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A numerical study on anisotropic strength of a rock containing fractures under uniaxial compression condition

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Abstract: Fractures in the form of micro cracks are commonly found in natural rocks. A rock behaves in a complex way due to fracture; in particular, the anisotropic strength of a rock material is significantly influenced by the presence of these fractures. Therefore, it is essential to understand the failure mechanism of a fractured rock. In this study, a fractured rock is formulated in terms of fabric tensor based on geometric and mechanical simplifications. In this way, position, density and shape of fractures can be determined by the fabric tensor so that rocks containing multi-fractures can successfully be modeled. Also an index to evaluate the degree of anisotropy of a fractured rock is proposed. Hence, anisotropic strength of a rock containing fractures under uniaxial compression condition is estimated through a series of numerical analyses for the multi-fractured model. Numerical investigations are carried out by varying the fracture angle from 0° to 90° and relationship between uniaxial compression strength and the degree of anisotropy is investigated. By comparing anisotropic strength of numerical analysis with analytic solution, this study attempts to understand the failure mechanism of rock containing fractures.

1. Introduction

Fractures are commonly found in natural rocks. These fractures complex the behaviour of rock masses; in particular, anisotropic strength and displacement of a rock material is significantly influenced by the presence of these fractures. For rock masses containing fractures, orientation and density of fractures influences anisotropic strength and displacement of rock masses. Hence it is essential to understand how a fractured rock fails.

Most of studies on predicting the behaviour of rock masses often assume that the medium is isotropic or predict the behaviour of rock mass containing joint. In this study, when a rock has multi-fractures, a fractured rock is presented in terms of fabric tensor based on geometric and mechanical simplifications and rocks containing multi-fractures are numerically modeled. In order for investigation the influence of orientation and density of fractures on compressive strength of rock, a numerical method is used.

2. Literature survey

Strength of rock masses containing discontinuities

The strength of rock masses containing discontinuities is influenced by the orientation of discontinuities (Jaeger, 1960). Therefore, the orientation of discontinuities needs to be understood for predicting anisotropic strength of rock masses (Lee, 2001).

Fig. 1 illustrates original Jaeger's criterion, in which there are two types of failure, such as a curved line to be slip along the discontinuity and a straight line to be failure of the intact rock. The original Jaeger's criterion assumes that rock masses are isotropic, and when orientation of discontinuity is 0° and 90°, the values of rock mass strength are identical.

However, many experimental studies show that these values are not identical. Therefore, the extended Jaeger's criterion divides the orientation into 0° and 90° and calculates the strength of rock masses separately (Donath, 1964; McLamore and Gray, 1967; Tien and Kuo, 2001).

Fabric tensor

Geometrical properties of discontinuities in rock masses can be described in terms of position and density, shape and dimension and orientation of discontinuities. These geometrical properties of discontinuities are expressed by the fabric tensor, such as equations (1) and (2) (Oda, 1984, 1984).

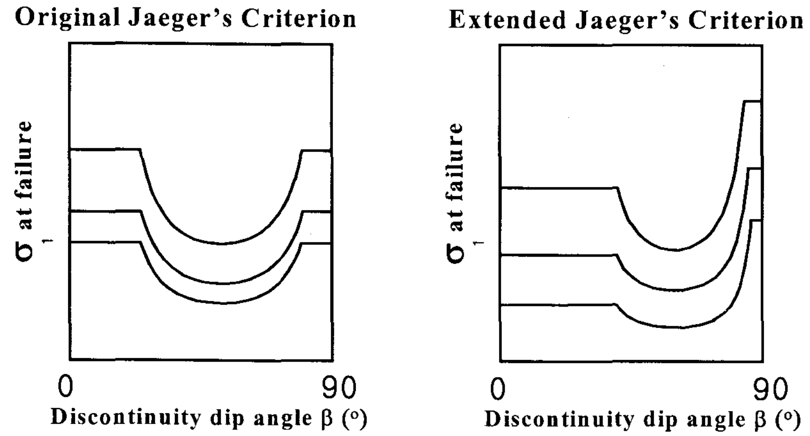

Fig. 1. Strength variation by original and extended Jaeger's criteria (after Tien and Kuo, 2001).

