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A Study on Similitude Law for Evaluation of Seismic Performance
내진성능평가를 위한 상사법칙에 관한 연구

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ABSTRACT

지진하중에 대한 구조물의 동적 거동과 성능을 예측 평가하기 위하여 실험적 방법들이 흔히 사용되고 있으나, 실험장비의 제약과 구조물의 규모 등으로 대부분 축소모형실험에 의존하고 있다. 그러나 일반적인 상사법칙(similitude law)은 탄성범위에서 유도된 것으로 지진거동과 같은 비탄성 거동을 예측하는 경우에는 한계가 있다. 또한 탄성범위 내에서도 크기효과(size effect)가 발생하므로 축소모형의 실험결과를 원형 구조물에 직접 적용하는 것은 많은 주의가 필요하다. 본 연구에서는 원형 구조물(prototype)과 축소모형(scaled model)을 모두 실험 대상으로 하여 실제 축소모형만을 실험하여 원형 구조물의 거동을 예측하는 경우의 문제점을 확인하고 그 해결방법을 모색하고자 한다. 실제로 축소모형실험에서는 원형 구조물의 경계조건을 정확히 재현하기 어려우며, 실험모형의 제작과정과 실험과정에서의 모든 오차가 강성의 변화로 반영되어 나타난다. 따라서 본 연구에서는 기하학적 상사율과 변화된 강성비(stiffness ratio)를 함께 고려하여 고유진동수의 오차를 보정하고 비탄성 거동 중에도 직접적인 실험결과의 비교가 가능한 상사법칙을 제안하였다. 더불어 제안된 상사법칙을 적용한 유사동적실험(pseudodynamic test)을 수행하여 실험오차보정(experimental error compensation)효과를 검증하였다.

1. INTRODUCTION

Although there are several experimental techniques to evaluate the seismic behavior and performance of structures, scaled models would be used due to the limitation of testing facilities or economic reasons in most of physical tests. The prediction of inelastic behavior under an earthquake loading condition has some discrepancies inherently because the similitude law is generally derived in the elastic range. Moreover size effect on the scaled models exists even in the elastic range. The evidence points to influence of size effect in steel beams was presented by C.W. Richards[1]. Thus, a special attention is required to regard the behavior of the scaled models as one of prototypes. In general, similitude law including geometric concept is the basis of performing scaled model tests. However, due to the discrepancy between the scaled model and prototype, it is basically influenced with the evaluation and application of experimental results obtained from the scaled models. By reason of the problems, M.Z. Zhang et al.[2-3] developed a new similitude law adaptable to seismic simulation tests on small-scale models and Q.L. Meng[4] made use of microconcrete material for the scaled model tests. As previous researchers, W. Kim et al.[5] and Y. Lu et al.[6] had some efforts on investigating reinforced concrete scaled models.

In experiments, it is difficult to simulate precisely the boundary conditions of a prototype by a scaled model due to the errors induced from test specimens. Also, the mechanical properties and experimental conditions could be different from each other. Therefore, the scaled model should satisfy an important similitude relationship of the prototype and reflect significant properties on test results. Consequently satisfying the similitude law, the scaled model tests could be reliable to predict the seismic performance of prototypes. In general, geometric similitude law in elastic range would be used for the scaled model tests. Thus, establishment of a similitude law considering inelastic behaviors and experimental errors may be an outstanding tool of the scaled model tests for exactly evaluating the seismic performance of structures. To avoid the limitation of small-scale models, pseudodynamic tests on large-scale models have been applied by many researchers[7-12]. By S. Kumar et al.[9], two choices corresponding to the selection of a convenient scale factor for mass or time, respectively, were examined for the pseudodynamic tests.

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In this study, consistency of three similitude laws based on mass, time or acceleration, respectively, are verified by the pseudodynamic scaled model tests under the same scale factors for length and force. And a modified similitude law considering both a scale factor for length and a stiffness ratio is proposed. It can compensate frequency errors of the scaled models and directly apply the scaled model test results to prototypes.

