Chaotic behavior analysis in the mobile robot : the case of Chua's equation

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ABSTRACT
In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Chua' s equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In the obstacle, we only assume that all obstacles in the chaos trajectory surface in which robot workspace has an unstable limit cycle with Van der Pol equation

Key words : chaos, chua's circuit, mobile robot, Lyapunov exponent

I. Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into the real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1-2], chaos synchronization and secure/crypto communication [3-7], Chemistry [8], Biology [9], and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot, where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot is represented by Arnold equation. They applied obstacle with chaotic trajectory, but they have not mentioned about the chaotic behavior except Lyapunov exponent.

In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Arnold equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In order to avoid obstacles, we assume that all obstacles in the chaos trajectory surface have an unstable limit cycle with Van der Pol equation. When chaos robots meet obstacles among the arbitrary wondering in the chaos trajectory, which is derived using chaos circuit equation such as Chua' s equation, obstacles pull out the chaos robots out of chaos trajectory because obstacles have unstable limit cycle with Van der Pol equation

II. Chaotic Mobile Robot embedding Chaos Equation

2.1 Mobile Robot

As the mathematical model of mobile robots, we assume a two- wheeled mobile robot as shown in Fig. 1.

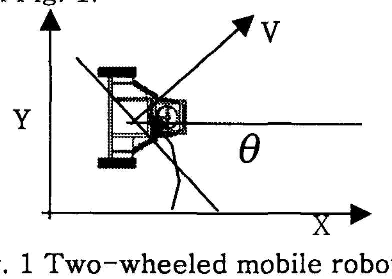


Fig. 1 Two-wheeled mobile robot

Let the linear velocity of the robot v[m/s] and angular velocity ω[rad/s] be the input to the system. The state equation of the four-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (1)$$

where (x,y) is the position of the robot and θ is the angle of the robot.

2.2 Chua' s Circuit Equation (2-Double Scroll)

Chua' s circuit (see Fig. 2 and 3) is one of the simplest physical models that have been widely investigated by mathematical, numerical and experimental methods. One of the main attractions of Chua' s circuit is that it can be easily built with less than a dozen standard circuit components, and has often been referred to as the poor man' s chaos generator. Since the Chua' s circuit is endowed with an unusually rich repertoire of nonlinear dynamical phenomena, it has become a universal paradigm for chaos.

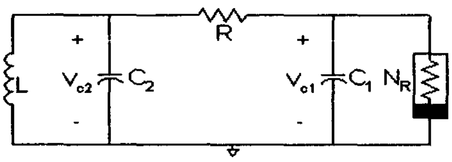


Fig. 2 Chua' s circuit

From the Fig. 1, we can derive the state equation of Chua' s circuit following as:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \quad (2)$$

where

$$g(x) = m_{2n-1}x + \frac{1}{2}\sum_{k=1}^{2n-1}(m_{k-1} - m_k)(|x + c_k| - |x - c_k|)$$

is shown in Fig. 3

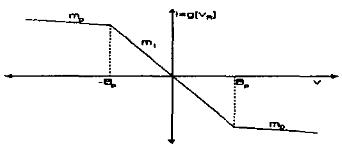


Fig. 3 $v_R - i_R$ characteristic of the nonlinear resistor

From equation (3), we can see chaos attractor as in Fig. 4

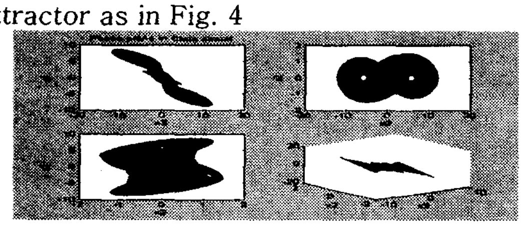


Fig. 4 Phase plane in Chua's circuit

2.3. Embedding of Chaos circuit in the Robot

In order to generate chaotic motions for the mobile robot, we apply chaos equation such as Chua' s equation. We define and use the following state variables:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ x_3 &= \theta \end{aligned} \quad (3)$$

We now design the inputs as follows:

$\omega = -\beta x_2$

v = arbitrary constant (4)

Finally, we can get the state equation of the Chua's circuit embedding in the mobile robot as follows:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= v\cos x_3 \\ \dot{y} &= v\sin x_3 \end{aligned} \quad (5)$$

Using the equation (5), we obtain the embedding chaos robot trajectories with Chua's circuit. Fig.5 and 6 show a chaos robot trajectories and phase plane of incremental component of mobile robot respectively.

