E_N^n 상의 비선형 퍼지 제어시스템에대한 제어가능성

The exact controllability for the nonlinear fuzzy control system in E_N^n.

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Abstract

This paper we study the exact controllability for the nonlinear fuzzy control system in E_N^n by using the concept of fuzzy number of dimension n whose values are normal, convex, upper semicontinuous and compactly supported surface in R^n.

Keywords and Phrases : fuzzy number of dimension n, fuzzy control, nonlinear fuzzy control system, exact controllability

1. Introduction

Many authors have studied several concepts of fuzzy systems. Kaleva [3] studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal, convex, upper semicontinuous and compactly supported fuzzy sets in R^n. Seikkala [5] proved the existence and uniqueness of fuzzy solution for the following equation:

$$\begin{cases} \dot{x}(t) = f(t, x(t)), \\ x(0) = x_0, \end{cases}$$

where f is a continuous mapping from $R^+ \times R$ into R and x_0 is a fuzzy number in E^1. Diamond and Kloeden [2] proved the fuzzy optimal control for the following system:

$$\begin{cases} \dot{x}(t) = a(t)x(t) + u(t), \\ x(0) = x_0 \end{cases}$$

where $x(\cdot)$, $u(\cdot)$ are nonempty compact interval-valued functions on E^1.

We consider the exact controllability for the following nonlinear fuzzy control system:

$$\text{(F.C.S.)} \quad \begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)), \\ x(0) = x_0, \end{cases}$$

where $a: [0, T] \to E_N^n$ is fuzzy coefficient, initial value $x_0 \in E_N^n$ and $f: [0, T] \times E_N^n \to E_N^n$ is nonlinear function and $u(t) \in E_N^n$ is control function.

Let E_N^n be the set of all fuzzy numbers in R^n with edges having bases parallel to axis $X_1, \cdots, X_n$.

For example, E_N^2 be the set of all fuzzy pyramidal numbers in R^2 with edges having rectangular bases parallel to the axis X_1 and X_2 [4].

2. Properties of fuzzy numbers

In this section, we give some definitions, properties and notations of the fuzzy number of dimension n.

Definition 2.1. We consider a fuzzy graph

$G \subset R^n$ that is a functional fuzzy relation in R^n such that its membership function

$\mu_G(x_1, \cdots, x_n) \in [0,1]$, $(x_1, \cdots, x_2) \in R^n$ has the following properties:

1. For all $x_i \in R$, $(i=1, \cdots, n)$,

$$\mu_G(x_1, \cdots, x_i, \cdots, x_n) \in [0,1]$$

is a convex membership function.

2. For all $\alpha \in [0,1]$,

$$\{(x_1, \cdots, x_n) \in R^n : \mu_G(x_1, \cdots, x_n) = \alpha\}$$

is a convex set.

3. There exists $(x_1, \cdots, x_n) \in R^n$,

$$\mu_G(x_1, \cdots, x_n) = 1 .$$

If the above conditions are satisfied, the fuzzy subset G is called a fuzzy number of dimension n.

The first projection of G is

$$\vee_{(x_2, \cdots, x_n)} \mu_G(x_1, \cdots, x_n) = \mu_{A_1}(x_1),$$

the second projection of G is

$$\vee_{(x_1, x_3, \cdots, x_n)} \mu_G(x_1, \cdots, x_n) = \mu_{A_2}(x_2)$$

and the i-th projection of G is

$$\vee_{(x_1, \cdots, x_{i-1}, x_{i-1}, \cdots, x_n)} \mu_G(x_1, \cdots, x_n) = \mu_{A_i}(x_i),$$

$(i=3, \cdots, n)$.

We denote by fuzzy number in

$$E_N^n \ A = (a_1, a_2, \cdots, a_n),$$

where a_i is projection of A to axis X_i $(i=1, \cdots, n)$, respectively.

And a_i $(i=1, \cdots, n)$s fuzzy number in R.

Definition 2.2. The α-level set of fuzzy number in E_N^n is defined by

$$[A]^\alpha = \{(x_1, \cdots, x_n) \in R^n : (x_1, \cdots, x_n) \in \prod_{i=1}^n [a_i]^\alpha\},$$

where notation $\prod$ is the Cartesian product of sets.