$$F_{ij} = \frac{1}{V}\sum^{m(V)}\left(\frac{1}{4}\pi r^3 n_i n_j\right) \tag{1}$$

$$F_{ij} = \frac{1}{A}\sum^{m(V)}\left((r^k)^2 \cos(\theta^k)\sin(\theta^k)\right) \tag{2}$$

where n is normal vector of a fracture, e_i (or e_j) is base vector, θ^k is the angle between n and e_i (or e_j), r^k is the fracture length and $m^{(V)}$ is number of fracture in a rock. As fabric tensor is symmetric ($F_{ij}=F_{ji}$), F_{ij} has three principal values, namely F_1, F_2 and F_3. Also Γ is proposed as an index to evaluate the degree of anisotropy of F_{ij} as follows

$$\Gamma = \sqrt{(F_1 - F_2)^2 + (F_2 - F_3)^2 + (F_3 - F_1)^2}\ . \tag{3}$$

3. Numerical analysis

In this study, FLAC is used to estimate a compressive strength of rock containing multi-fractures, and Fig. 2 illustrates a basic numerical model of 60 by 120mm with the fracture length of 12mm. The properties used in simulation are listed in Table 1.

Table 1. Properties of a rock material.

Cohesion	Tensile strength	Friction angle	Elastic modulus	Poisson's ratio
7.5 MPa	4.2 MPa	50°	33GPa	0.27

The effect of fracture angle on the uniaxial compressive strength

Numerical investigations are carried out by varying the fracture angle from 0° to 90°. Fig .3 illustrates the stress-strain curve of a fracture rock and Table 2 summarizes uniaxial compressive strength that is determined by the stress-strain curve. The uniaxial compressive strength of rock with fractures decrease from 30° to 60°, and increase after 60°.

The uniaxial compressive strength of rock is defined by the stress when a rock is entirely destructed.

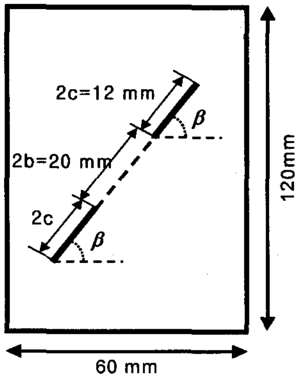

Fig. 2. A fractured rock sample used for numerical experiment.

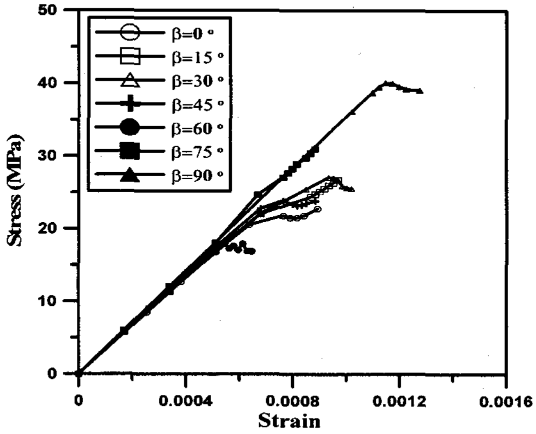

Fig. 3. Stress-strain curves for different fracture angle (β).

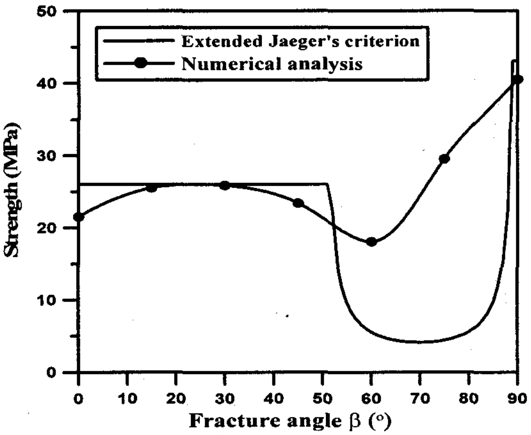

Fig. 4. Compression strength varying with fracture angle

Table 2. The uniaxial compressive strength corresponding to model with different angles.

