Lee, Y., Moon, H. K., A study on the estimation of temperature distribution around gas storage cavern.
International Symposium on the Fusion Technology of Geosystem Engineering, Rock Engineering and Geophysical Exploration
Seoul, Korea, Nov. 18-19, 2003

A study on the estimation of temperature distribution around gas storage cavern

Yang Lee, Hyun-Koo Moon
Hanyang University, Seoul, Korea

Abstract: As there are many advantages on underground caverns, such as safety and operation, they can also be used for gas storage purpose. When liquefied gas is stored underground, the cryogenic temperature of the gas will affect the stability of the storage cavern. In order to store the liquefied gas successfully, it is essential to estimate the exact temperature distribution of the rock mass around the cavern. In this study, an analytic solution and a conceptual model that can estimate three-dimensional temperature distribution around the storage cavern are suggested. When calculating the heat transfer within a solid, it is likely to consider the solid as the intersection of two or more infinite or semi-infinite geometries. Therefore heat transfer solution for the solid is expressed by the product of the dimensionless temperatures of the geometries, which are used to form the combined solid. Based on the multi-dimensional transient heat transfer theory, the analytic solution is successfully derived by assuming the cavern shape to be of simplified geometry. Also, a conceptual model is developed by using the analytic solution of this study. By performing numerical experiments of this multi-dimensional model, the temperature distribution of the analytic solution is compared with that of numerical analysis and theoretical solutions.

1. Introduction

Underground gas storage facility is effective for safety, security, operation and preservation of environment. However, when low temperature gas is stored in underground cavern, cooling down of the rock around the cavern induces shrinkage and distinct effects causing relief of compressive stress, thermal cracking, opening of the existing crack and change of the rock properties, which will affect the safety of the rock masses. For the successful construction of the storage cavern it is important to investigate thermal behavior of rock masses around the cavern and to estimate temperature distribution around the storage cavern after gas is stored. In this study, an analytic solution applied transient heat transfer theory is proposed.

From 1970s, there have been many attempts to use unlined cavern for LNG storage but most of them were not successful because the cryogenic temperature (-162℃) of the LNG induced excessive thermal cracking due to thermal stress and excessive boil-off. Recently the studies on the lined cavern for LNG storage have been performed, and it was proved that the containment system of the insulating panel lining is useful. By using the insulating panel, the temperature that is contact on rock mass is kept above –50 degrees and the boil-off rate become below 0.1%/day (Kim et al., 2003). Synn et al. (1999) showed that thermal conductivity of in-situ rock is smaller about 23% and specific heat of the in-situ rock is larger about 25% than intact rock specimen by performing a study on the thermal properties of rock, distribution of joint and water condition. Glamheden (2001) investigated the change of mechanical properties of rock mass after cooling down. He showed that the results of temperature dependency test of tensile strength for air-dry specimens were scattered and the trend became uncertain. On the contrary to the air-dry samples, the results for water saturated samples showed a clear trend of temperature dependency In the test temperature between 20 and -50℃, the tensile strength of water saturated samples increased by rate 1%/℃ as temperature decrease, and the Young's modulus increased a little as temperature decrease whereas the temperature dependency of Poisson's ratio was very weak. Lee and Kim (2002) proved the change of boundary condition was the effective factor to estimate temperature distribution by comparing analytic solution with numerical calculation and measured data. They proposed the reasonable analysis procedure for prediction of temperature distribution around cold cavern. Lee and Moon (2003) estimated the temperature distribution around refrigerated cavern in 2-D by the analytic solution applied transient heat conduction in multidimensional system

2. Transient heat conduction theory

Transient heat conduction in large plane walls

Considering a plane wall initially at a uniform temperature T_i, as shown in Fig. 1, at time t=0, the plane wall is symmetric about its center plane (x=0) if the wall is placed in a large medium that is at a constant temperature T_∞

and heat transfer coefficient between the wall and the medium is h. However, the solution of the one-dimensional transient temperature distribution *T(x,t)* in a wall normally involves the parameters, such as distance (x), the thickness of the wall (L), time (t), thermal conductivity (*k*), thermal diffusivity (α), heat transfer coefficient (h), the initial temperature of solid (T_i) and the temperature of medium (T_∞). In order to reduce the number of parameters, the dimensionless temperature $\theta(x,t)$ is introduced so that transient temperature distribution *T(x,t)* in a wall can be represented in a simply form, such as

$$\theta(x,t) = \frac{T(x,t) - T_\infty}{T_i - T_\infty}. \tag{1}$$

Transient heat conduction in semi-infinite solids

Considering a semi-infinite solid at a uniform temperature T_i (Fig. 2), at time t=0, the surface of the solid at x=0 in exposed to convection by a fluid at a constant temperature T_∞, with a heat transfer coefficient h. This problem can be solved analytically for the transient temperature distribution *T(x,t)* using dimensionless temperature $\theta^*(x,t)$, such as

$$\theta^*(x,t) = 1 - \theta(x,t) = 1 - \frac{T(x,t) - T_\infty}{T_i - T_\infty} = \frac{T(x,t) - T_i}{T_\infty - T_i}. \tag{2}$$