2. GENERAL SIMILITUDE LAW

Similitude law is generally applied to define a specimen for scaled model tests. A proper similitude law should be selected for satisfying a specific test objective or method. Typically in time-dependent loading problems, three independent scale factors, which represent three fundamental dimensions, namely, mass, length and time, need to be selected for designing the scaled models. Thus selecting three dimensions, other scale factors can be derived from the principles of dimensional analysis[13]. Scale factors may be determined from consideration of the capacity of testing facilities in the scaled model tests. When the same materials on both a prototype and a scaled model are used, a scale factor for stress becomes unity. Thus, various derivatives can be obtained based on the selected dimensions. Considering an adequate added mass, three similitude laws with the same material could be normally derived as shown in Table 1. From Table 1, a scale factor for length is S as a basic dimension.

Table 1. Three similitude laws

Quantity	Dimension	Scale Factor		
		Method I (Mass based)	Method II (Time based)	Method III (Acceleration based)
Length	L	S	S	S
Mass	M	S^3	S	S^2
Time	T	S	1	$S^{1/2}$
Stress	$ML^{-1}T^{-2}$	1	1	1
Velocity	LT^{-1}	1	S	$S^{1/2}$
Acceleration	LT^{-2}	$1/S$	S	1
Force	MLT^{-2}	S^2	S^2	S^2
Stiffness	MT^{-2}	S	S	S
Damping	MT^{-1}	S^2	S	$S^{3/4}$
Frequency	T^{-1}	$1/S$	1	$S^{-1/2}$

2.1 Method I - Mass Based

When the effect on gravity loads plays an important role, it is convenient to select a scale factor for mass as S^3. In Method I, mass distribution of prototypes is accurately simulated in scaled models and there is no need to consider an added mass. However, a scale factor for time is defined as S. Such a compression of time would have complicated the test conditions. In particular, using a conventional dynamic testing method like shaking table tests, the limitation on shaking speed could be occurred. But pseudodynamic tests being carried out in a static manner may be satisfied with Method I.

2.2 Method II - Time Based

If gravity loads can be negligible on evaluating the seismic performance of the scaled models, a scale factor for time can be chosen as a basic dimension. This method has been justified by stating that since the frequency effects are preserved, qualitative information can be obtained regarding the seismic performance of the structure subjected to the given earthquake[9]. However, it is possible in the elastic range. In the inelastic range, it should be realized that since the forces are no longer proportional to the displacement, the exact response of the structure couldn't be obtained. Method II has been applied to the pseudodynamic tests[8-10,12]. In case of the shaking table tests, an added mass is needed because a scale factor for mass is S.

2.3 Method III - Acceleration Based

Although acceleration inputs as an artificial loading could be controlled, the acceleration of gravity is not controlled artificially. Thus, a scale factor for acceleration should be unity to simulate both gravity and inertia forces. In Method III, added mass and compressed time are needed for performing the real-time dynamic tests because scale factors for mass and time correspond to S^2 and $S^{1/2}$, respectively. However, it is an ideal method for the pseudodynamic tests that deals with mass and time numerically assumed in a computer.

3. PRELIMINARY TEST

This study is to verify the problems of scaled model tests and then search the feasible relationship between scaled model and prototype. The specimens used are cantilever steel columns and the dimensions of prototype are shown in Figure 1. The specimens were fabricated of SS400 steel[14] and the material properties determined from tensile coupon tests[15] are presented in Table 2. The similitude law applied is Method III and the detail dimensions and characteristics of prototype and scaled model are summarized in Table 3. Figure 2 shows the test setup for the specimens.

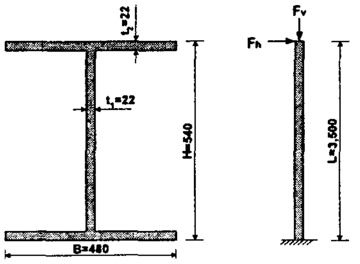


Figure 1. Dimensions of specimen(prototype)