Definition 2.3. Let A and B in E_N^n, for all $\alpha \in (0,1]$,

(2.1) $$A = B \Leftrightarrow [A]^\alpha = [B]^\alpha .$$

(2.2) $$[A *_n B]^\alpha = \prod_{i=1}^n [a_i * b_i]^\alpha ,$$

where $*_n$ is operation in E_N^n and $*$ is operation in E_N.

Definition 2.4. The derivative $x'(t)$ of a fuzzy process $x \in E_N^n$ is defined by

$$[x'(t)]^\alpha = \prod_{i=1}^n [(x_{il}^\alpha)'(t), (x_{ir}^\alpha)'(t)], \ 0 < \alpha \le 1$$

provided that is equation defines a fuzzy $x'(t) \in E_N^n$.

The fuzzy integral $\int_a^b x(t)dt$, $a, b \in I$ is defined by

$$[\int_a^b x(t)dt]^\alpha = \prod_{i=1}^n [\int_a^b x_{il}^\alpha(t)\,dt, \int_a^b x_{ir}^\alpha(t)\,dt]$$

provided that the Lebesgue integrals on the right exist.

Let $\prod_{i=1}^n [a_i]^\alpha$, $0 < \alpha \le 1$, be a given family of nonempty areas.

If

(2.3) $\prod_{i=1}^n [a_i]^\beta \subset \prod_{i=1}^n [a_i]^\alpha$ for $0 < \alpha < \beta < 1$ and

(2.4) $\prod_{i=1}^n \lim_{k \to \infty} [a_i]^{\alpha_k} = \prod_{i=1}^n [a_i]^\alpha$

whenever (α_k) is a nondecreasing sequence converging to $\alpha \in (0,1]$, then the family $\prod_{i=1}^n [a_i]^\alpha$, $0 < \alpha \le 1$, represents the α-level sets of a fuzzy number $A \in E_N^n$.

Conversely, if $\prod_{i=1}^n [a_i]^\alpha$, $0 < \alpha \le 1$, are the α-level sets of a fuzzy number in R^n, then the conditions (2.3) and (2.4) hold true.

We define the metric d_∞ on E_N^n.

Definition 2.5. Let $A, B \in E_N^n$.

$$d_\infty(A, B) = \sup\{d_H([A]^\alpha, [B]^\alpha) : \alpha \in (0,1]\}$$

$$= \sup\{d_H(\prod_{i=1}^n [a_i]^\alpha, \prod_{i=1}^n [b_i]^\alpha) : \alpha \in (0,1]\}$$

$$= \sup\{\sqrt{\sum_{i=1}^n (d_H([a_i^\alpha], [b_i^\alpha]))^2} : \alpha \in (0,1]\}$$

where d_H is the Hausdorff distance.

The supremum metric H on $C([0,T] : E_N^n)$

is defined by

$$H(x,y)=\sup\{d_\infty(x(t),y(t)): t\in[0,T]\}$$

for all $x, y\in C([0,T]:E_N^n)$.

3. The exact controllability

In this section, we show the exact controllability for the following nonlinear fuzzy control system:

$$\text{(F.C.S.)}\quad \begin{cases} \dot{x}(t)=a(t)x(t)+f(t,x(t)), \\ x(0)=x_0, \end{cases}$$

with fuzzy coefficient $a:[0,T]\to E_N^n$, initial value $x_0\in E_N^n$, control $u:[0,T]\to E_N^n$ and inhomogeneous term $f:[0,T]\times E_N^n\to E_N^n$ satisfies a global Lipschitz condition.

The (F.C.S.) is related to the following fuzzy integral system:

$$\text{(F.I.S.)}\quad \begin{cases} x(t)=S(t)x_0+\int_0^t S(t-s)f(s,x(s))ds \\ \qquad +\int_0^t S(t-s)u(s)ds, \\ x(0)=x_0\in E_N^n, \end{cases}$$

where $S(t)$ is fuzzy number of dimension n and

$$[S(t)]^\alpha=\prod_{i=1}^n[S_i(t)]^\alpha=\prod_{i=1}^n[S_{il}^\alpha(t),\ S_{ir}^\alpha(t)]$$

where $S_{il}^\alpha(t)$ is $\exp\{\int_0^t a_l^\alpha(s)ds\}$ and $S_{ir}^\alpha(t)$ is $\exp\{\int_0^t a_r^\alpha(s)ds\}$. $S_{ij}^\alpha(t)$ $(j=l,r)$ is continuous. That is, there exists a constant $c>0$ such that $|S_{ij}^\alpha(t)|\le c$ for all $t\in[0,T]$.

