4:1 평면수축유동에서 보이는 Phan-Thien Tanner 모델의 수치 모사

정창권, 송선진, 안경현, 이승종
서울대학교 응용화학부

Numerical Simulation of linear Phan-Thien Tanner model in a 4:1 planar contraction flow

Changkwon Chung, Sunjin Song, Kyung Hyun Ahn, Seung Jong Lee
School of Chemical Engineering, Seoul National University

Introduction

Phan-Thien Tanner model was derived from network theories of concentrated polymer solutions and melts, and was first introduced by Phan-Thien and Tanner[1]. As the Phan-Thien Tanner model is one of the nonlinear rheological models, it has been used to describe the rheological properties of shear-thinning solutions such as PAA, PIB/C14, LDPE and so on. Among them, the linear version of Phan-Thien Tanner model can be written like below.

$$\tau + \varepsilon \frac{\lambda}{\eta} tr(\tau)\tau + \lambda \overset{\nabla}{\tau} = 2\eta \mathbf{D} \tag{1}$$

τ and $\mathbf{D}$ are the extra-stress tensor and the deformation-rate tensor, λ is the relaxation time and η is a constant viscosity. And $\overset{\nabla}{\tau}$ denotes the upper-convected derivative.

$$\overset{\nabla}{\tau} = \frac{\partial \tau}{\partial t} + \mathbf{u} \cdot \nabla \tau - \tau \cdot \nabla \mathbf{u} - (\nabla \mathbf{u})^T \cdot \tau \tag{2}$$

Though shear parameter, ξ is omitted in the linear version, (1) can tell the shear-thinning behavior of polymer solutions with ε only, where ε is from Trouton viscosity data and linked to the extensional experiments[1].

Theory

To begin the numerical approach the flow of viscoelastic material in a sudden contraction geometry, we should solve the simultaneous equations including the mass conservation, momentum conservation and constitutive equation. By using the stabilizing techniques, we could obtain more accurate solutions. DEVSS-G/DG methods play an important role of stabilizing the system[2, 3]. With a fractional step method, it is possible to analyze the unsteady motions of contraction flow[4]. The final forms of governing equations as a FEM formulation are written as follows:

$$\left\langle \psi ; \boldsymbol{\tau} + tr(\boldsymbol{\tau})\boldsymbol{\tau} + We\left(\frac{\partial \boldsymbol{\tau}}{\partial t} + \mathbf{u}\cdot\nabla\boldsymbol{\tau} - \boldsymbol{\tau}\cdot\mathbf{G}^T - \mathbf{G}\right) - \eta(1-\beta)(\mathbf{G}^T + \mathbf{G}) \right\rangle - \sum_{e=1}^{N}\int_{\Gamma_e} \psi : \mathbf{u}\cdot\mathbf{n}(\boldsymbol{\tau} - \boldsymbol{\tau}^{ext})d\Gamma = 0 \tag{3}$$

where $\langle A; B\rangle$ denotes $\int_\Omega ABd\Omega$ on domain Ω, $\mathbf{n}$ is a normal vector which has the outward direction at the boundary of finite elements. $\boldsymbol{\tau}^{ext}$ takes the upstream additional stress value in the region of $\mathbf{u}\cdot\mathbf{n} < 0$..

$$\left\langle \phi ; Re\left(\frac{\breve{\mathbf{u}} - \mathbf{u}^n}{\Delta t} + \frac{1}{2}\left(\breve{\mathbf{u}}\cdot\nabla\breve{\mathbf{u}} + \mathbf{u}^n\cdot\nabla\mathbf{u}^n\right)\right)\right\rangle = -\langle\nabla\cdot\phi ; p^n\rangle + \left\langle(\nabla\phi)^T ; \frac{1}{2}\left(\boldsymbol{\tau}^{n+1} + \boldsymbol{\tau}^n\right) + \frac{1}{2}\left(\nabla\breve{\mathbf{u}} + \nabla\mathbf{u}^n\right)\right\rangle$$

