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Application of Weibull Distribution Function to Analysis of Breakthrough Curves from Push Pull Tracer Test

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Abstract

In the case of the remediation studies, push pull test is a more time and cost effective method than multi-well tracer test. It also gives just as much or more information than the traditionally used methods. But the data analysis for the hydraulic parameters, there have been some defections such as underestimation of dispersivity, requirement for effective porosity, and calculation of recovery of center of mass to estimate linear velocity. In this research, Weibull distribution function is proposed to estimate the center of mass of breakthrough curve for Push pull test. The hydraulic parameter estimation using Weibull function showed more exact values of center of mass than those of exponential regression for field test data.

key word : push pull test, hydraulic parameter, weibull distribution function, exponential regression.

1. Introduction

The single well push pull test (Istock et al., 1997) is useful method to estimate various aquifer physical, chemical, and microbiological characteristics in situ. The method consists of three parts. The first step is the injection (or push) of prepared solution into the aquifer through injection well. Then, resting phase follows, and the extraction (or pull) for the test solution is performed from the well where it was injected. Recently, The method has been used to determine microbial metabolic activities, aerobic respiration, denitrification, sulfate reduction, methanogenesis (Istok et. al., 1997) and detect for NAPL in the subsurface (Istok et. al., 2002). Not only the active research in evaluating the microbial reaction rate, but there has also been much attention in estimating the hydraulic parameters such as dispersivity, linear velocity, and effective porosity. Some researchers who have studied on estimating the hydraulic parameters mentioned that the dispersivity of the tested aquifer cannot be independently estimated due to the dimensionless form of the solution. The objective of this study is to present a method to analyze the breakthrough curve for push pull test with Weibull distribution function. In this study, it is applied to construct the relationship between hydraulic parameters from breakthrough curve and Weibull distribution parameters.

2. Analysis of push pull data

2.1 Weibull distribution function

The Weibull PDF(probability density function) is

$$f(T) = \frac{\beta}{\eta}\left(\frac{T-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{T-\gamma}{\eta}\right)}$$

$$\eta = \textit{scale parameter} \qquad (1)$$
$$\beta = \textit{shape parameter}$$
$$\gamma = \textit{location parameter}$$

When breakthrough curve for push pull test is regressed with Weibull Distribution function, the location parameter, γ, is zero. The two-parameter Weibull y function is appropriately used for the regression.

$$f(T) = \frac{\beta}{\eta}\left(\frac{T}{\eta}\right)^{\beta-1} e^{-\left(\frac{T}{\eta}\right)} \qquad (2)$$

and the Weibull CDF(cumulative density function) is

$$F(T) = 1-e^{-\left(\frac{T}{\eta}\right)^{\beta}} \qquad (3)$$

The estimation of the parameters of the Weibull distribution can be found graphically via probability plotting paper, or analytically, either using least squares or maximum likehood. In this research, the analytical method, least squares, is used to estimate Weibull parameters, β and η. Performing rank regression using Weibull CDF(cumulative density function) on Y or X requires that a straight line should be fitted mathematically to a breakthrough curve for push pull test, such that the sum of the squares of the vertical deviations from the points to the line is minimized.

2.2 Experimental research

A Push Pull Test was conducted at chunchon, kangwon-do, South Korea from 20 to 23 December 2002. In order to estimate Weibull parameters, the breakthrough curve should also be transformed into cumulative mass of tracer obtained as a function of time.

