Target Tracking Accuracy and Performance Bound

Dong-Hun Yoon*, Seok-Won Yeom, Dong-Uk Yoon, Han-Seok Ko
II&SP lab. Dept. of Electronics Engineering Korea University
5-1 Anam-dong Sungbuk-ku Seoul 136-701 Korea.
{koppie, swyeom, duyoon, hsko}@ispl.korea.ac.kr

Abstract

This paper proposes a simple method to measure system's performance in target tracking problems. Essentially employing the Cramer-Rao Lower Bound (CRLB) on tracking accuracy, an algorithm of predicting system's performance under various scenarios is developed. The input data is a collection of measurements over time from sensors embedded in Gaussian noise. The target of interest may not maneuver over the processing time interval while the own ship observing platform may maneuver in an arbitrary fashion. The proposed approach is demonstrated and discussed through simulation results.

Keywords : Cramer-Rao Lower Bound (CRLB), Observability, Target Motion Analysis (TMA).

1. Introduction

The tracking problem of unknown marine platforms using measurements is generally referred to as target motion analysis (TMA). The aim of TMA is to estimate the parameters such as position, course, and speed of a (maneuvering) platform, given a time sequence of measurements. A basic requirement for the TMA is the system's observability, i.e., the existence of a unique tracking solution.

Previous works on observability of target tracking have been widely investigated mainly of ocean environment. Nardone solved a third-order nonlinear differential equation explicitly and established the necessary and sufficient conditions for TMA observability [1]. Torrieri provided analysis statistically for stationary transmitters [2]. Many explicit investigations were performed [3-5] with mathematical rigor. But it is difficult to maintain good physical insight into the problem via these approaches under various conditions and with a yes-no type answer [6].

When designing a tracking system it is important to be able to predict the system performance under a number of conditions. A technique is needed which quickly answers such questions without requiring the design and testing of an actual tracking system. A method for system performance measure employing the Cramer-Rao Lower Bound (CRLB) on tracking accuracy is proposed. Its ease of implementation is demonstrated while requiring fewer system resources.

We will first describe the concept through the "Performance Bound" in Section 2 and formulate the problem to be worked on. And then we present relevant examples by reflecting the performance measurement procedure in Section 3. A concluding remark is given in the Section 4.

2. Performance bound

2.1 Problem formulation

The conventional system for tracking a target can be considered to be a mathematical function that maps an input vector to an output vector. The input vector is a set of error free measurements to which the measurement noise and bias are added. The measurement occurs at arbitrary times and may be of different types (bearing, time delays, frequencies, etc.). In this investigation we assume there are *m* measurements, the measurement noise is zero mean Gaussian of known variance, and the measurement noise is independent for different measurements.

The output is usually a four-element state vector, which describes the position and motion of the target. This assumes straight line non-maneuvering tracks. The size of the state vector can be increased by adding accelerations or other unknown quantities. We assume the four state vector as $[x, y, \dot{x}, \dot{y}]$.

In addition to a $m*1$ measurement vector for an input and a $4*1$ state vector for an output, two covariance matrices are included. Measurement error statistics are contained in a $m*m$ matrix. For independent measurement errors this matrix is diagonal with the i-th diagonal element being the variance of the i-th measuerment. The output covariance matrix contains the statistics for the output state vector. It is seldom diagonal, and the off diagonal terms are influenced by correlation between the estimated state elements.

The problem is depicted in Fig. 1. ***r*** is a measurement vector with covariance matrix, R, which contains measurement variance. $\boldsymbol{x}$ is a state vector with covariance matrix P, which contains the necessary information to describe the accuracy of the tracker. Variance of the estimated state elements are represented by diagonal terms, and ellipses of position uncertainty can be plotted. Therefore, a method to compute P is sufficient for allowing a study of tracker performance.