Figure 2. Test setup for specimens

Table 2. Material properties of steel

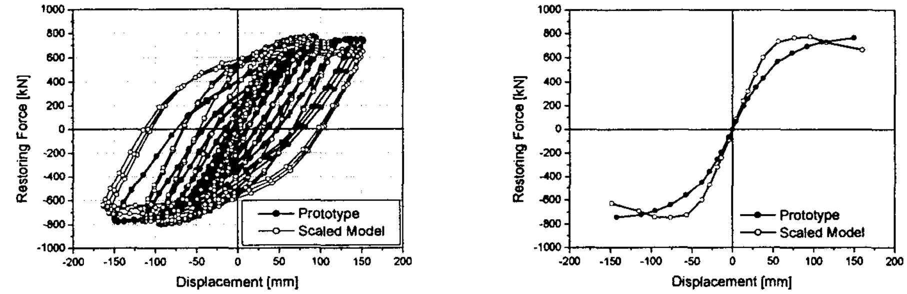
Coupon	E [Gpa]	σ_y [Mpa]	ε_y [%]
Prototype	203	311	0.153
Scaled Model	196	324	0.165

Table 3. Dimensions & characteristics of specimens

Item	Prototype	Scaled Model (S=3.79)
Height H [mm]	540	142.36
Width B [mm]	480	126.55
Thickness t [mm]	22	5.8
Length L [mm]	3500	922.73
Mass M [kg]	77.67×10^3	5.40×10^3
Stiffness K [N/m]	23.66×10^6	6.24×10^6
Frequency f [Hz]	2.78	5.41
Yield Force F_y [N]	458.98×10^3	31.90×10^3
Yielding Displ. δ_y [m]	19.4×10^{-3}	5.11×10^{-3}

3.1 Quasistatic Tests

At first, hysteretic behavior of the specimens was obtained from the quasistatic tests. In this study, constant axial force corresponding to structural mass is applied to be 15% of the compressive strength of steel columns. Also, the cyclic loadings in displacement control are exerted to the specimens horizontally. In an initial stage up to $1.0\delta_y$, the number of cycles is only one in each step and the displacement increment is $0.25\delta_y$. Beyond this stage, in each step three cycles with the displacement increment of $1.0\delta_y$ are applied up to $8.0\delta_y$. The quasistatic test results of the specimens are presented in Figure 3.



(a) Hysteresis loop (b) Envelope

Figure 3. Quasistatic test results of the prototype and scaled model

Since the scaled model was designed with a length scale factor of 3.79, stiffness of the prototype could be calculated to be 3.79 times higher than the scaled model. However, the stiffness values measured directly from both the prototype and the scaled model appeared as lower than the designed values. The stiffness decrease ratio of the prototype was higher than the scaled model. Therefore, the stiffness ratio of the prototype to the scaled model was estimated as 3.07. It can be presumed that stiffness reduction is reasoned by an excessive welding.

From the scaled model tests, only scale factor is generally considered to estimate the structural performance of prototype. Figure 3 shows a comparison between the prototype test results and the estimated results from scaled model test using the designed scale factor(S=3.79). According to Figure 3, it is noticed that over-yield-strength is expected as estimating the response of prototype with the designed scale factor. However, it is not appropriate to compare because the stiffness of prototype and the estimated value are not identical in the elastic range.

3.2 Observation of Plastic Hinge Zone

The behavior of plastic hinge zone is observed by strain gauges during the quasistatic tests. Figures 4 and 5 show the locations of strain gauges attached on the plastic hinge zone and their appearances. In this study, it can be assumed that plastic hinge zone is located within 1.5B high from a clamped end.

Both the prototype and the scaled model show nearly elastic behavior at strain gauges levels 6 and 7 throughout the whole quasistatic testing procedure. Thus, from this observation, it could be confirmed that assuming the plastic hinge zone to be 1.0B is appropriate, based on the measured strain variations along the strain gauge levels and the buckling location of test specimens.

Failure modes due to local buckling in flange are shown in Figure 5. It can be inferred from comparison of the measured strain values that locations of local buckling observed from the prototype and the scaled model could be different.