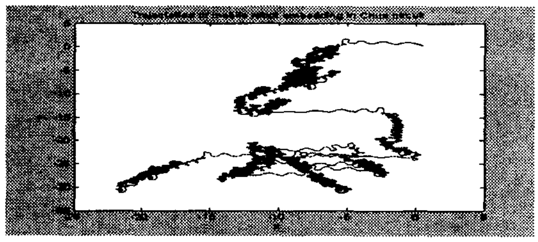


Fig. 5 Trajectories of mobile robot embedding in Chua' s circuit

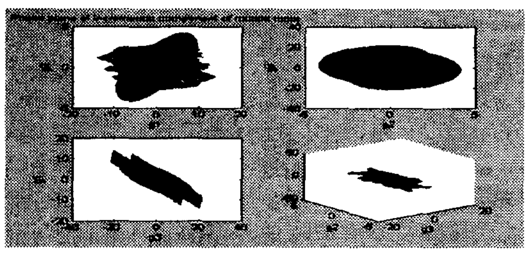


Fig. 6 Phase portrait of variation component (dx, dy,dz) trajectories in Chua' s circuit.

2.4 Mirror Mapping

Basically, equation (5) is assumed that the mobile robot moves in a smooth state space without boundary. However, real robot movies in space with boundary like walls or surfaces of obstacles. To avoid a boundary or obstacle, we consider mirror mapping when the robot approach walls or obstacles using the Eq. (6) and (7).

Whenever the robot approaches a wall or obstacle, we calculated the robot new position

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \quad (6)$$

$$A = 1/1+m \begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \quad (7)$$

We can use equation (6) when slope is infinitive such as $\theta = 90$ and also use equation (7) when slope is not infinitive.

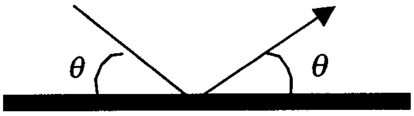


Fig. 7 Mirror mapping

3. The Chaotic Behavior of embedding Chaos Robot with obstacle

3.1 Fixed obstacle

In this section, we will study the chaotic behavior of a chaos robot with mirror mapping relay on Chua' s equation.

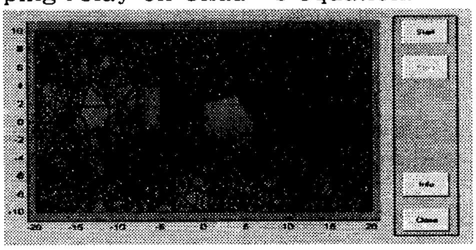


Fig. 8 Trajectories of chaos robot with obstacle embedding Chua' s equation

Fig. 8 show the trajectory of chaos robot can avoid obstacles to which mirror-mapping is applied by Eq (6) and (7).

3.2. VDP equation as a obstacle

In order to represent obstacle of the mobile robot, we employ the VDP, which is written as follows:

$$\dot{x} = y$$
$$\dot{y} = (1 - y^2)y - x \quad (8)$$

From equation (8), we can get the following limit cycle such as Fig. 9

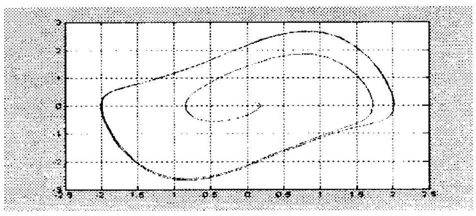


Fig. 9 Limit cycle of VDP

In Fig. 10, computer simulation result show that 1 VDP obstacle at the origin and 1 robot of Chua' s equation is working well.