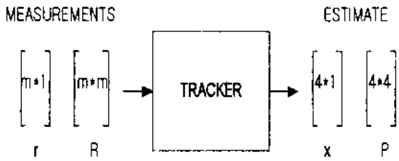

Figure 1. Tracker model

2.2 Cramer-Rao Lower Bound

The theoretical bounds on tracking performance can be computed from the Cramer-Rao lower bounds (CRLB) [7]. The CRLB is defined as the inverse of the Fisher information matrix J, where the elements of J are computed as

$$J_{ij} = E\left[\frac{\partial \ln p_{r|x}(\boldsymbol{r} \mid \boldsymbol{x})}{\partial \boldsymbol{x}_i}\frac{\partial \ln p_{r|x}(\boldsymbol{r} \mid \boldsymbol{x})}{\partial \boldsymbol{x}_j}\right] \quad (1)$$

$$= -E\left[\frac{\partial^2 \ln p_{r|x}(\boldsymbol{r} \mid \boldsymbol{x})}{\partial \boldsymbol{x}_i \partial \boldsymbol{x}_j}\right] \quad (2)$$

where the vector $\boldsymbol{x}$ is the target state and $p_{r|x}(\boldsymbol{r} \mid \boldsymbol{x})$ is the probability density function (pdf) of the obseravtion given state, also known as the *likelihood function*. If $p_{r|x}(\boldsymbol{r} \mid \boldsymbol{x})$ is taken to be Gaussian, then the natural logarithm of the pdf is proportional to the mean square error (MSE), and the computation effort is simplified considerably. Usually the measurements are nonlinear with respect to the target state and the linearization of the measurements with respect to the target state $\boldsymbol{x}$ is required.

In concept, if all measurements are error free and the motion model for the target track is correct, the tracker will produce an error free state estimate. However, it would be necessary to have a sufficient number of measurements; i.e. $m>4$. Even in this case there exists a relationship between each of the relationship between each of the m measurements and the *4*-state elements. Were there some error in any measurement it would cause some perturbation in the elements of the state vector, although not necessarily all of them. To determine the tracking accuracy we need to calculate the influence of each measurement error on errors in the estimated state and then combine the effects of all of the measurements.

The relationship between changes in measuremets caused by changes in the state is contained in a *mx4* matrix of partial derivatives [2,3]:

$$M = E\begin{bmatrix} \frac{\partial r_1}{\partial x_1} & \frac{\partial r_1}{\partial x_2} & \cdots & \frac{\partial r_1}{\partial x_4} \\ \frac{\partial r_2}{\partial x_1} & \frac{\partial r_2}{\partial x_2} & & \\ \vdots & & \ddots & \\ \frac{\partial r_m}{\partial x_1} & & & \frac{\partial r_m}{\partial x_4} \end{bmatrix} \quad (3)$$

In the non-linear system, these derivatives must be evaluated with knowledge of the state. In an actual tracker the estimated state is used for this purpose and will contain some error, which causes somewhat incorrect derivatives. In our analysis we use the true state value and introduce no error from incorrect derivative values.

At this point we avoid a derivation of tracking equations but will use one relationship that is a byproduct. This important formula relates deviations in the estimated state to deviations in the measurements.

$$\delta \boldsymbol{x} = [M^T R^{-1} M]^{-1} M^T R^{-1} \delta \boldsymbol{r} \quad (4)$$

where $\delta \boldsymbol{x}$ is a *4*1* vector of changes in state and $\delta \boldsymbol{r}$ is a *m*1* vector of changes in measurements. Now P, the covariance matrix for the estimated state, is $E\{\delta \boldsymbol{x}\, \delta \boldsymbol{x}^T\}$ where E is an expected value.

$$\begin{aligned} P &= E\{[M^T R^{-1} M]^{-1} M^T R^{-1} \delta \boldsymbol{r} \\ &\qquad \delta \boldsymbol{r}^T R^{-1} M [M^T R^{-1} M]^{-1}\} \quad (5) \\ &= [M^T R^{-1} M]^{-1} [M^T R^{-1} M][M^T R^{-1} M]^{-1} \\ &= [M^T R^{-1} M]^{-1} \end{aligned}$$

where $E\{\delta y\, \delta y^T\} = R$ by definition.