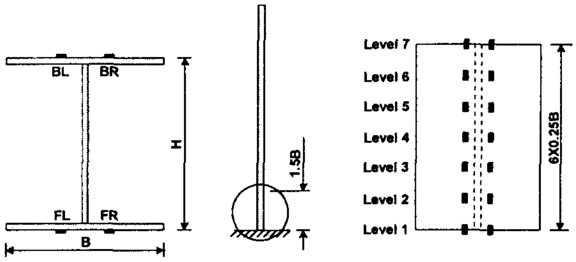
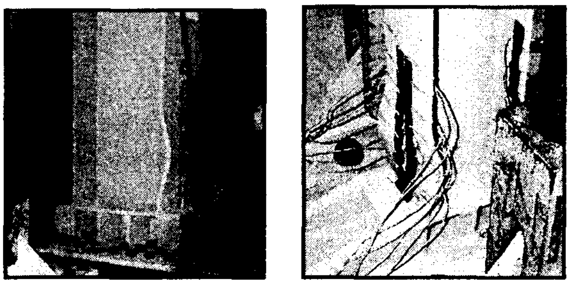


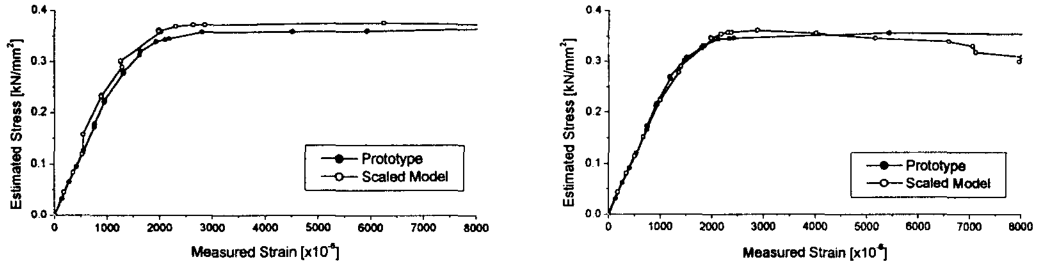
Figure 4. Location of strain gauge levels



(a) Prototype (b) Scaled Model

Figure 5. Test specimens attached with strain gauges

Stress-strain curves on strain gauge levels 5 and 6 at the location of FR are plotted in Figure 6. At each gauge level, the estimated stress can be calculated by using flexural moment derived from the measured forces.



(a) Gauge level 5(=1.0B) (b) Gauge level 6(=1.25B)

Figure 6. Stress-strain curves at the location of FR

From the test results, yield strains are obtained near about 1,500με, which is a little higher than nominal value of 1,200με. The test results from other locations are almost similar with this phenomenon.

The above results are nearly identical and particularly the higher yield stress on the scaled model is not examined. Thus, it can be observed that there is no material-based size effect in the test specimens. Stress-strain curves obtained from the quasistatic tests and the coupon tests are compared in Figure 7.

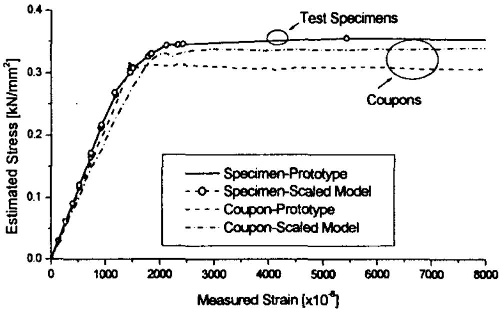


Figure 7. Comparisons of stress-strain curves

4. PSEUDODYNAMIC TEST

4.1 Verification of Three Similitude Laws

To verify the feasibility of three similitude laws presented in Table 1, pseudodynamic tests were performed with the same specimens used in the quasistatic tests. The earthquake accelerogram as an input load is two-times intensity of the 1940 El Centro earthquake(N-S Component) record shown in Figure 8. As previous researchers, S. Kumar et al.[9] conducted an experimental study using methods I and II on the concrete filled steel pier specimens. They made a conclusion that the responses of each similitude law are not different from the same scale factor for length.

