# Prioritization of Pipe Replacements by Using a New Water Main Break Prediction Model

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## 1. Introduction

In this paper a new mathematical model to predict pipe breaks – prediction of number of breaks in the future time of a pipe - in water distribution systems is developed. It is composed of linear and exponential prediction models and determines the dominance of past break trends – linear, exponential or in-between – by using a weighting factor. Thus, the model is coined as "General Pipe Break Prediction Model" to reflect its capability of fitting the three types of break trends. As can be found in the literature[Shamir and Howard (1979), Walski and Pelliccia (1982), and Clark et al. (1982)], the mathematical model for the pipe failure prediction has been mostly based on the exponential model. The newly developed model overcomes the shortcomings of the exponential model, which tends to over-predict future number of breaks of a pipe, and is coupled with the optimal threshold break rate developed by Loganathan et al.(2002) to obtain the optimal replacement time of a pipe. This paper also suggests a methodology for prioritizing pipe replacements under limited budget.

## 2. General Pipe Break Prediction Model

The exponential part of the General Pipe Break Prediction Model has the following form:

$$N_c(t) = Be \cdot e^{Ae(t-t_0)} \quad (1)$$

where $N_c(t)$ is the cumulative number of breaks of a pipe in year $t$, $t$ is time in years, $t_0$ is the base year for analysis(pipe installation year, or the first year for which data are available), $Ae$ and $Be$ are the coefficients of the exponential model.

The linear part of the General Pipe Break Prediction Model has following form:

$$N_c(t) = Bl + Al(t - t_0) \quad (2)$$

where $Al$ and $Bl$ are the coefficients of the linear model.

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By combining the exponential and the linear model we define the General Pipe Break Prediction Model as the following form:

$$N_c(t) = (1-WF)(Bl + Al(t-t_0)) + WF \cdot Be \cdot exp(Ae(t-t_0)) \tag{3}$$

where $WF$ is the weighting factor to determine the best model for the data given( $0 \le WF \le 1$ ).

The coefficients ($Al$, $Bl$, $Ae$, and $Be$) are determined by using ordinary least squares estimation for each model, separately. Then, the weighting factor is determined between 0 and 1 that would result in the least sum of the squared errors in the General Pipe Break Prediction Model. The mathematical programming of obtaining $WF$ can be respresented as follows:

$$\text{Minimize } SSE = \sum_{j=n-k}^{n} \left(O_j - C_j\right)^2 \tag{4}$$

$$\text{Subject to: } C_j = (1-\omega_i)L(t) + \omega_i E(t)$$

$$\omega_i = i \cdot \varepsilon$$

$$i = \{0, 1, \ldots, 1/\varepsilon\}$$

$$1/\varepsilon \text{ is an integer and } 0 < \varepsilon < 1$$

where $SSE$ is sum of the squared errors for each $i$, $O_j$ is each observed break time($t_j$), $C_j$ is computed value from the General Pipe Break Prediction Model for each $t_j$, $\omega_i$ is weighting factor for each $i$, $E(t)$ is the exponential model (Eq. (1)), $L(t)$ is the linear model (Eq. (2)), $n$ is number of the total breaks, $k$ is the starting data point from the most recent break incident at which an analyst wants to include the break time data in the analysis. An analyst could only use $k$ recent data to reflect latest break trend in the model. Therefore, $k$ is an integer and could take values between $n-1$ and 3, for example.

## 3. Optimal Replacement Time by Using the General Pipe Break Prediction Model

The General Pipe Break Prediction Model (Eq. (3)) can be used to determine the optimal replacement time of a pipe by considering the equivalence relationship with the Threshold Break Rate developed by Loganathan et al. (2002):

$$\frac{dN_c(t)}{dt} = Brk_{th} \tag{5}$$

where the left-hand side of Eq. (5) is the pipe breakage rate at time $t$ and the right-hand side of Eq. (5) represents the Threshold Break Rate of a pipe, which is

$$Brk_{th} = \frac{\ln(1+R)}{\ln\left(1 + \dfrac{C}{F_l \cdot L}\right)} \tag{6}$$

where $F_l$ is replacement cost per unit length of a pipe, $L$ is the length of a pipe, C is repair cost of a pipe, and R is discount ratio. Therefore, the optimal replacement time of a pipe can be determined by taking a derivative of Eq. (3) with respect to time $t$ and substituting it to Eq. (5). The optimal replacement time is obtained as

$$t^* = \frac{1}{Ae}\ln\left(\frac{Brk_{th} - (1-WF)\cdot Al}{WF\cdot Ae\cdot Be\cdot e^{-Ae\cdot t_0}}\right). \tag{7}$$