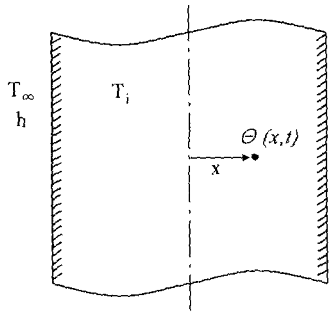


Fig. 1. Transient heat conduction in large plane wall.

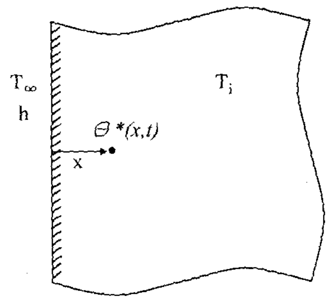


Fig. 2. Transient heat conduction in semi-infinite solid.

The exact solution of the transient one-dimensional heat conduction problem in a semi-infinite medium initially at a uniform temperature of T_i and suddenly subjected to convection at time t=0 can be obtained by heat transfer theory, as follows

$$\theta^*(x,t) = \operatorname{erfc}(z_1) - \exp\left[z_2(2z_1 + z_2)\right]\operatorname{erfc}(z_1 + z_2), \tag{3}$$

Where erfc(z) is the complementary error function, and z_1, z_2 are the dimensionless variables, expressed by $z_1 = x/2(\sqrt{\alpha t})$ and $z_2 = h\sqrt{\alpha t}/k$, respectively.

Thermal penetration depth

When the thermal boundary conditions are imposed on the surface of semi-infinite solid, the depth at which it starts to undergoes temperature change after time t is called the thermal penetration depth, and it can be approximately calculated by

$$l = \sqrt{\alpha t}, \tag{4}$$

Where α is thermal diffusivity, defined as $\alpha = k/(C_p \times \rho)$.

Transient heat conduction in multidimensional systems

The solutions for 1-D conduction for infinite and semi-infinite solids can be applied to 2-D and 3-D conduction in a variety of regular geometrical solids. The solution for a long solid bar with cross section of $a \times b$ rectangle is the intersection of the two infinite plane walls of thickness a and b, as shown in Fig. 3.

The mathematical basis of obtaining the solutions for 2-D and 3-D conduction from the intersection of two or three infinite or semi-infinite geometries lies in the fact that a partial differential equation can be solved by the separation of variables method, in which the solutions are assumed to be the product of solutions of 1-D cases, therefore, the solution for a particular finite geometry is expressed by the product of the dimensionless temperatures for the each infinite or the semi-infinite geometry. Hence, the transient temperature distribution for the rectangular bar can be formulated as

$$\theta(x,y,t) = \theta(x,t)\theta(y,t). \tag{5}$$

As the same, the solution for a cube is obtained in the form of a product of three solutions for 1-D case, such as

$$\theta(x,y,z,t) = \theta(x,t)\theta(y,t)\theta(z,t). \tag{6}$$

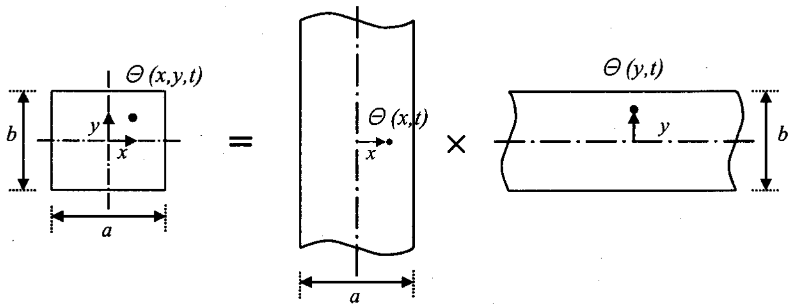


Fig. 3. Transient heat conduction in multidimensional system.

