Composite Adaptive Dual Fuzzy Control of Nonlinear Systems

비선형 시스템의 이원적 합성 적응 퍼지 제어

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Abstract – A composite adaptive dual fuzzy controller combining the approximate mathematical model, linguistic model description, linguistic control rules and identification modeling error into a single adaptive fuzzy controller is developed for a nonlinear system. It ensures the system output tracks the desired reference value and excites the plant sufficiently for accelerating the parameter estimation process so that the control performances are greatly improved. Using the Lyapunov synthesis approach, proposed controller is analyzed and simulation results verify the effectiveness of the proposed control algorithm.

Keywords: Adaptive dual fuzzy Control, Identification modeling, Lyapunov synthesis approach

1.Introduction

In some control tasks, such as those in robot manipulation, the systems to be controlled have parameter uncertainty at the beginning of the control operation. Unless such parameter uncertainty is gradually reduced on-line by an adaptation or estimation mechanism, it may cause inaccurate or instability for the control systems. Fuzzy controllers are supposed to work in situations where there is a large uncertainty or unknown variation in plant parameters and structures.[1][2].The undesirable phenomenon of parameter drift can occur, leading to bursting and even instability when parameter convergence is absent. Nonetheless, most researches of adaptive nonlinear control have been focused on the convergence of tracking error[6][7].

One of the motivation in this research is that how can we control the nonlinear plant output to track a desired reference trajectory and identifies a model of the plant at the same time. Thus, considering adaptive dual properties, it can provide a number of advantages compared to the conventional adaptive control schemes. In this paper, we combine direct and indirect adaptive fuzzy controller that can incorporate both types of linguistic information. And also we parameterized both the plant model and controller using a common set of parameters in order to be able to combine the two types of errors for adaptation. The problem formulation is first made in section2. Composite adaptive dual controller design is given in section3. Simulation is provided to illustrate the performance of the proposed controller in section4. Concluding remarks are finally made in section5

2. Problem Formulation

Suppose that the plant is a *n'th* order nonlinear system described by the differential equation

$$x^{(n)} = f(x, \dot{x}, \ldots, x^{(n-1)}) + g(x, \dot{x}, \ldots, x^{(n-1)})u$$
$$y = x \tag{1}$$

where f and g are unknown functions, $u \in R$ and $y \in R$ are the input and output of the plant, respectively, and $X = (x_1, x_2, \ldots, x_n)^T = (x, \dot{x}, \ldots, x^{(n-1)})^T \in R^n$ is the state vector of the system that is assumed to be available for measurement.

In order for (1) to be controllable, we require that $g(x) \neq 0$. Without loss of generality we assume that $g(x)$ positive constant.

Information1: Fuzzy IF-THEN rules describing the input-output behavior of $f(x)$

If x_1 is F_1^τ and $\ldots$ and x_n is F_n^τ, then $f(x)$ is C^τ (2)

Information2: Fuzzy IF-THEN rules describing the input-output behavior of $g(x)$

If x_1 is G_1^s and $\ldots$ and x_n is G_n^s, then $g(x)$ is D^s (3)

Information3: Fuzzy IF-THEN rules describing human control actions

If x_1 is P_1^r and $\ldots$ and x_n is P_n^r, then u is Q^r. (4)

where $F_i^\tau, C^\tau, G_i^s, D^s, P_i^r$ and Q^r are fuzz sets, $\tau = 1,2,\ldots,L_f$, $s = 1,2,\ldots,L_g$, and $r = 1,2,\ldots,L_u$

3. Design of the Fuzzy controller

If the nonlinear functions f(x) and g(x) are known, then we have the control law as

$$u^* = \frac{1}{g(x)}[-f(x) + y_m^{(n)} + K^T e] \tag{5}$$

($s^n + k_1 s^{(n-1)} + \ldots + k_n$ are in the open left-half complex plane).

To utilize Information 1 and 2, we should use the controller

$$u_{12} = \frac{1}{\hat{g}(X \mid \theta_g)}(-\hat{f}(X \mid \theta_f) + y_m^{(n)} + K^T e)$$

Since Information 3 consists of a set of fuzzy control rules, to use it we should consider the controller

$$u_3 = u_D(X \mid \theta_D)$$

A good choice of the final controller is a weighted average of u_{12} and u_3, that is the final controller is

$$u = \alpha u_{12} + (1-\alpha)u_3$$

where $\alpha \in [0,1]$ is a weighting factor. If the plant knowledge, that is Informations 1 and 2, is more important and reliable than the control knowledge Information 3, we should choose a larger α; otherwise, a smaller α should be chosen.