## 5. Replacement Prioritization

When utilities face limited budget for replacing all of the pipes which reached optimal replacement time, they need to prioritize pipe replacements. Prioritization of pipe replacements can be accomplished by using the saved cost concept presented in this section. The saved cost represents a sum of the repair costs incurred during the next year(or some interval) if the pipe is replaced this year. Prediction of number of breaks of a pipe could be obtained by using the General Pipe Break Prediction Model presented in this paper. If the saved cost of a replacement candidate pipe during the next given period of time is greater than other replacement candidates, the pipe is assigned higher priority than others. The method is shown in the following example.

Table 1. Replacement Candidate Pipes in Year 1.

| Costs / Pipe | Unit Repair Cost ($) | Predicted Number of Breaks in Year 2 | Saved Cost ($) | Replacement Cost ($) |
|---|---|---|---|---|
| Pipe A | 10,000 | 2 | 20,000 | 100,000 |
| Pipe B | 9,000 | 3 | 27,000 | 100,000 |
| Pipe C | 8,000 | 1 | 8,000 | 200,000 |
| Pipe D | 7,000 | 8 | 56,000 | 250,000 |
| Pipe E | 6,000 | 6 | 36,000 | 300,000 |

Note: Reapir and replacement costs and predicted number of breaks are assumed for illustration purposes.

In Table 1 there are 5 pipes for replacement at year 1 which are Pipe A, B, C, D, and E. The utility plans a limited budget of about $650,000 for pipe replacements. The first step in prioritization is to obtain predicted number of breaks for the next given period which is assumed as year 2 for simplification. Then, the saved costs of each pipe for year 2 can be computed as: Saved Cost = [Unit Repair Cost ($/break)] * [Predicted Number of Breaks for Year 2]. We now order the pipes according to the computed saved costs as shown in Table 2. Since the total cost of replacements must be less than or equal to the limited budget, we can prioritize replacements by adding a pipe from the top of the ordered table until just before the accumulated replacement costs exceeds the limited budget. As a result, the prioritized list of pipe replacement for year 1 is obtained as Pipe D, E, and B.

For replacement prioritization in year 2, consider the following: Pipe A and C are not replaced in year 1 and, therefore, they are carried over to year 2. There may be new pipes entering the replacement candidate list in year 2 competing for replacement with the pipes that are carried over from year 1. Prediction of number of breaks in year 3 are obtained for all of the pipes in the new replacement candidate list for year 2 and saved costs for each of the pipes for year 3 are computed. Following the same procedure as for year 1, a new list of prioritized pipe replacements can be generated for year 2.

Table 2. Prioritization of Replacements for Year 1.

| Costs / Pipe | Unit Repair Cost ($) | Predicted Number of Breaks in Year 2 | Saved Cost ($) | Replacement Cost ($) | Accumulated Replacement Cost ($) | Replace ? |
|---|---|---|---|---|---|---|
| **Pipe D** | 7,000 | 8 | 56,000 | 250,000 | 250,000 | **Yes** |
| **Pipe E** | 6,000 | 6 | 36,000 | 300,000 | 550,000 | **Yes** |
| **Pipe B** | 9,000 | 3 | 27,000 | 100,000 | 650,000 | **Yes** |
| Pipe A | 10,000 | 2 | 20,000 | 100,000 | 750,000 | No |
| Pipe C | 8,000 | 1 | 8,000 | 200,000 | 950,000 | No |

## 6. Summary

A new pipe break prediction model, which is coined as "General Pipe Break Prediction Model" that can accommodate linear, exponential or in-between of linear and exponential break trend is developed. The model is used with the Threshold Break Rate(Loganathan et al., 2002) to obtain the optimal replacement time of a pipe. The newly developed model and replacement analysis method is expected to contribute to the reduction of pipe maintenance costs in water distribution systems. In addition, a pipe replacement prioritization methodology based on saved cost concept is presented. It provides a systematic procedure of assigning replacement orders under limited budget condition.

## 7. References


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