# 3D AE source location considering the anisotropy of elastic wave velocity under triaxial compression

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Abstract: We considered the variation of elastic wave velocity due to the anisotropy of rock materials and stress level for acoustic emission (AE) source location in cylindrical rock specimens. Elastic wave velocity and AE were measured for Keochang granite and Yeosan marble under various axial stresses and confining pressures. Partition approximation method was suggested and it was compared with the difference approximation method and the least square method.

#### 1. Introduction

The source location by acoustic emission (AE) is a method used to find an initiated/propagated fracture position by using the AE velocity and time to sensors. AE is transmitted as an elastic wave such as compression wave (P-wave) and shear wave (S-wave), and its velocity is changed according to the stress in the medium material (Lockner et al., 1977). The variable velocity of the elastic wave has been applied to source location by many researchers (Ko, 1983, Kim, 1990, Lee et al., 1997).

Most in-situ rocks show anisotropy in the traverse velocity of the elastic wave (Hartmut et al., 1997, Park, 1995, Sayers et al., 1990) and the in-situ rocks are also under triaxial stress rather than uniaxial stress.

In this study, we observed the variation of the elastic wave velocity due to the variation of stress and anisotropy of rock specimens undergoing triaxial tests and applied the variation of elastic wave velocity to the AE source location. We also modified the table look-off method of the AE source location and compared its efficiency with other methods.

# 2. Theory for AE source location

#### Table look-off method

After dividing a specimen into virtual unit boxes, an error in traverse time at each AE sensor is calculated assuming the AE source to be the center of each box. The center position of the box having the minimum error is considered as the AE source(佐藤嘉晃, 1985). The traverse time is calculated by dividing the distance between a sensor and the virtual box center by the elastic wave velocity. The difference between the calculated time and a measured one is the time error. The time error summed for the all sensors is compared with the previous cases where different virtual boxes were taken into account.

This method has an advantage because its theory is very simple and programming is easy, and local solutions or boundary problems can be avoided. One drawback, however, is that it can be very time consuming when the specimen size increases or higher accuracy is required.

#### Least square method

All of the observed values include some amount of measurement error, and many of the engineering problems are over-determined, which means that the number of measured values is bigger than the number of equations necessary to define the problem. The least square method (Marquardt, 1963) gives a solution of these problems, which is obtained by taking the smallest sum of the squared errors. The error is, of course, the difference between the theoretical value and the measured value.

An initial value for the AE source location by the least square method, however, should be carefully determined so that the calculation result does not give a local solution. This initial value problem has been solved in this study by adopting the table look-off method, which finds a proper starting point among the center points of the roughly-divided boxes.

## Difference approximation method

In this method, the source location is not affected by the position of AE sensors, and the accuracy of estimation increases as the number of the sensors increases (勝山邦久,1985). The most outstanding merit of this method is that the amount of calculation is relatively small.

Because an initial value problem exists in the difference approximation method, as in the least square method, the initial position has been set by a central point of a rock specimen in this study.

## Partition approximation method

This method is a kind of table look-off method modified by authors to reduce the computer memory use and enhance the calculation speed. The specimen is virtually divided into small regular hexahedron boxes and then, a box that shows the minimum error in terms of the traverse time of an elastic wave is selected. This procedure is repeated on the currently redefined domain including the minimum error box and its surrounding boxes until the solution accuracy meets a pre-defined criterion.

One of the merits of this method is that it does not require any initial values.

# 3. Equipments for experiments

# Rock specimen

NX size cores were taken from Keochang granite and Yeosan marble in Korea. The cores of the same rock type were managed to have the same boring direction considering the anisotropic characteristics of the rock. The specimens having a length/diameter ratio of 2 were kept in room condition for more than one week after they were dried at 105 °C for 24 hours.

Fundamental physical properties of the rock such as density and porosity were checked, and the horizontal traverse velocity of the elastic wave was measured at every 30° along the circumferences of specimens.

## Measurement of the elastic wave velocity under triaxial stress

P-waves having frequencies of 1 kHz ~ 10 MHz were transmitted through each rock specimen under triaxial stress in one vertical direction and in two horizontal directions to check its traverse velocity. The two horizontal directions were the directions in which the P-wave velocity was the maximum and minimum, respectively. The load and strain were recorded at every 0.5 seconds. Figure 1 shows the whole cross section of the triaxial test chamber, a rock specimen, and the elastic wave measuring equipment.

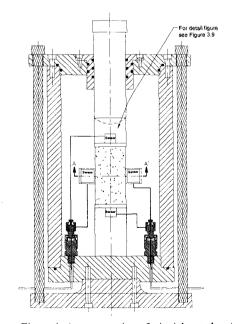


Figure 1. A cross section of triaxial test chamber.

The transducer holding boxes were designed to protect the P-wave transducers from axial and confined stresses in the triaxial test chamber. Two kinds of holding boxes were made: one for the vertical measurement of the P-wave and the other for both horizontal measurements. The vertical holding box was made of heat-treated carbon steel, SN45C, and the horizontal holding boxes of aluminium. The cross sectional view of the transducer holing boxes is shown in Figure 2.