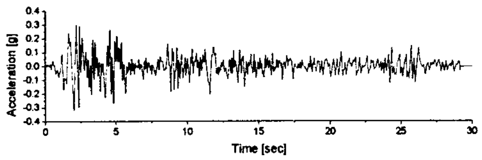
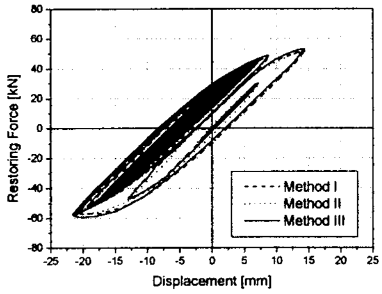
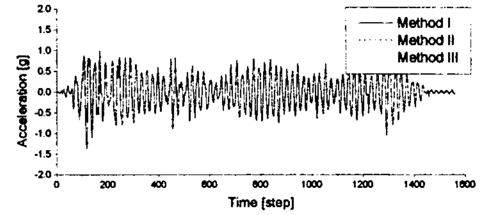


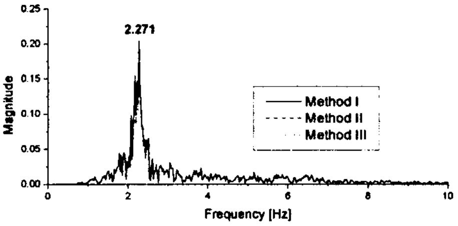
Figure 8. The 1940 El Centro earthquake ground acceleration (PGA=0.319g)



(a) Hysteresis loop



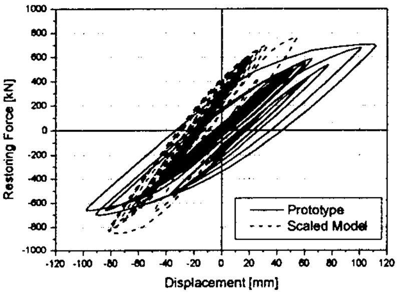
(b) Acceleration response



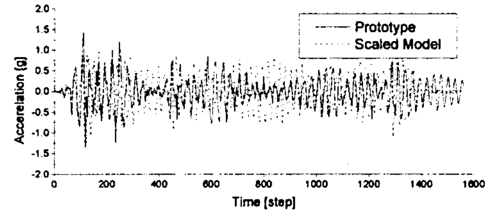
(c) Power spectrum

Figure 9. Pseudodynamic test results of scaled models using three similitude laws

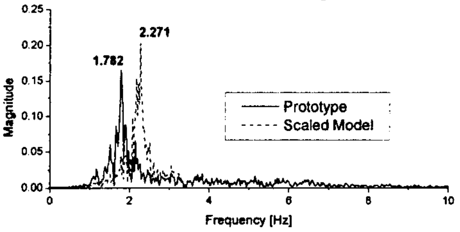
This study expanded the previous works to compare three similitude laws(Methods I, II & III), which also have the same scale factors for length and force. The test results of three similitude laws are presented in Figure 9. From the comparison of pseudodynamic test results, it can be confirmed that the inelastic responses are practically coincident, when the same scale factors for length and force are used even in different similitude laws.



(a) Hysteresis loop



(b) Acceleration response



(c) Power spectrum

Figure 10. Pseudodynamic test results of the prototype and scaled model

4.2 Comparison of Prototype and Scaled Model

Due to stiffness distortion induced from fabrication errors and test setup conditions, fundamental frequencies on the specimens were varied with their stiffness reductions. Consequently, it is difficult to directly compare the test results from the prototype and the scaled model because there may be a phase shift in the inelastic responses.

Seismic responses of the prototype and the scaled model are compared in Figure 10. It can be unreasonable that the seismic performance of prototype structures is evaluated using the scaled models with a distorted stiffness.

5. MODIFIED SIMILITUDE LAW

In the scaled model test, it is not easy to avoid stiffness distortion of specimens. Thus, most of experimental errors including the testing procedure can be reflected to stiffness distortion of specimens. Also, it is difficult to simulate the boundary conditions of the prototype in the scaled model, precisely.

