}$ (H) has exponential objects.

Key words and phrases : intuitionistic H-fuzzy proximity(resp. preorder, equivalence) relation, cartesian closed category, topological universe.

0. Introduction

In [7], K.Hur investigated categorical structures of $\mathbf{IRel}_{\mathbf{R}}$ (H) consisting of H-fuzzy reflexive relational spaces in the sense of a topological universe, defined by L.D.Nel(cf. [12]).

Also, we studied categorical structures of the categories **ISet** (H), **IRel** (H) and $\mathbf{IRel}_{\mathbf{R}}$ (H) in the similar viewpoint.

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In this paper, we introduce the subcategories $\mathbf{IRel_{PR}}(H)$, $\mathbf{IRel_{P}}(H)$ and $\mathbf{IRel_{E}}(H)$ of $\mathbf{IRel_{R}}(H)$ and investigate some categorical structures of theses categories in a viewpoint of the topological universe. Moreover, we show that $\mathbf{IRel_{E}}(H)$ has exponential objects.

For general background for lattice theory, we refer to [1,11] and
for general categorical background to [6,12].

1. Preliminaries

We introduce some well-known results [6,12] which are needed in a later section.

Result 1.A[12, Theorem 2.4 ; 6, Proposition 36.10 and 36.11]. Let **A** be a well-powered and co-(well-powered) topological category and let **B** a subcategory of **A**. Then the following are equivalent :

(1) **B** is epireflective in **A**.

(2) **B** is closed under the formation of initial monosources.

(3) **B** is closed under the formation of products and pullbacks in **A**.

Result 1.B[12, Theorem 2.5]. Let **A** be a well-powered and co-(well-powered) topological category and let **B** a subcategory of **A**. Then the following are equivalent :

(1) **B** is bireflective in **A**.

(2) **B** is closed under the formation of initial sources.

Result 1.C[12, Theorem 2.6]. If **A** is a (property fibred, resp.) topological category and **B** is a bireflective subcategory of **A**, then **B** is also a (property fibred, resp.) topological category. Moreover, every source in **B** which is initial in **A** is initial in **B**.

Throughout this paper, we use H as a complete Heyting algebra.

2. Subcategory of $\mathbf{IRel_{R}}(H)$

We introduce some subcategories of $\mathbf{IRel_{R}}(H)$ which are topological universes over **Set**.

Definition 2.1. Let R be an IHFR on a set X. Them R is said to be :

(1) *symmetric* if $\mu_R(x,y)=\mu_R(y,x)$ and $\nu_R(x,y)=\nu_R(y,x)$ for each $x,y\in X$.

(2) *antisymmetric* if for each $(x,y)\in X\times X$ with $x\neq y$, $\mu_R(x,y)\neq\mu_R(y,x)$, $\nu_R(x,y)\neq\nu_R(y,x)$ and $\pi_R(x,y)=\pi_R(y,x)$, where $\pi_R(x,y)=1-\mu_R(y,x)-\nu_R(x,y)$.

(3) *transitive* if $\mu_{R\cdot R}\leq\mu_R$ and $\nu_{R\cdot R}\geq\nu_R$, where

$$\mu_{R\cdot R}(x,z)=\bigvee_{y\in X}[\mu_R(x,y)\wedge\mu_R(y,z)]$$

and

$$\nu_{R\cdot R}(x,z)=\bigwedge_{y\in X}[\nu_R(x,y)\vee\nu_R(y,z)]$$

for any $(x,y)\in X\times X$.

(4) an *intuitionistic H-fuzzy proximity relation* if it is reflexive and symmetric.

(5) an *intuitionistic H-fuzzy preorder relation* if it is reflexive and transitive.

(6) an *intuitionistic H-fuzzy equivalence relation* if it is reflexive, symmetric and tran-

sitive.

(7) R is called an *intuitionistic H-fuzzy order relation* if it is reflexive, transitive and antisymmetric.

Notation 2.1. (1) $\mathbf{IRel_S}\ (H)$ denotes the full subcategory of $\mathbf{IRel}\ (H)$ determined by all intuitionistic H-fuzzy symmetric relational spaces.