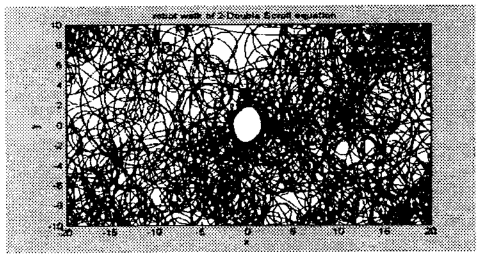


Fig. 10 Computer simulation result of obstacle avoidance with 1 robot and 1 obstacle in Chua' s equation

4. Chaotic behavior analysis in the Mobile Robot

4.1. Embedding method

In order to reconstruct phase plane from data of robot' s single variable, we applied an embedding method proposed by Takens [12]. The embedding method is referring to the process in which a representation of the attractor can be constructed from a set of scalar time-series. The form of such reconstructed state is given as follows:

$$X_t = [x(t), x(t+\tau), \ldots\ldots, x(t+(m-1)\tau] \quad (9)$$

Where $x(t)$ is a robot trajectory data, τ is a delay time, and m is an embedding dimension. It is significant factor to get reasonable embedding phase plane. In chaos mobile case, we choose τ is 400 using an auto-correlation time and m is chosen 5 because nearest false neighbor disappears in that dimension. Fig. 11 shows the timeseries of chaos robot from equation (5)

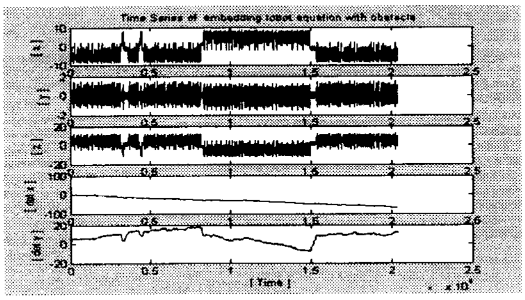
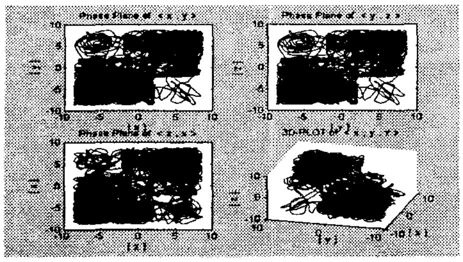


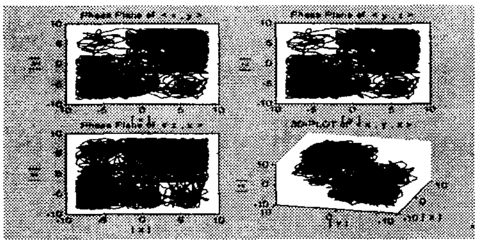
Fig. 11 Chaos robot time-series

4.2. Qualitative Analysis

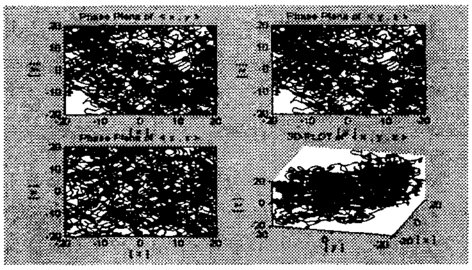
With reconstructed state, the qualitative chaotic degree of chaotic robot path is analyzed in this section using embedding phase plane. Fig. 12 shows phase plane of these embedding state which are originally robot paths when robot has a (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.



(a)



(b)

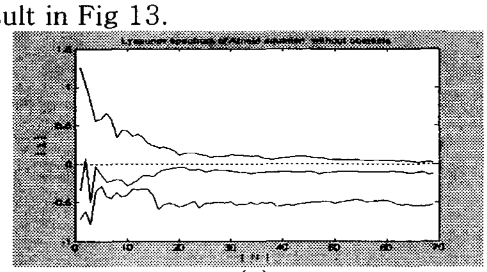


(c)

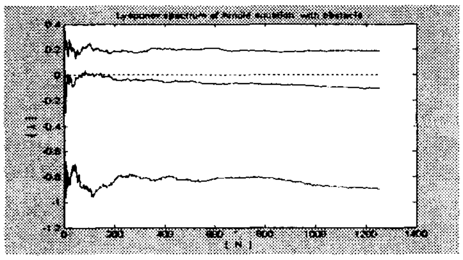
Fig. 12 Reconstructed phase plane
(a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.

4.3. Quantitative Analysis

In this section, we evaluate Lyapunov spectrum [13] in the mobile robot as a quantitative chaos analysis and show the result in Fig 13.



(a)



(b)

Fig. 13 Lyapunov spectrum of mobile robot
(a) without obstacle, (b) with obstacle

6. Conclusion

In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Chua's equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In the obstacle, we only assume that all obstacles in the chaos trajectory surface in which robot workspace has an unstable limit cycle with Van der Pol equation.

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