3. Heat conduction around cavern

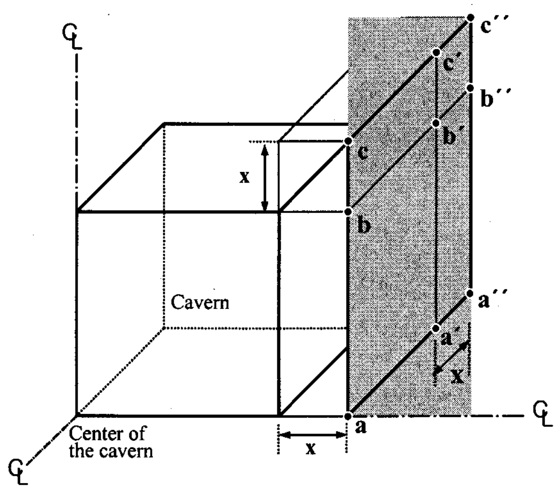


Fig. 4. Conceptual model for analytic 3-D solution.

In order to obtain heat conduction solution around refrigerated cavern, the shape of the cavern is simplified by a cube, as shown in Fig. 4. Because the cavern is a cube and the temperatures of heat source at each wall are identical of the model is considered, the geometry of the model is symmetric about the center of the cavern. In this study, therefore, the region of $x \geq 0$, $y \geq 0$ and $z \geq 0$ is considered. Using the model, the temperature of the plane at x from the cavern wall ($\square\, acc''a''$) can be predicted by the following procedure.

Initially, the temperature of the plane $\square\, abb'a'$ is $T(x,t)$ according to the semi-infinite solid solution. Then the temperature down by ambient temperature (T_i) of the rock can be calculated. Finally, the temperatures at points c, c' and c'' are determined by the multidimensional conduction theory. This process is summarized in Table 1.

Table 1. Procedure of analytic solution.

Target position	Target Temp.	Ini. Temp. of solid	Temp. of heat source	time	Applied theory
b (a′)	T(b,t)	T(x,t)	T_i	t′	Semi-infinite
a	T(a,t)	T(x,t)	T(b,t)	t′	Semi-infinite
c (a′′)	T(c,t)	T_i	T(b,t)	t	Multidimensional system
b′	T(b′,t)	T(x,t)	T_i	t′	Multidimensional system
c′ (b′′)	T(c′,t)	T_i	T(b′,t)	t	Multidimensional system
c′′	T(c′′,t)	T_i	T(c′,t)	t	Multidimensional system

Here, the penetration time (t') can be calculated by equation (4).

In this study, when the cavern ($10m \times 10m \times 10m$) is constructed in rock masses with the initial temperature of 12℃, the temperature distribution around the cavern is estimated. The temperature of heat source is -50℃ based on the containment system. The unit weight, thermal conductivity and specific heat are 2660kg/m^3, 2.08W/m℃, 885J/kg℃ respectively (Synn et al., 1999), and heat transfer coefficient is 3W/m^2℃ (Lee and Kim, 2002). The analytic solution of this study is compared with the numerical results based on the heat flux boundary condition.

4. Results

The temperature distribution for the plane of 1~6m from the wall is summarized in Tables 2, 3 and 4, and the contour line of temperature on the plane that is 4m from the wall is shown in Fig. 5.

Table 2. Temperature distribution after 90 days cooling down.

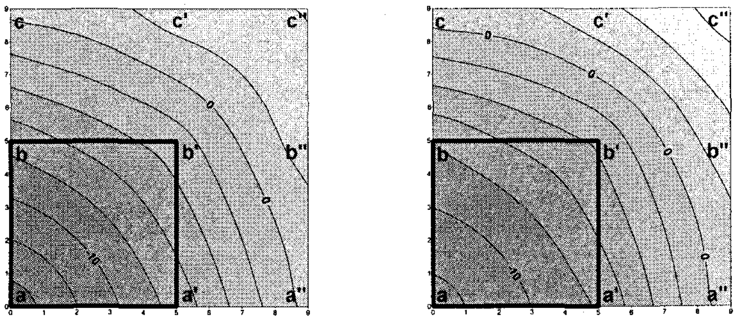
Distance (m)	Model	Temperature (℃)					
		a	b (a′)	b′	c (a′′)	c′ (b′′)	c′′
1	Analytic	-25.5	-17.4	-13.9	-9.3	-6.7	-6
	Numerical	-27.4	-18.3	-10.3	-10.7	-4.1	-0.1
2	Analytic	-13.8	-7	-1.8	-0.3	3.1	4.4
	Numerical	-15.9	-8.3	-2.3	0.7	4.3	7.7
4	Analytic	1	4.2	9	7.4	10.3	11.1
	Numerical	0.1	3.9	6.5	9.8	10.6	11.6
6	Analytic	8.2	9.4	11.6	10.5	11.8	12
	Numerical	7.9	9.3	10.2	11.7	11.8	12

Table 3. Temperature distribution after 180 days cooling down.