$$-\left\langle(\nabla\phi)^T ; (1-\beta)\left(\frac{3}{2}\left(\mathbf{G}^n + (\mathbf{G}^n)^T\right) - \frac{1}{2}\left(\mathbf{G}^{n-1} + (\mathbf{G}^{n-1})^T\right)\right)\right\rangle \tag{4-1}$$

$$\left\langle \phi ; Re\frac{\mathbf{u}^* - \breve{\mathbf{u}}}{\Delta t}\right\rangle = \left\langle \nabla\cdot\phi ; \frac{1}{2}p^n\right\rangle \tag{4-2}$$

$$\left\langle \nabla\cdot\phi ; \frac{1}{2}\nabla p^{n+1}\right\rangle = \left\langle \nabla\cdot\phi ; \frac{Re}{\Delta t}\mathbf{u}^*\right\rangle \tag{4-3}$$

$$\left\langle \phi ; Re\frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t}\right\rangle = -\left\langle \nabla\cdot\phi ; \frac{1}{2}p^{n+1}\right\rangle \tag{4-4}$$

$$\left\langle \varphi ; \mathbf{G}^{n+1} - \left(\nabla\mathbf{u}^{n+1}\right)^T\right\rangle = 0 \tag{5}$$

where ψ, ϕ and φ are linear basis function, Reynolds number and Weissenberg number are the dimensionless groups like as $Re = \dfrac{L\bar{u}\rho}{\eta}$, $We = \dfrac{\bar{u}\lambda}{L}$ respectively. L is the downstream width and $\bar{u}$ is the average velocity in a downstream. With above algorithms, we can show the different phenomena in a complicated flow predicted by linear Phan-Thien Tanner model and Oldroyd-B model. And β is assumed to be 1/9 in both cases.

Results & Discussion

We obtained the steady solutions of high *We* flow with relative finer mesh which consists of 2641 elements. The time increment was $\Delta t = 2 \times 10^{-3}$ and the relative error for steady criterion was order of ten to –6.

The differences of the numerical results predicted by another constitutive models are found at a glimpse of vortex dynamics evidently. In Fig. 1, linear Phan-Thien Tanner model shows the corner vortex enhancement mechanism. Contrary to those, the so-called lip vortex enhancement mechanism is discovered in case of the Oldroyd-B model in Fig. 2.

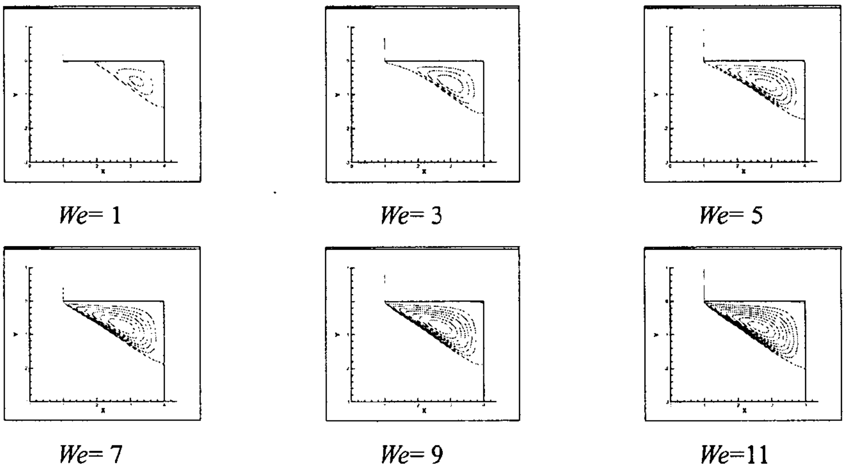

Fig. 1. Vortex dynamics of LPTT model ($\varepsilon = 0.25$) in case of $Re = 10^{-1}$ ($\Delta\psi = 5 \times 10^{-4}$).