Specifically, using the product inference engine, singleton fuzzifier and center average defuzzifier, we obtain

$$\hat{f}(x \mid \theta_f) = \theta_f^T \xi(x)$$
$$\hat{g}(x \mid \theta_g) = \theta_g^T \eta(x)$$
$$u_D(x \mid \theta_D) = \theta_D^T \pi(x) \tag{6}$$

where $\varsigma(x), \eta(x), \pi(x)$, are basis functions

$$\frac{\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i)}{\sum_{l_1=1}^{p_1} \ldots \sum_{l_n=1}^{p_n} (\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i)}$$

3.1 Design of Adaptive Law

Substituting u into (1) and some manipulation, we obtain error equation

$$e^{(n)} = -K^T e + \alpha(\hat{f} - f) + g(1-\alpha)(u^* - u_D)$$
$$+ (\hat{g} - g)(u - (1-\alpha)u_D)$$
$$= -K^T e + \alpha(\hat{f} - f) + (\hat{g} - g)u + (1-\alpha)(gu^* - \hat{g}u_D)$$
$$= -K^T e + \alpha(\hat{f} - f) + \alpha(\hat{g} - g)u_{12} + (1-\alpha)g(u^* - u_D)$$
$$\dot{e} = \Lambda e + b[\alpha(\hat{f} - f) + \alpha(\hat{g} - g)u_{12} + (1-\alpha)g(u^* - u_D)] \tag{7}$$

where, $\Lambda = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -k_n & -k_{n-1} & -k_{n-2} & \ldots & -k_1 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$

Define the optimal parameters as

$$\theta_f^* = \arg\min_{\theta_f \in R^M}[\sup_{X \in R^n}|\hat{f}(x|\theta_f) - f(x)|]$$
$$\theta_g^* = \arg\min_{\theta_f \in R^M}[\sup_{X \in R^n}|\hat{g}(x|\theta_g) - g(x)|]$$
$$\theta_D^* = \arg\min_{\theta_f \in R^N}[\sup_{X \in R^n}|u^*(x) - u_D(x|\theta_D)|]\cdot$$

Let w be the minimum approximation error defined by

$$w = \alpha(\hat{f}(X|\theta_f^*) - f) + \alpha(\hat{g}(X|\theta_g^*) - g)u_{12} + (1-\alpha)g(u^* - u_D(X|\theta_D^*))$$

Using (7), (6) and w, we can rewrite error equation

$$\dot{e} = \Lambda e + b[\alpha(\theta_f - \theta_f^*)^T\xi(x) + \alpha(\theta_g - \theta_g^*)^T\eta(x)u_{12} - (1-\alpha)g(\theta_D - \theta_D^*)^T\pi(x) + w]$$

Let's consider Lyapunov candidate

$$V = \frac{1}{2}e^T Pe + \frac{\alpha}{2\gamma_1}\phi_f^T\phi_f + \frac{\alpha}{2\gamma_2}\phi_g^T\phi_g + \frac{(1-\alpha)}{2\gamma_3}g\phi_D^T\phi_D$$

(g is positive constant) , (8)

If we choose the adaptation law

$$\dot{\theta}_f = -\gamma_1 e^T Pb\xi(x)$$
$$\dot{\theta}_g = -\gamma_2 e^T Pb\eta(x)u_{12}$$
$$\dot{\theta}_D = \gamma_3 e^T Pb\pi(x)$$

The derivative of Lyapunov candidate

$$\dot{V} = \frac{1}{2}(\dot{e}^T Pe + e^T P\dot{e}) + \frac{\alpha}{\gamma_1}\phi_f^T\dot{\phi}_f + \frac{\alpha}{\gamma_2}\phi_g^T\dot{\phi}_g + \frac{1-\alpha}{\gamma_3}g\phi_D^T\dot{\phi}_D = -\frac{1}{2}e^T Qe + e^T Pbw$$
$$\approx -\frac{1}{2}e^T Qe \le 0$$

Let's consider when $\alpha = 1$ that is $u = u_{12}$ we define the series-parallel identification model for identifying the nonlinear plant[8].

$$\dot{\hat{x}}_1 = x_2$$
$$\dot{\hat{x}}_2 = x_3$$
$$\vdots$$
$$\dot{\hat{x}}_n = \hat{f} + \hat{g}u + p(x_n - \hat{x}_n)$$

(where p is positive constant)

modeling error $e_1 = \dot{\hat{x}}_n - \dot{x}_n$, $e_1 = -\dot{\mu}$

$$e_1 = w + \phi_f^T\xi(x) + \phi_g^T\eta(x)u_{12} + p\mu$$

$$\dot{\mu} = -p\mu - \phi_f^T\xi(x) - \phi_g^T\eta(x)u \quad (9)$$

Considering proposed adaptive law,

$$\dot{\theta}_f = -\gamma_1 e^T Pb\xi(x) + \gamma_1 r\dot{\mu}\xi(x)$$
$$\dot{\theta}_g = -\gamma_2 e^T Pb\eta(x)u + \gamma_2 r\dot{\mu}\eta(x)u \quad (10)$$

where r is positive constant.