Fracture angle β (°)	0	15	30	45	60	75	90
Compressive strength (MPa)	21.5	25.5	25.9	23.4	18.1	29.5	40.5

Fig. 4 illustrates an anisotropic strength curve by the numerical and theory analyses. Both numerical and theory analyses show different strength values when the fracture angle is 0°and 90°. But minimum strength appears from different fracture angles. Although a difference exit in the minimum strength values, on the whole, both curves behave in a similar way qualitatively.

The effect of fracture length on the uniaxial compressive strength

In order for investigation the influence of fracture length, in which the fracture angle is fixed at 45°, on the compressive strength of a rock, numerical analyses are carried out by varying the fracture length. Table 3 and Fig. 5 illustrate the stress-strain curve and uniaxial compressive strength by fracture length.

When fractures are in a rock, a density of fractures, ρ, is defined by equation (4).

$$\rho = \frac{Area\ of\ fractures}{Area\ of\ rock} \tag{4}$$

Numerical investigations are carried out by varying the fracture angle from 0° to 90°. Table 2 summarizes uniaxial compressive strength determined by the stress-strain curve. The uniaxial compressive strength of rock with fractures decrease from 30° to 60°, and increase after 60°.

Table 3. Compression strength corresponding to model with different length.

Fracture length (mm)	0	6	12	24	42	60	78
Fracture density ($\times 10^{-3}$)	0	0.83	1.67	3.33	5.83	8.33	10.83
Compressive strength (MPa)	30.50	28.12	27.13	21.55	5.69	3.80	1.26
Degree of anisotropy Γ	0	0.0018	0.0071	0.0283	0.0866	0.1768	0.2987

Table 3 shows that the more fracture length increases, the more a rock compressive strength decrease. Also, Γ increases with increasing the fracture length. Therefore, it can be drawn that the compressive strength of a rock decreases, as Γ increases.

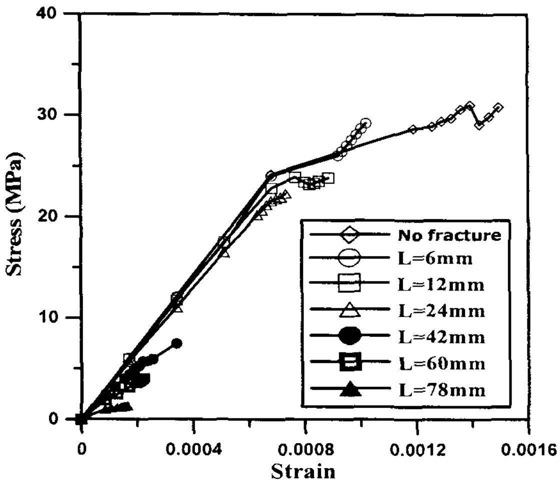

Fig. 5. Stress-strain curves for different fracture length.

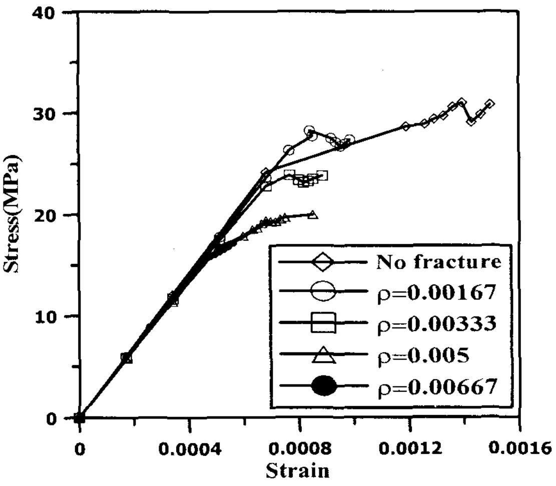

Fig. 6. Stress-strain curves for different fracture density.