To compensate the experimental errors, a scale factor for stiffness, K_r, can be substituted by a stiffness ratio, S^*, which means the measured stiffness ratio of prototype to scaled model. Therefore, it is desirable that a stiffness ratio, S^*, is considered to compensate the scaled model in order to estimate the seismic performance of the prototype properly. Defining a stiffness ratio as S^*, a scale factor for force can be modified as given in Equation (1).

$$F_p = K_p \cdot \delta_p = S^* K_m \cdot S\delta_m = S^* S \cdot K_m \delta_m = S^* S \cdot F_m \tag{1}$$

The subscripts, p and m, mean quantities of prototype and scaled model, respectively. And the subscript, r, means a scale factor of prototype to scaled model. In this way, using S and S^*, scale factors for acceleration and frequency can be expressed as Equations (2) and (3).

$$a_r = \frac{a_p}{a_m} = \frac{F_p / M_p}{F_m / M_m} = \frac{F_p}{F_m}\frac{M_m}{M_p} = S^* S \cdot \frac{1}{S^2} = S^* S^{-1} \tag{2}$$

$$f_r = \frac{f_p}{f_m} = \sqrt{\frac{K_p / M_p}{K_m / M_m}} = \sqrt{\frac{K_p}{K_m}\frac{M_m}{M_p}} = \sqrt{S^* \cdot \frac{1}{S^2}} = S^{*0.5} S^{-1} \tag{3}$$

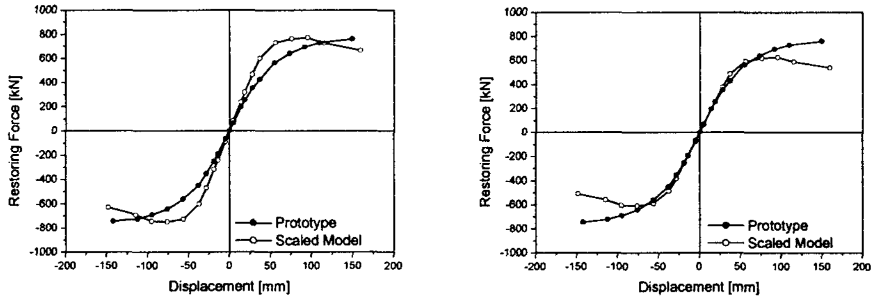
Based on a stiffness ratio, S^*, the quantities derived are summarized in Table 4. The modified similitude law proposed in this study has a problem that a scale factor for stress is not unity. It is reasoned that S and S^* is not identical. However, considering the difficulties in matching the stiffness ratio to S, the modified similitude law may be more appropriate in engineering perspective.

Table 4. Modified similitude law considering stiffness ratio

Quantities	Scale Factors	Quantities	Scale Factors
Force F_r	$S^* S$	Acceleration a_r	$S^* S^{-1}$
Time T_r	$S^{*-0.5} S$	Velocity v_r	$S^{*0.5}$
Frequency f_r	$S^{*0.5} S^{-1}$	Stress σ_r	$S^* S^{-1} \neq 1$

5.1 Compensation of Stiffness Distortion

When frequency shift is caused by stiffness distortion, it is difficult to directly compare the test results of the prototype and the scaled model with distorted dynamic properties. By compensating of considering the stiffness ratio, the elastic stiffness of the scaled model can be closed to the prototype.

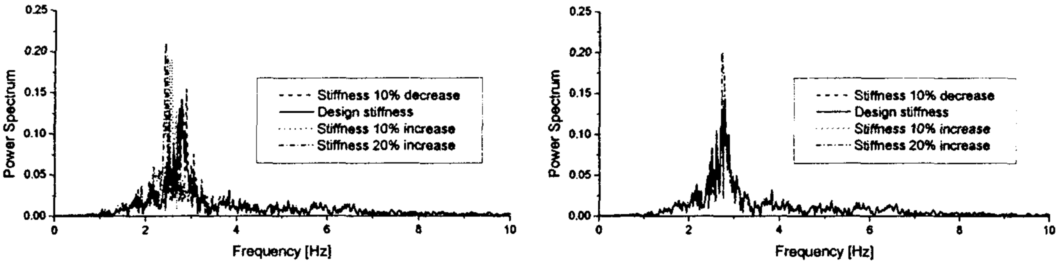


(a) Uncompensated (b) Compensated

Figure 11. Compensation of the quasistatic test results by considering stiffness ratio

From Figure 11(a), it is shown that the scaled model has 8 to 13 percent of higher yield stress than the prototype and a failure of local buckling after yielding happens to the scaled model.