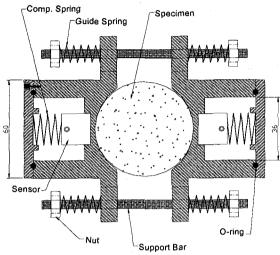


Figure 2. A cross section of transducer holding box for horizontal measurement (unit : DII).

#### AE source location

MISTRAS 2001 system of Physical Acoustics Corporation (PAC) was used for AE signal processing. MISTRAS 2001 has functions of measurement, storing and analysis of AE signals. The AE from a rock specimen under triaxial stress is perceived by piezoelectric sensors and stored in a computer after being amplified by  $40 \text{ db} \sim 60 \text{ db}$ .

Circular piezoelectric sensors having a diameter of 7mm and a thickness of 1.5mm have been adopted considering the AE frequency range of  $100 \text{ kHz} \sim 2 \text{ MHz}$ . Both sides of the piezoelectric material are connected to copper electrodes and one side is glued onto the rock specimen by an epoxy.



Figure 3. Piezoelectric material sensor used in AE test.

Sampling rate of the AE signal was set to 8MHz, which means that the electric wave is sampled every  $1/8 \mu s$ . Lower and upper limits of a filter were set to 100 kHz and 1200 kHz, respectively. Each AE event was stored for 256  $\mu s$  starting at 60  $\mu s$  before the trigger point of the event.

## 4. Test and discussion

#### P-wave velocity according to the confining pressure

To confirm the anisotropy of rock material in the horizontal direction, P-wave traverse velocity was checked at every 30° around the circumference of each specimen. After this, a horizontal transducer with its holing boxes was installed along the maximum or minimum direction of the traverse velocity. The sensor-installed specimen was

concealed with silicon to prevent the pressurized oil from penetrating the specimen or sensor holding boxes. Tests were conducted after drying the silicon for 24 hours.

The initial axial stress was set to 17.15 MPa to reduce the influence of silicon intervened between the sensors and the upper/bottom sides of a specimen. The confining pressure was set to a value a little less than the initial axial stress and kept constant during the triaxial tests. The axial load increased by 1 ton per minute and the traverse velocity of the P-wave was measured at every 0.5 tons of increment of the axial load.

Eighteen specimens for each rock type-Keochang granite and Yeonsan marble- were used for the test. The confining pressure was set to 4.9 MPa, 9.8 MPa or 14.7MPa. For each confining pressure, tests were carried out; one was for the maximum horizontal velocity and the other for the minimum horizontal velocity. The horizontal traverse velocities from both tests were compared with each other at every percentage of the axial stress.

#### 1) Yeosan marble

At three stages of the confining pressure, the vertical and horizontal traverse velocities of P-wave were measured Difference in the initial velocity in the three directions in Figure 4 and Figure 5 was due to the fact that the initial axial stress was higher than the confining pressure and the inherent anisotropy existed in the rock specimen.

Three kinds of the P-wave velocity increased as the axial stress increased. The P-wave velocity increased relatively steeply at the initial stage, considered as an elastic deformation stage. In the elastic deformation stage, small fissures of the rock are closed as the (compressive) stress increases, and thus increasing the rock density. After the initial stage, the velocity slope decreased in the vertical direction and became low or flat in the horizontal direction before the failure. In the latter stage, fractures were initiated and propagated in the specimen, reducing the traverse velocity.

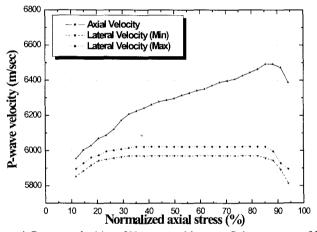


Figure 4. P-wave velocities of Yeosan marble at confining pressure of 2.45 MPa.

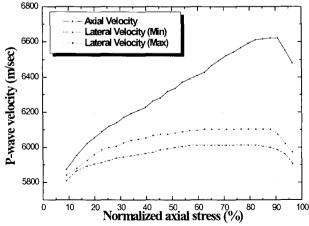


Figure 5. P-wave velocities of Yeosan marble at confining pressure of 3.98 MPa.

We can see that the three kinds of traverse velocity increased as the confining pressure increased. The anisotropy confirmed at the initial measurement before the triaxial tests is shown in the figures, but the degree of anisotropy seems much lowered. The difference between the maximum and minimum horizontal velocities was about 1,000 m/sec for the initial measurement but decreased to 100 m/sec, at best, in Figures 4 and 5.

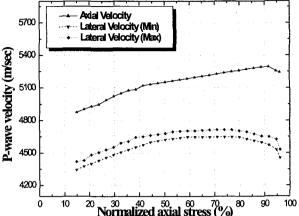


Figure 6. P-wave velocities of Keochang granite at confining pressure of 4.9 Ma.

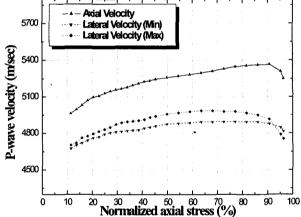


Figure 7 P-wave velocities of Keochang granite at confining pressure of 9.8 Ma.