Using (9) and (10), derivative of (8) is

$$\dot{V} = -\frac{1}{2}e^T Qe + e^T Pbw - r\phi_f^T\xi(x)w - r\phi_f^{T^2}\xi(x)^2 - r\phi_f^T\xi(x)\phi_g^T\eta(x)u - r\phi_f^T\xi(x)p\mu - r\phi_g^T\eta(x)uw - r\phi_f^T\xi(x)\phi_g^T\eta(x)u - r\phi_g^{T^2}\eta(x)^2u^2 - r\phi_g^T\eta(x)up\mu$$
$$\approx -\frac{1}{2}e^T Qe + e^T Pbw - r[\phi_f^T\xi + \phi_g^T\eta u]^2$$
$$\approx -\frac{1}{2}e^T Qe - r[\phi_f^T\xi + \phi_g^T\eta u]^2 \le 0$$

4.Simulation

Consider the problem of balancing and swing up of an inverted pendulum systems.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\sin x_1 - \dfrac{ml x_2^2\cos x_1\sin x_1}{m_c + m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2 x_1}{m_c + m}\right)} + \frac{\left(\dfrac{\cos x_1}{m_c + m}\right)u(t)}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2 x_1}{m_c + m}\right)}$$

we choose $g = 9.8m/s^2$, $m_c = 1kg$, $m = 0.1kg$ $l = 0.5m$, $k_1 = 2, k_2 = 1$ and $Q = diag(10,10)$ and $A_i^l = B_i^l$ (i=1,2)

with $\mu_{A_i^1}(x_i) = \exp[-(\frac{x_i + \pi/6}{\pi/10})^2]$,

$\mu_{A_i^2}(x_i) = \exp[-(\frac{x_i}{\pi/10})^2]$ $\mu_{A_i^3}(x_i) = \exp[-(\frac{x_i - \pi/6}{\pi/10})^2]$

$\mu_{C_i^1}(x_i) = \frac{1}{1 + e^{-30x_i}}$ $\mu_{C_i^2}(x_i) = \frac{1}{1 + e^{30x_i}}$

Also, $\gamma_1 = 5, \gamma_2 = 1, \gamma_3 = -1$ and $r = 2$ $\alpha = 0.85$ In our control problem, closed-loop x_1 together with the ideal output, $y_m = (0,0)^T$

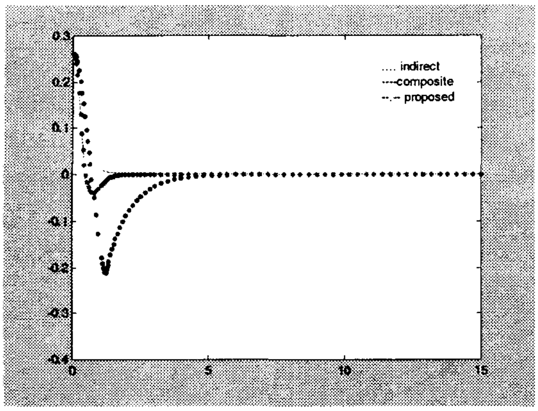

fig.1 shows output tracking using indirect, composite, proposed adaptive law for initial condition $X(0) = (\pi/12, 0)^T$.

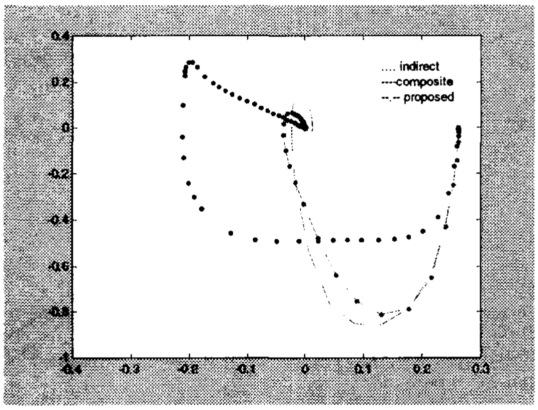

fig.2 phase plane (x_1, x_2) of the system

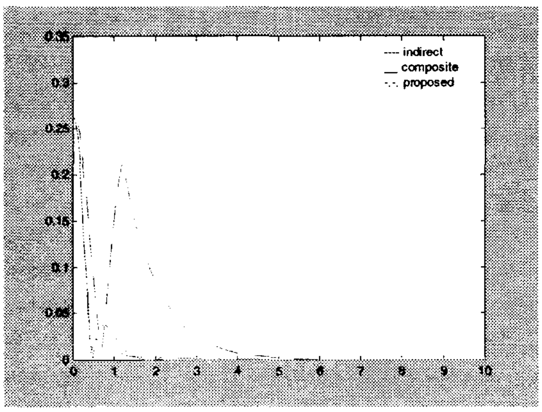

fig.3 Norm of the tracking error

5. Conclusions

Composite (direct/indirect) adaptive fuzzy controller has been considered and also adaptive law are proposed for dual properties. Proposed law involves the effect of the parameter error, so parameter adjustment will continues until both $e = 0$ and $e_1 = 0$. This means that proposed law improves parameter convergence. Therefore, it controls the nonlinear plant output to track a desired reference trajectory, and identifies a model of the plant at the same time. Fast decreasing of Lyapunov function makes the more error reduction. By comparing indirect adaptive fuzzy control , composite adptive fuzzy control, and proposed control, we could see the better transient performances of the system.

6. References

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