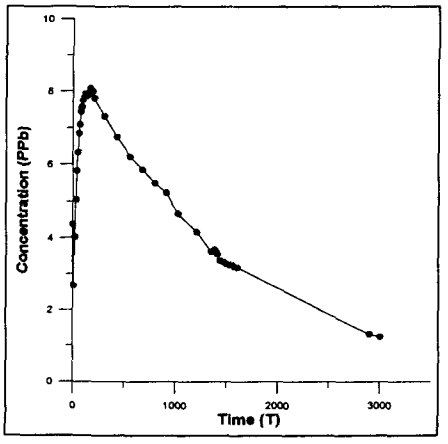

Fig.1 Tracer breakthrough curve

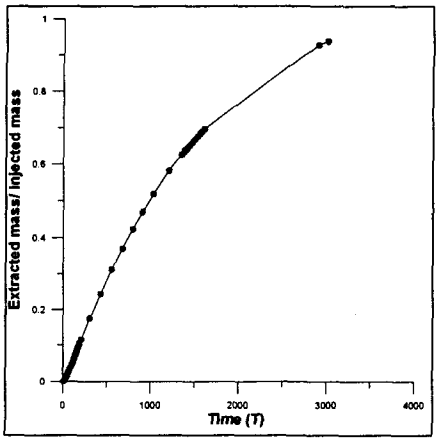

Fig.2 Cumulative mass recovery

2.3 Breakthrough curve fitting

There are many kinds of methods to estimate hydraulic parameters from the

breakthrough curve. After performing rank regression using Eq.(2) on Y or X, the calculated parameters, β and η, are used to fit the breakthrough curve. The values of estimated β and η are 1.26 and 1283.18 min, respectively. The Webull PDF can be plotted with these estimated parameters.

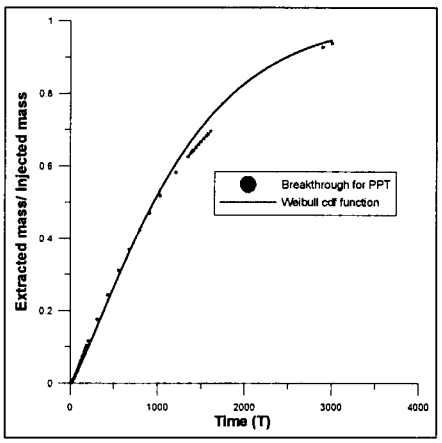

Fig.3 Weibull rank regression

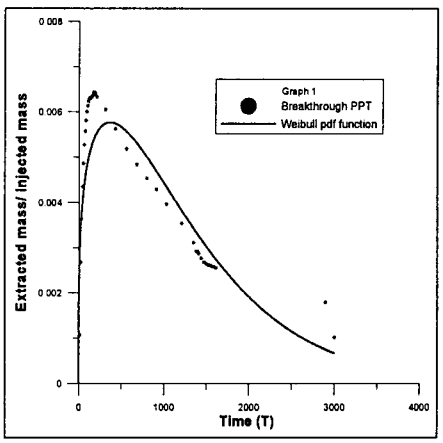

Fig.4 Estimated Weibull function

These parameters (β and η) can be analogous to the transport parameters such as dispersivity, and linear velocity, which can be measured from the elapsed time of obtaining the center of mass of injected tracer and the distance it moved.

3. Results and Discussion

In the case of exponential regression, which has been used as a regression method for push pull data, the calculated value of T_{50}, the arrival time of center of mass, was 1019 minute, but this method caused a problem of data loss (shaded area in Fig.5). Therefore, T_{50} and linear velocity must be overestimated and underestimated, respectively. Using Weibull regression, T_{50} would be estimated at 753 min without data loss and it could also cover the breakthrough curve tailing. (see Fig.6)

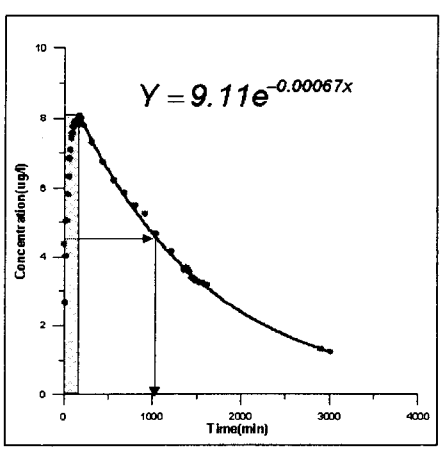

Fig.5 Exponential regression

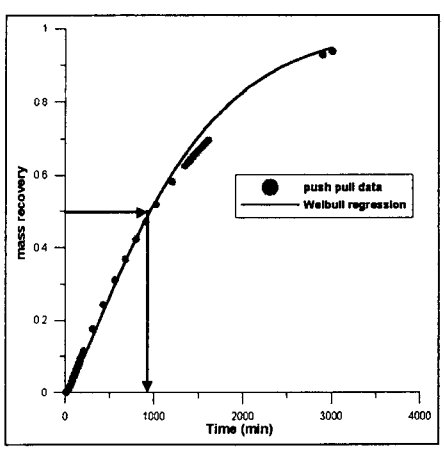

Fig.6 Weibull regression

The Weibull function showed to be more suitable method for breakthrough curve analysis for push pull test than other methods. If the physical meanings of Weibull

parameters (β and η) are known, one can also estimate the hydraulic parameters (dispersivity and degree of tailing) without performing numerical simulation. Consequently, a research on the relationship between the hydraulic parameters and Weibull parameters should be needed.

4. References

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