Definition 3.1. The (F.I.S.) is exact controllable if, there exists $u(t)$ such that the fuzzy solution $x(t)$ of (F.I.S.) satisfies $x(T)=_\alpha x^1$ (i.e.,

$$[x(T)]^\alpha=\prod_{i=1}^n[x_i(T)]^\alpha=\prod_{i=1}^n[(x^1)_i]^\alpha=[x^1]^\alpha\ )$$

where x^1 is target set.

We assume that the following linear fuzzy control system with respect to nonlinear fuzzy control system (F.C.S.):

$$\text{(F.C.S. 1)}\quad \begin{cases} \dot{x}(t)=a(t)x(t)+u(t), \\ x(0)=x_0\in E_N^n \end{cases}$$

is exact controllable. Then

$$x(T)=S(T)x_0+\int_0^T S(T-s)u(s)ds=_\alpha x^1$$

and

$$\begin{aligned}
&[x(T)]^\alpha \\
&=\prod_{i=1}^n[S_i(T)(x_0)_i+\int_0^T S_i(T-s)u_i(s)ds]^\alpha \\
&=\prod_{i=1}^n[S_{il}^\alpha(T)(x_0)_{il}^\alpha+\int_0^T S_{il}^\alpha(T-s)u_{il}^\alpha(s)ds, \\
&\qquad S_{ir}^\alpha(T)(x_0)_{ir}^\alpha+\int_0^T S_{ir}^\alpha(T-s)u_{ir}^\alpha(s)ds] \\
&=\prod_{i=1}^n[(x^1)_{il}^\alpha,(x^1)_{ir}^\alpha]=[x^1]^\alpha.
\end{aligned}$$

Defined the fuzzy mapping $\tilde{g}: \mathcal{P}(R^n)\to E_N^n$ by

$$\tilde{g}^\alpha(v)=\begin{cases} \int_0^T S^\alpha(T-s)v(s)ds, & v\subset\overline{\Gamma_u}, \\ 0, & \text{otherwise}. \end{cases}$$

Then there exists $\tilde{g}_i: \mathcal{P}(R)\to E_N (i=1,2,\cdots,n)$ such that

$$\tilde{g}_i^\alpha(v_i)=\begin{cases} \int_0^T S_i^\alpha(T-s)v_i(s)ds, & v_i(s)\subset\overline{\Gamma_{u_i}}, \\ 0, & \text{otherwise} \end{cases}$$

where u_i is projection of u to axis $X_i, (i=1,\cdots,n)$ respectively and there exists $\tilde{g}_{ij^\alpha}$ $(j=l,r)$

$$\tilde{g}_{il}^\alpha(v_{il})=\int_0^T S_{il}^\alpha(T-s)v_{il}(s)ds,$$
$$v_{il}(s)\in[u_{il}^\alpha(s),\ u_i^1(s)],$$

$$\tilde{g}_{ir}^\alpha(v_{ir})=\int_0^T S_{ir}^\alpha(T-s)v_{ir}(s)ds,$$
$$v_{ir}(s)\in[u_i^1(s),\ u_{ir}^\alpha(s)].$$

We assume that $\tilde{g}_{il}^\alpha$, $\tilde{g}_{ir}^\alpha$ are bijective mappings.

Hence α-level of $u(s)$ are

$$\begin{aligned}
[u(s)]^\alpha&=\prod_{i=1}^n[u_i(s)]^\alpha=\prod_{i=1}^n[u_{il}^\alpha(s),u_{ir}^\alpha(s)] \\
&=\prod_{i=1}^n[(\tilde{g}_{il}^\alpha)^{-1}((x^1)_{il}^\alpha-S_{il}^\alpha(T)(x_0)_{il}^\alpha),
\end{aligned}$$

$(\tilde{g}_{ir}{}^{\alpha})^{-1}((x^1)^\alpha_{ir} - S^\alpha_{ir}(T)(x_0)^\alpha_{ir})]$.