This simple relationship calculates the tracking accuracy as a function of measurement errors and the geometric relationships that control the derivatives that make up the M matrix.

2.3 Issues and implementation

To calculate P, the *4x4* matrix $M^T R^{-1} M$ must be inverted. If it is singular (rank less than 4) P cannot be calculated, and the state is referred to as "unobservable." Theory may indicate that the state is observable, yet on a computer the $M^T R^{-1} M$ matrix may not invert. Numerical problems can be treated in a number of ways. Common solutions are double precision calculations, forcing $M^T R^{-1} M$ to be symmetric by averaging the *ij* with *ji* elements, rotating the state so that measurements relate directly to state elements, and scaling the state so that all diagonal elements of $M^T R^{-1} M$ are of comparable value. There is very useful information in the eigenvalues of $M^T R^{-1} M$. A zero or near zero eigenvalue indicates a non-observable state. The eigenvectors give a geometric picture of the tracking solution.

Sequential implementation is possible by inspecting the property of noise. In the *P* matrix, $M^T R^{-1} M$, *R* is a diagonal matrix if the measurement noise is independent from measurement to measurement. Then

$$P = \left[\sum_{i=1}^{m} \frac{1}{\sigma^2} \begin{bmatrix} M_{i1} \\ M_{i2} \\ M_{i3} \\ M_{i4} \end{bmatrix} \begin{bmatrix} M_{i1} & M_{i2} & M_{i3} & M_{i4} \end{bmatrix} \right]^{-1} \quad (6)$$

We have taken advantage of the diagonal structure of *R* to separate the contributions from different measurements.

Let a *4x4* matrix, *Z*, be the matrix without inverse in Eqn. (6) which can be thought of as the Fisher Information matrix. Initialize all elements of *Z* to be zero before measurement indicating no information available. For the first measurement we can calculate *Z*, $i-1$ in Eqn. (6), and add it to previous value. We then do the procedure recursively. Any time we desire to know *P*, we can invert the *Z* matrix. Note that the running sum contained in *Z* requires no matrix inversion; only the calculation of *P* requires an inversion.

3. Examples

The tracker observability plot of bearing only tracking system in Fig. 2 was presented as an example. The two-dimensional location problem can be formulated as follows. Let $\boldsymbol{x} = [x, y, \dot{x}, \dot{y}]$ be the state vector in Cartesian coordinates. The discrete-time equation for the target state assuming constant velocity is given by

$$\boldsymbol{x}(k+1) = \Phi(k+1, k)\boldsymbol{x} \quad (7)$$

where

$$\Phi = \begin{bmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

The bearing to the target is defined by the relationship.

$$\tan \beta(k) = \frac{r_x}{r_y} \quad (9)$$

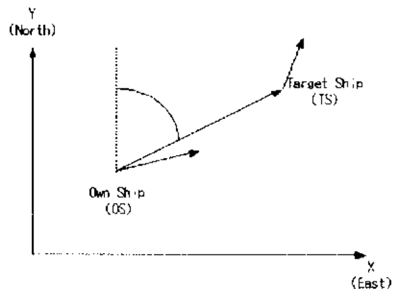

Figure 2. Problem geometry

where $\boldsymbol{r}$ is the range between own ship (OS) and target ship (TS).

Figs. 3 and 5 show the OS and TS motion scenarios. The first scenario is one-leg case. Total simulation time is 20 minutes. Target speed is 9 knots, target course is 90 degrees. Surveying area is 40000 (yards) x 40000 (yards) and each cellsize is 300 (yards) x 300 (yards). Measurement bearing uncertainty is 2.6 degrees. The output is semi-major axis in yards. All of the area is unobservable with large value in observability plot. It is a well match to the previous analysis on TMA (Fig. 4).