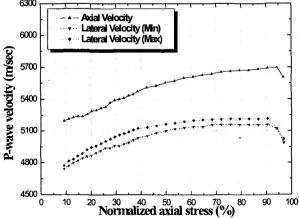


Figure 8. P-wave velocities of Keochang granite at confining pressure of 14.7 Ma.

## 2) Keochang granite

As in the case of Yeosan marble, P-wave velocity was measured in each direction under three kinds of constant confining pressures.

Figure 6~8 show that the P-wave velocity increases as the axial stress increases making their relationship similar to a bi-linear function as in the case of Yeosan marble. The transition points between the linear elastic deformation and the plastic deformation (fracture-occurring) are observed at about 40% of the normalized axial stress.

# Comparison of source location algorithms

Six piezoelectric sensors were glued on the rock specimens. Each sensor was horizontally  $60^{\circ}$  apart from adjacent ones. Three of them were vertically 75mm apart from the bottom face and the other three were 30mm apart from the same face. Source location tests were conducted after 24 hours of drying silicon brushed on the sensor-installed specimen. As in the P-wave velocity measurement, the axial load was increased by one ton per minute until the break down point and the confining pressure was set to 4.9MPa, 9.8MPa or 14.7MPa for Keochang granite, and 2.45MPa or 3.92MPa for Yeosan marble.

The AE source location was analyzed by 4 kinds of algorithms: difference approximation method, partition approximation method, least square method, and an internal default method of MISTRAS 2001. Variation of the elastic wave velocity in the anisotropic specimens under the various stress conditions was considered in the application of each algorithm except for the MISTRAS 2001' internal algorithm. The MISTRAS 2001 algorithm assumed a constant traverse velocity, regardless of the material anisotropy and stress condition.

Figure 9 shows the time error distributions of each method. The partition approximation method seems superior to the other three, and the mean error of each method increased in order of the least square method, difference approximation, and MISTRAS 2001's. The widely dispersed error distribution of MISTRAS 2001's disproves the suitability of assuming the traverse velocity to be constant in the triaxial stress condition.

As for the partition approximation method, the great portion of time error is under 1.5  $\mu$ s and all events have the time error less than 5  $\mu$ s.

Three kinds of assumptions on the traverse velocity of the elastic wave were tested for the AE source location. The first one adopts an isotropic velocity that varies according to stress condition, and the second the anisotropic velocity previously applied to the three source location algorithms. The last assumption is that the traverse velocity is constant without regard to the stress and orientation. In the comparison test of these three assumptions, two kinds of source location algorithms were employed: the partition approximation method (Figure 10) and the difference approximation method (Figure 11).

In Figures 10 and 11, the constant velocity case showed the biggest error, and the isotropic velocity and the anisotropic velocity presented opposite results when the source location algorithm had been changed. Considering all the cases including the cases of Figures 10 and 11, the anisotropic velocity generally showed less time error than the isotropic velocity did. The little difference in error calculation by the two kinds of traverse velocities seems to be related to the result that the degree of anisotropy of the horizontal traverse velocity in triaxial tests was smaller than those in the initial state without any stress.

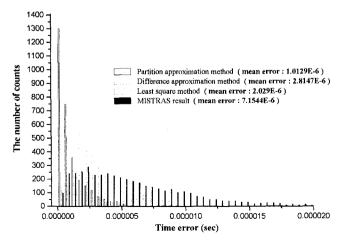


Figure 9. Time error of 4 kinds of source location methods.

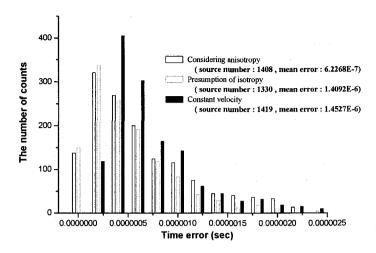


Figure 10. Time error of 3 kinds of S-wave velocity conditions by partition approximation method.

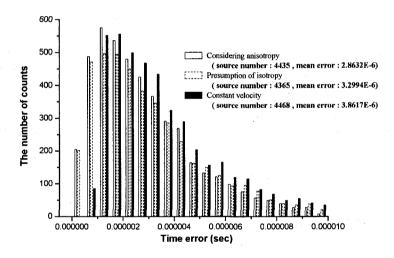


Figure 11. Time error of 3 kinds of S-wave velocity conditions by difference approximation methods.

# 4. Conclusion

The traverse velocity of elastic wave in rock specimens under triaxial stress increased as the confining pressure increased. At a constant confining pressure, P-wave velocity bi-linearly increased as the axial stress increased. The slope at the initial elastic deformation stage was steep and the slope at the fracture-occurring stage slack.

The accuracies of the partition approximation method, difference approximation method, and least square method were compared by AE source location tests. The partition approximation method showed the least time error and the difference approximation method was advantageous in terms of calculation speed.

Among the three kinds of assumptions on the traverse velocity, the anisotropic velocity presented the smallest time error, whereas the constant velocity gave the largest time error in AE source location tests.

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