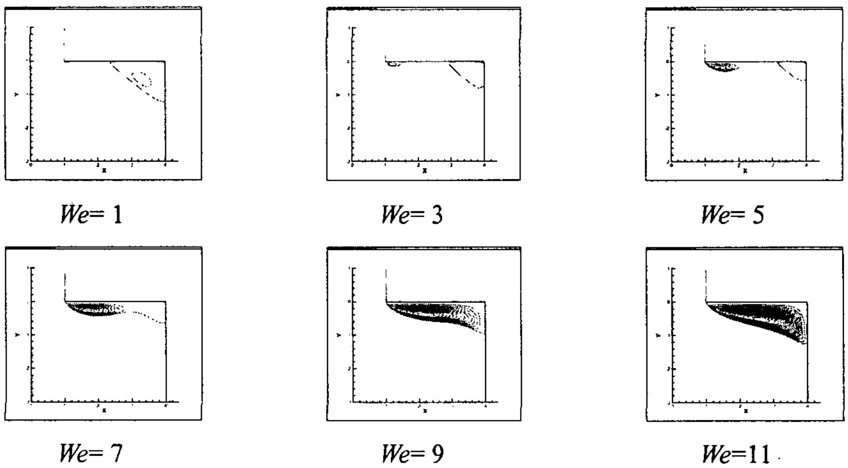

Fig. 2. Vortex dynamics of Oldroyd-B model in case of $Re = 10^{-1}$ ($\Delta\psi = 5 \times 10^{-4}$).

Also the notable differences between two models can be observed in case of the stress distributions near the contraction plane. The normalized first normal stress difference, $N_1 (= \tau_{yy} - \tau_{xx})$ with $T_w = 3\eta\bar{u} / L$ are shown in Fig. 3.

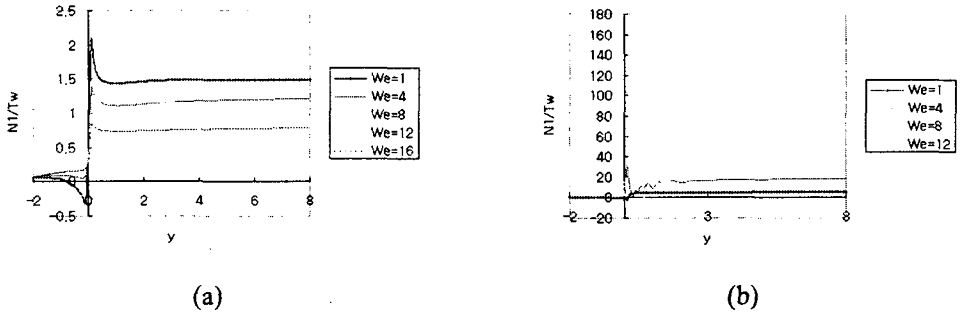

Fig. 3. First normal stress difference along the downstream wall of LPTT model (a) and Oldroy-B model (b).

References

[1] N. Phan-Thien and R. I. Tanner, A new constitutive equation derived from network theory, J. Non-Newtonian Fluid Mech. 2 (1977) 353-365.

[2] A. W. Liu, D. E. Bornside, R. C. Armstrong and R. A. Brown, Viscoelastic flow of polymer solutions around a periodic, linear array of cylinders: comparisons of predictions for microstructure and flow fields, J. Non-Newtonian Fluid Mech. 77 (1998) 153-190.

[3] M. Fortin and A. Fortin, A new approach for the fem simulation of viscoelastic flows, J. Non-Newtonian Fluid Mechanics 32 (1989) 295-310.

[4] H. G. Choi, H. Choi and J. Y. Yoo, A fractional four-step finite element formulation of the unsteady incompressible Navier-Stokes equations using SUPG and linear equal-order element methods, Comput. Methods Appl. Mech. Engrg. 143 (1997) 333-348.

Acknowledgement

The authors wish to acknowledge the Korean Science and Engineering Foundation (KOSEF) for the financial support through the Applied Rheology Center, an official engineering research center (ERC) in Korea.