Thus we can be introduced $u(s)$ of nonlinear system

$$
\begin{aligned}
[u(s)]^\alpha &= \prod_{i=1}^n [u_i(s)]^\alpha = \prod_{i=1}^n [u^\alpha_{il}(s),\ u^\alpha_{ir}(s)] \\
&= \prod_{i=1}^n [(\tilde{g}_{il}{}^{\alpha})^{-1}((x^1)^\alpha_{il} - S^\alpha_{il}(T)(x_0)^\alpha_{il} \\
&\quad - \int_0^T S^\alpha_{il}(T-s) f^\alpha_{il}(s, x^\alpha_{il}(s))\,ds), \\
&\quad (\tilde{g}_{ir}{}^{\alpha})^{-1}((x^1)^\alpha_{ir} - S^\alpha_{ir}(T)(x_0)^\alpha_{ir} \\
&\quad - \int_0^T S^\alpha_{ir}(T-s) f^\alpha_{ir}(s, x^\alpha_{ir}(s))\,ds)].
\end{aligned}
$$

Then substituting this expression into the (F.I.S.) yields α-level of $x(T)$. For each $i = 1, \cdots, n$,

$$
\begin{aligned}
&[x_i(T)]^\alpha \\
&= [S^\alpha_{il}(T)(x_0)^\alpha_{il} + \int_0^T S^\alpha_{il}(T-s) f^\alpha_{il}(s, x^\alpha_{il}(s))\,ds \\
&\quad + \int_0^T S^\alpha_{il}(T-s)(\tilde{g}_{il}{}^{\alpha})^{-1}((x^1)^\alpha_{il} - S^\alpha_{il}(T)(x_0)^\alpha_{il} \\
&\quad - \int_0^T S^\alpha_{il}(T-s) f^\alpha_{il}(s, x^\alpha_{il}(s))\,ds)ds, \\
&\quad S^\alpha_{ir}(T)(x_0)^\alpha_{ir} + \int_0^T S^\alpha_{ir}(T-s) f^\alpha_{ir}(s, x^\alpha_{ir}(s))\,ds \\
&\quad + \int_0^T S^\alpha_{ir}(T-s)(\tilde{g}_{ir}{}^{\alpha})^{-1}((x^1)^\alpha_{ir} - S^\alpha_{ir}(T)(x_0)^\alpha_{ir} \\
&\quad - \int_0^T S^\alpha_{ir}(T-s) f^\alpha_{ir}(s, x^\alpha_{ir}(s))\,ds)ds] \\
&= [(x^1)^\alpha_{il},\ (x^1)^\alpha_{ir}] = [(x^1)_i]^\alpha
\end{aligned}
$$

Therefore

$$[x(T)]^\alpha = \prod_{i=1}^n [x_i(T)]^\alpha = \prod_{i=1}^n [(x^1)_i]^\alpha = [x^1]^\alpha.$$

We now set

$$
\begin{aligned}
(\Phi x)(t) &=_\alpha S(t)x_0 + \int_0^t S(t-s) f(s, x(s))\,ds \\
&\quad + \int_0^t S(t-s)\ \tilde{g}^{-1}(x^1 - S(T)x_0 \\
&\quad - \int_0^T S(T-s) f(s, x(s))\,ds)ds.
\end{aligned}
$$

where the fuzzy mappings $\tilde{g}^{-1}$ satisfied above statements.

Notice that $(\Phi x)(T) =_\alpha x^1$, which means that the control $u(t)$ steers the (F.C.S.) from the origine to x^1 in time T provided we can obtain a fixed point of the operator Φ.

Assume that the following hypotheses:

(H1) (F.C.S. 1) is exact controllable.

(H2) Inhomogeneous term $f: [0, T] \times E^n_N \to E^n_N$ satisfies a global Lipschitz condition, there exists a finite constant $k_i > 0$ such that

$$
\begin{aligned}
& d_H([f_i(s, x(s))]^\alpha, [f_i(s, y(s))]^\alpha) \\
& \le k_i\ d_H([x_i(s)]^\alpha, [y_i(s)]^\alpha)
\end{aligned}
\tag{3.2}
$$

for all $x_i(s), y_i(s) \in E_N$ and $f_i: [0, T] \times E_N \to E_N$ $(i = 1, \cdots, n)$ is the i-th projection of f.

We denote $k = \max\{k_i | i = 1, \cdots, n\}$.

Theorem 3.1. Suppose that hypotheses (H1), (H2) are satisfied. Then the state of the (F.I.S.) can be steered from the initial value x_0 to any final state x^1 in time T.

Proof. Omitted.

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