A simulation study is conducted numerically using a bilinear hysteretic model in order to investigate the influence of stiffness distortion. The inelastic responses of the target system with design stiffness and the distorted systems with stiffness variation are converted to power spectra and then compared in Figure 12(a). Figure 12(b) shows that considering the stiffness ratio due to stiffness variations can effectively compensate the inelastic response.



(a) Uncompensated (b) Compensated

Figure 12. Numerical simulation results according to stiffness variations

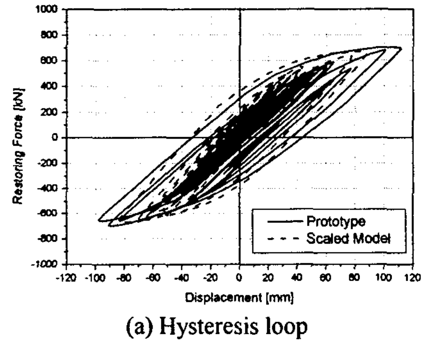
The inelastic responses compensated are nearly close to the behavior of target system although there are some differences in peak value according to stiffness variations. However, the inelastic responses uncompensated show that peak values and hysteretic properties in each distorted system are not comparable with target system. Finally, numerical simulation results according to stiffness variations are summarized in Table 5.

Table 5. Numerical simulation results according to stiffness variations

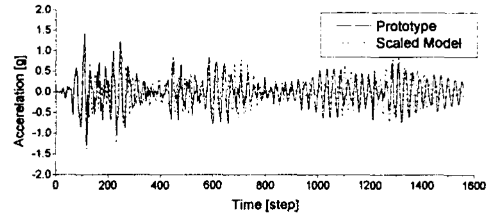
Cases	Fundamental frequencies according to stiffness variations [Hz]			
	10% decrease	Design value	10% increase	20% increase
Uncompensated	2.881	2.783	2.564	2.417
Compensated	2.783	2.783	2.783	2.734

5.2 Verification Test of Modified Similitude Law

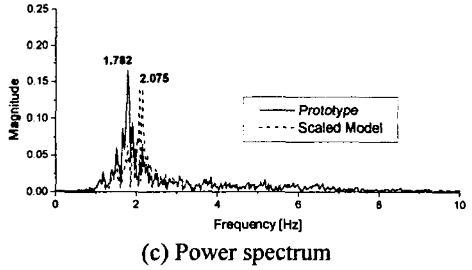
Based on the modified similitude law proposed in this study, pseudodynamic test was carried out with the same specimen as the scaled model tests before. Using the modified similitude law, scale factors for dynamic parameters of the scaled model are adjusted depending on the measured stiffness ratio, S^*. Therefore, the modified scaled model can be applied to pseudodynamic test algorithm. In this study, the elastic stiffness ratio of 3.13, which corresponds to 17.4 percent of stiffness variation, was obtained experimentally from the specimen used.



(a) Hysteresis loop



(b) Acceleration response



(c) Power spectrum

Figure 13. Pseudodynamic test results by using the modified similitude law

According to pseudodynamic test results shown in Figure 13, the seismic responses of the modified scaled model are much improved as compared with the results in Figure 10. Overall, it is believed that the modified similitude law considering stiffness ratio could be effective in simulating the seismic response of prototype structures.

CONCLUSIONS

In this study, consistency of three similitude laws based on mass, time or acceleration, respectively, are verified by the pseudodynamic scaled model tests under the same scale factors for length and force. From the comparisons of pseudodynamic test results with three similitude laws, it can be confirmed that the inelastic responses are practically coincident, when the same scale factors for length and force are used even in different similitude laws. And a modified similitude law considering both a scale factor for length and a stiffness ratio is proposed. It can compensate frequency errors of scaled models and directly apply the scaled model test results to prototypes. Overall, it is believed that the modified similitude law considering stiffness ratio could be effective in simulating the seismic response of prototype structures. Also, this application provides an opportunity to use the results in designing the scaled models for shaking table tests.

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