Runoff Forecasting by Using Transfer Function-Noise Model and State-Space Model with Kalman Filtering

Kalman필터링 기법을 적용한 전이함수모형과 상태공간모형을 이용한 유출량예측

。Chang-yong Cha[1], Jun-Haeng Heo[2], Taesoon Kim[3]

1. Introduction

Within the circulation of water, the most important factor to human life is runoff. Therefore, the main focus in hydrology is in accurate analysis and forecasting of runoff. In this case, if we can utilize the rainfall and runoff data in real-time on-line, we could compose the rainfall-runoff process into a rather simple black-box type input-output system model as well as apply the filtering theory, and as a result, forecast the flood runoff with promptitude and accuracy in case of precipitation.

In this research, 22 satisfactory rainfall events from the year 1987 through 1996 were chosen for the purpose of carrying out the construction of models and the runoff forecasting in the basin near Chung-Ju Dam. During the process, model formation and forecasting were implemented using six hourly average rainfall in the basin and inflow into Chung-Ju Dam. Transfer function-noise model and state-space model were applied along with the Kalman filtering method for the forecasting and compared to each other. Global statistics package SAS system was used to construct models. Transfer function-noise model was identified by Box & Jenkins'(1994) method. Impulse response function was estimated by prewhitening method. Porte Manteau statistics were used to goodness of fit test. In case of constructing state-space model, Akaike's(1974) canonical correlations analysis was used to identify the model and the model was estimated and tested a goodness of fit.

2. Basic Theories

2.1 TFN Model

The stream flow discharge Q_t observed at the exit of basin is generally the linear sum of deterministic component q_t and stochastic component η_t, as stated in the following Eq. (1).

$$Q_t = q_t + \eta_t \tag{1}$$

In stochastic analysis of input and output time series, the combination of process model and noise model called transfer function noise(TFN) can be derived as follows:(Choi. 1995)

$$Q_t = \frac{\omega(B)}{\delta(B)} P_{t-b} + \frac{\theta(B)}{\Phi(B)} a_t \tag{2}$$

1) Graduate Student, Civil Eng, Yonsei University

2) Professor, School of Civil and Environmental Eng., Yonsei University

3) Ph. D. Candidate, Civil Eng, Yonsei University

2.2 State-Space Model

In a system with dynamic behavior and $s = 1, 2, 3, \cdots$, if the future vector of the system $X(t+s)$ is expressed by the information of $X(t)$ and $X(t-s)$, we call this system as Gauss-Markov system. Considering the nature of Gauss-Markov system, $X(t)$ at time period t can be expressed in the following mathematical equation utilizing $X(t-1)$, $X(t-2)$, $\cdots$. This is called system equation.(Park. 1995)

$$X(t) = \Phi\{ X(t-1), t-1 \} + \Gamma\{ w(t), t \} \tag{3}$$

where $\Phi\{\cdot\}$ and $\Gamma\{\cdot\}$ are called transition function, whereas $w(t)$ is called system noise. The state vector $X(t)$ must be estimated through observation system. The observation vector $Z(t)$ at a time period t is a linear combination of function $X(t)$ and observation noise $V(t)$ which can be expressed as following measurement equation.

$$Z(t) = H\{ X(t), t \} + V(t) \tag{4}$$

For reference $H\{\cdot\}$ is the transition function of observation, and the process of estimating $X(t)$ by overcoming the existence of $V(t)$ through observation vector $Z(t)$ is called filtering. In addition, considering both the system and observation equation as linear, and assuming Φ, Γ and H as known matrix, the equation (3) and (4) can be simplified as follows.

$$X(t) = \Phi(t-1)\cdot X(t-1) + \Gamma(t)\cdot W(t) \tag{5}$$

$$Z(t) = H(t)\cdot X(t) + V(t) \tag{6}$$

3. Data Applications

In this research, the data of 22 satisfactory rainfall events from the year 1987 through 1996 were chosen. These are six hourly mean rainfall using the Kriging method of Surfer Program and inflow in Chung-Ju Dam. Events 1 through 14 were used to analyze the basic structure and initial value of real-time rainfall-runoff model. Events 15 through 22 were used to apply to the constructed model. In order to compose a transfer function-noise model including the mean rainfall of basin as the input time series and inflow of Chung-Ju Dam as the output time series, PROC ARIMA of SAS system has been utilized. Thus, the optimum TFN is as follows:

$$w_t = 0.015572\frac{(0.17323-0.0016B+0.05637B^2)}{(1-0.82333B)}z_t + (1-0.84679B)e_t \tag{7}$$

where, $z_t = (1-0.67183B+0.08795B^2)(1-B)^2 x_t$, e_t: white noise process

$w_t = (1-0.67183B+0.08795B^2)(1-B)^2 y_t$

Table 1. Rainfall Events

Event	Starting date	Duration (6hrs)	Event	Starting date	Duration (6hrs)	Event	Starting date	Duration (6hrs)
1	1987. 7. 15	36	9	1990. 7. 11	40	15	1993. 6. 28	12
2	1987. 7. 26	20	10	1990. 9. 8	28	16	1993. 7. 8	48
3	1987. 8. 2	44	11	1991. 7. 15	44	17	1993. 8. 20	28
4	1987. 8. 24	40	12	1991. 9. 3	16	18	1995. 7. 8	40
5	1988. 7. 6	92	13	1992. 7. 10	36	19	1995. 8. 8	8
6	1989. 7. 25	20	14	1992. 8. 22	24	20	1995. 8. 19	12
7	1989. 9. 13	24				21	1995. 8. 23	24
8	1990. 6. 19	48				22	1996. 8. 25	16