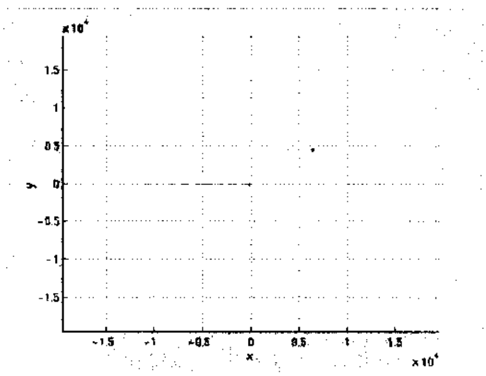

Figure 3. Scenario 1: one-leg (TS: 90 degrees)

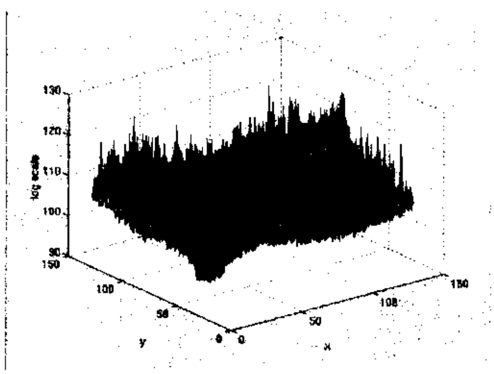

Figure 4. Observability of one-leg case

The second scenario is two-leg case. TS moving 315 degrees. The maneuver of OS is necessary in bearing only system. Fig. 5 shows the OS and TS motion with observability plot. It is easily discernable as the highly observable region and poorly observable region. The uncertainty in the highly observable region is about 100 yards to 500 yards (Fig. 6).

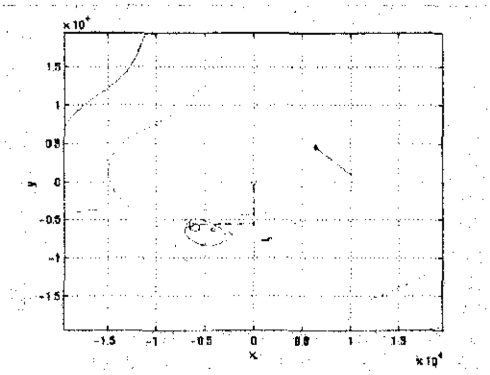

Figure 5. Scenario 2: two-leg (TS: 315 degrees) and observability plot

The techniques outlined in these paper yield a CRLB on tracking accuracy. Achieving this accuracy in a real systme depends on how many of the underlying assumptions have been violated. The assumptions are as follows. No measurement bias is present. Measurement errors are independent. Partial derivatives were evaluated at true value of state, not estimated values.

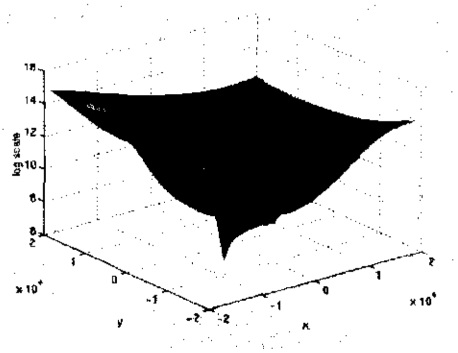

Figure 6. Observability of two-leg

4. Conclusions

Tracker observability analysis is presented. The tracking accuracy bound can be obtained via this approach and can be used to determine the tracker's performance measures.

Knowledge of the CRLB is very useful. It shows what makes the target track observable, how many measurements are needed, what accuracy is required, and what geometry is favorable for solutions, etc. Although it is hard to achieve CRLB in a real tracking system, the values show whether the attempt is justified.

The proposed tracking analysis tool is very easy to implement and use. It is far simpler than a Monte Carlo analysis in which a complete tracker must be designed along with a measurement simulator and then exercised hundreds of times. In designing an efficient tracking system, the proposed method should be used first and then followed by a complete design with Monte Carlo analysis.

Acknowledgement

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