(2) $\mathbf{IRel_T}\ (H)$ denotes the full subcategory of $\mathbf{IRel}\ (H)$ determined by all intuitionistic H-fuzzy transitive relational spaces.

(3) $\mathbf{IRel_{AS}}\ (H)$ denotes the full subcategory of $\mathbf{IRel}\ (H)$ determined by all intuitionistic H-fuzzy antisymmetric relational spaces.

(4) $\mathbf{IRel_{PR}}\ (H) = \mathbf{IRel_R}\ (H) \cap \mathbf{IRel_S}\ (H)$ denotes the full subcategory of $\mathbf{IRel_R}\ (H)$ determined by all intuitionistic H-fuzzy proximity relational spaces.

(5) $\mathbf{IRel_P}\ (H) = \mathbf{IRel_R}\ (H) \cap \mathbf{IRel_T}\ (H)$ denotes the full subcategory of $\mathbf{IRel_R}\ (H)$ determined by all intuitionistic H-fuzzy preorder relational spaces.

(6) $\mathbf{IRel_E}\ (H) = \mathbf{IRel_R}\ (H) \cap \mathbf{IRel_S}\ (H) \cap \mathbf{IRel_T}\ (H)$ denotes the full subcategory of $\mathbf{IRel_R}\ (H)$ determined by all intuitionistic H-fuzzy equivalence relational spaces.

(7) $\mathbf{IRel_O}\ (H) = \mathbf{IRel_R}\ (H) \cap \mathbf{IRel_{AS}}\ (H) \cap \mathbf{IRel_T}\ (H)$ denotes the full subcategory of $\mathbf{IRel_R}\ (H)$ determined by all intuitionistic H-fuzzy order relational spaces.

It is easy to show that the following result holds.

Proposition 2.2. The category $\mathbf{IRel_{PR}}\ (H)$(resp. $\mathbf{IRel_P}\ (H)$, $\mathbf{IRel_O}\ (H)$ and $\mathbf{IRel_E}\ (H)$ is properly fibred over **Set**.

Lemma 2.3. $\mathbf{IRel_{PR}}\ (H)$(resp. $\mathbf{IRel_P}\ (H)$ and $\mathbf{IRel_E}\ (H)$) is closed under the formation of initial sources in $\mathbf{IRel_R}\ (H)$.

Theorem 2.4. $\mathbf{IRel_{PR}}\ (H)$, $\mathbf{IRel_P}\ (H)$ and $\mathbf{IRel_E}\ (H)$ are bireflective subcategories of $\mathbf{IRel_R}\ (H)$ and hence topological categories over **Set**.

Theorem 2.5. $\mathbf{IRel_{PR}}\ (H)$, $\mathbf{IRel_P}\ (H)$ and $\mathbf{IRel_E}\ (H)$ are closed under the formation of final structures in $\mathbf{IRel_R}\ (H)$ and hence all of them are bicoreflective subcategories of $\mathbf{IRel_R}\ (H)$.

Lemma 2.6. $\mathbf{IRel_S}\ (H)$ (resp. $\mathbf{IRel_T}\ (H)$) is closed under the formation of pullback in $\mathbf{IRel}\ (H)$.

Theorem 2.7. Final episink in $\mathbf{IRel_{PR}}\ (H)$ (resp. $\mathbf{IRel_P}\ (H)$ and $\mathbf{IRel_E}\ (H)$) preserved by pullbacks.

By Theorem 2.4 and Theorem 2.7 we obtain the following result.

Theorem 2.8. $\mathbf{IRel_{PR}}\ (H)$(resp. $\mathbf{IRel_P}\ (H)$ and $\mathbf{IRel_E}\ (H)$) is topological universe over **Set**. Hence each category is a concrete quasi-topos in the sense of E.J.Dubuc [4].

Lemma 2.9. $\mathbf{IRel_{PR}}\ (H)$ has exponential objects. Hence $\mathbf{IRel_{PR}}\ (H)$ is cartesian closed over **Set**.

Theorem 2.10. $\mathbf{IRel_E}\ (H)$ is cartesian closed over **Set**.

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