In order to compose state-space model with a two-dimensional vector to exhibit average basin rainfall and runoff, PROC STATESPACE of SAS system was utilized. The final state-space model derived is as follows:

$$y_{1,t} = \text{Rainfall(t)} \quad , z_{t+1} = Fz_t + Gv_{t+1} \quad \text{where, } z_t = (y_{1,t}, y_{2,t})'$$

$$y_{2,t} = \text{Discharge(t)} \quad y_t = z_t \tag{8}$$

$$F = \begin{bmatrix} 0.616259 & 0 \\ 0.184217 & 0.866277 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 0.618981 & 0.043981 \\ 0.043981 & 0.17672 \end{bmatrix}$$

4. Forecasting Results with Kalman Filtering

The representative forecast result at 1 step ahead from a real-time rainfall-runoff model, which applied Kalman filter algorithm after state conversing the input and output system data, is as shown:

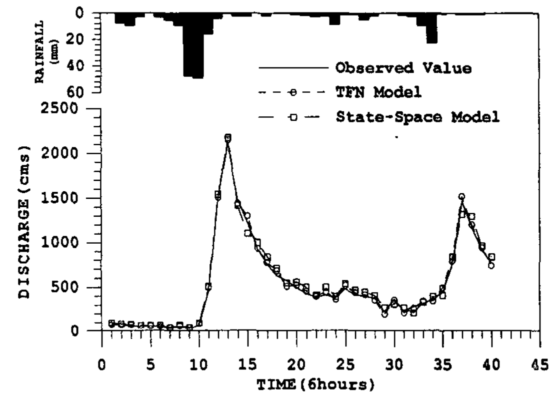

Fig1. Forcasting result of event 18

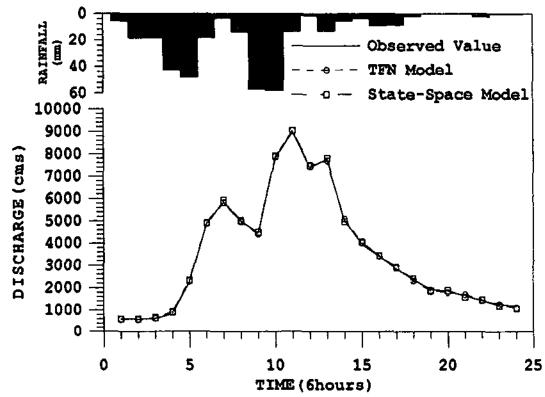

Fig2. Forcasting result of event 21

The error analysis pertaining to the runoff forecast at real-time 1-step ahead is shown on Table 2. The absolute, relative, mean sequence, and root mean squared errors were calculated respectively.

Table 2. The Errors for TFN and State-Space Model

Event	TFN Model				State-Space Model			
	Error (%)		MSE	RMSE	Error (%)		MSE	RMSE
	ABS	REL			ABS	REL		
15	84.27	30.17	156.82	12.52	91.76	-37.27	229.81	15.16
16	211.55	0.36	746.01	27.31	406.63	-284.48	2075.05	45.55
17	205.98	-120.42	1402.13	37.44	364.79	-128.22	3689.82	60.74
18	404.02	-244.3	1014.99	31.86	649.12	-495.69	3728.89	61.06
19	121.86	-52.45	1596.91	39.96	154.14	-119.39	2791.76	52.84
20	73.64	0.38	2476.37	49.76	103.65	-19.02	4759.56	68.99
21	54.44	1.83	2137.14	46.23	67.22	-15.86	3507.96	59.23
22	327.45	-121.95	1548.14	39.35	504.39	-409.61	3371.53	58.06
Average	185.4	-63.3	1384.81	35.55	292.71	-188.69	3019.3	52.7

As a result of error analysis, we can see that both TFN model and state-space model give out accurate forecast of runoff amount in the basin near Chung-Ju Dam due to the precipitation. However, as you can see with absolute error and relative error, the forecasting result by state-space model tends to either overestimate or underestimate compared to that of transfer function-noise model. This is due to the fact that despite applying the restrict condition to have casuality, the state-space model does not sufficiently satisfy the condition. In addition, pertaining to MSE and RMSE, TFN model is the more appropriate one compared to state-space model when it comes to forecasting the runoff at the exit of basin near Chung-Ju Dam due to precipitation.

5. Conclusions

As a result of constructing transfer function-noise model and state-space model to the basin near Chung-Ju Dam and applying the recursive adaptive estimate Kalman filtering method, both the transfer function-noise model and state-space model showed a rather accurate outcome. This result is from the fact that basin near Chung-Ju Dam has no dam in upper basin. Thus precipitation on upper basin naturally incomes to Chung-Ju Dam.

As you can see with the results from the forecast and error analysis, the state-space model tends to either overestimate or underestimate streamflow compared to transfer function-noise model. Since transfer function-noise model was formed under the presumption that satisfy the casuality condition, the rainfall data affected the runoff, not the rainfall resulting from the variance in runoff. However, since state-space model does not satisfy the casuality condition, the variance in the runoff under the influence of rainfall once again affected the rainfall despite applying the restrict condition to have casuality. Therefore, in case of state-space model, a more desirable result can be achieved by improving the way to restrict the casuality condition.

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