Distance (m)	Model	Temperature (℃)					
		a	b (a′)	b′	c (a′′)	c′ (b′′)	c′′
1	Analytic	-30.9	-21.5	-19.5	-12.4	-10.8	-10.5
	Numerical	-31.8	-23.2	-15.3	-16.4	-9.5	-5.4
2	Analytic	-21.1	-12.4	-8.7	-3.8	-1.4	-0.6
	Numerical	-22.2	-14.4	-7.8	-5.4	-0.8	3.4
4	Analytic	-7.3	-1.7	3.7	3.9	7.1	8.3
	Numerical	-7.1	-2.2	1.6	6.1	7.7	10.1
6	Analytic	1.6	4.8	9.3	7.9	10.5	11.2
	Numerical	2.3	4.9	6.8	10.3	10.8	11.7

Table 4. Temperature distribution after 360 days cooling down.

Distance (m)	Model	Temperature (℃)					
		a	b (a′)	b′	c (a′′)	c′ (b′′)	c′′
1	Analytic	-35	-24.8	-23.7	-14.7	-13.8	-13.7
	Numerical	-34.6	-26.9	-19.5	-20.9	-14.2	-10.7
2	Analytic	-26.9	-16.7	-14.4	-6.5	-5.1	-4.7
	Numerical	-26.4	-19.1	-12.7	-10.8	-5.9	-1.6
4	Analytic	-15.3	-7.3	-3	0.6	3.1	4.1
	Numerical	-12.9	-7.8	-3.7	1.3	3.5	7.1
6	Analytic	-6.6	-1	4.2	4.6	7.6	8.7
	Numerical	-3.4	-0.3	2.2	7.2	8.2	10.4



(a) Analytic (b) Numerical

Fig. 5. The contour line at x=4m after 360 days cooling down.

Except position b′ and c′′, the analytic solution close around the cavern shows higher temperature than numerical solution but the temperature of the analytic solution par from the wall is smaller than the numerical solution. This tendency increases as cooling time elapses. On the contrary, analytic solution is smaller near the wall and larger far from the wall at position b′. Generally, the temperature distribution of the numerical solution is estimated smaller than that of the analytic solution and the maximum difference between the analytic and numerical occurs at position c′′.

5. Conclusions

In order to predict the 3-D temperature distribution around refrigerated cavern, the geometry of cavern is simplified by a cube and the analytic solution applied transient heat transfer in multidimensional system is suggested. By comparing the results of the analytic solution suggested in this study and those of the numerical analysis allowed simulation for transient heat conduction in material, it is shown that between the results is less than 5% within the temperature range from –50 to 12℃. It is clear that the analytic solution applied transient heat conduction in multidimensional system can be useful for prediction of temperature around storage cavern and design the supports in initial construction stage

As the time elapses and the distance increases, it is reasonable to assume that the overlap of heat sources on the cavern walls affects the temperature distribution. Therefore there is a limit that the complicated heat transfer in 3-D is solved by analytic proposed in this study. However, if the conduction theory of semi-infinite conduction with limited heat source and multidimensional system which has different heat source on each surface is established, the analytic solution containing the actual cavern shape and the thermal anisotropy of rock mass can be developed.

The temperature around gas storage cavern is affected of the size of cavern, boundary conditions and the temperature dependency of thermal properties of rock masses, which are not mentioned in this study, but in near future, a study about the effects of those will be achieved.

References

Glamheden, R., 2001, Thermo-mechanical behaviour of refrigerated caverns in hard rock, Ph. D. Thesis, Department of Geotechnical Engineering, Charmers University of Technology, Sweden.

Kim, H. Y., Amantini, E., Chanfreau, E., 2003, Pilot project: Lined cavern LNG storage, Journal of Korean Society for Geosystem Engineering, Vol. 40, No. 2, 140-145.

Kirk D. Hagen, 1999, Heat transfer with applications, Prentice-Hall.

Lee, D. H., Kim, H. Y., 2002, Analytic and numerical model study to predict the temperature distribution around a underground food cold storage pilot cavern, Tunnel & Underground, Journal of Korean Society for Rock Mechanics, Vol. 12, No. 3, 27-35.

Lee, Y., Moon, H. K., 2003, A study on the estimation of temperature distribution around low temperature gas storage cavern. Proceedings of 80^{th} Symposium of Korean Society for Geosystem Engineering, 229-233.

Synn, J. H., Park, Y. J., Kim, H. Y., Park, C., 1999, Transient heat conduction in rock mass around arch shape cold storage cavern and estimation of in-situ thermal properties, Tunnel & Underground, Journal of Korean Society for Rock Mechanics, Vol. 9, No. 1, 27-35.

Yunus A. Çengel, 1999, Heat transfer; A practical approach, McGraw-Hill.