The effect of fracture density on uniaxial compressive strength

Numerical investigations are carried out by varying number of fractures, fixing the fracture angle and length at 45° and 12mm, respectively. Fig. 6 and Table 4 show the stress-strain curve and the uniaxial compressive strength of numerical results.

Table 4. The compression strength corresponding to model with different fracture number.

Number of fractures	0	1	2	3	4
Fracture density	0	0.00167	0.00333	0.00500	0.00667
Compressive strength (MPa)	30.58	27.13	13.39	19.31	16.29
Degree of anisotropy Γ	0	0.0071	0.0141	0.0212	0.0283

It is described in Table 4 that the more number of fracture increase, the more the compressive strength of strength decrease (or Γ increase). Hence the numerical results indicate that the compressive strength of a rock decrease as the degree of anisotropy increases.

Under identical density condition, in order for investigation the influence of fracture number on the compressive strength of a rock, numerical analyses are carried out by varying the fracture number, fixing the fracture angle and density at 45°and 0.0033. Table 5 and Fig. 7 show the stress-strain curve and the uniaxial compressive strength of numerical results.

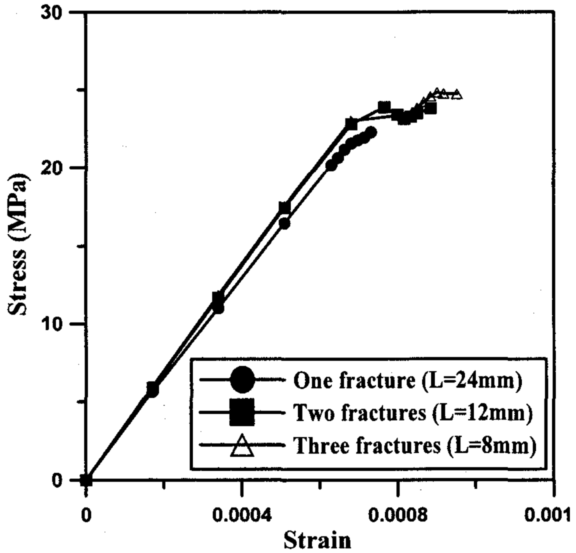

Fig. 7. Stress-strain curves for different number of fractures maintaining the same fracture density.

Table 5. The compressive strength corresponding to model with equal density.

Number of fractures	1	2	3
Fracture length (mm)	24	12	8
Fracture density	0.0033	0.0033	0.0033
Compressive strength (MPa)	21.55	23.39	24.75
Degree of anisotropy Γ	0.0283	0.0141	0.0094

Table 5 shows that in spite of equal density of fractures, the more number of fracture increase, the more the compressive strength decrease (or Γ increase). It is founded that bridge(or intact rock) between fractures is one of the most important factors, influencing the strength of a rock.

4. Conclusions

In this study, a fractured rock is presented in terms of fabric tensor. Thus position, density and shape of fractures are determined by the fabric tensor so that rocks containing multi-fractures is modeled by numerical method. The significant findings are as follows

(1) By comparing anisotropic strength curve of numerical analysis and analytic solution, it is shown that the failure mechanism of rock containing fractures and rock masses containing discontinuities are similar.

(2) On the contrary to the previous conclusion, a rock containing fractures is influenced by intact rock between fractures. Under the identical fracture density, bridge(or intact rock) between fractures improves the compressive strength of rock. This means that intact rock between fractures is one of the most important factors that have influence on the strength of rock.

(3) The uniaxial compressive strength increases as the degree of anisotropy decreases, when the orientation of fractures in a rock is identical. This result indicates that the fabric tensor represents a relative strength of fractured rocks by Γ.

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