

## MODEL BASED DIAGNOSTICS FOR A GEARBOX USING INFORMATION THEORY

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This article discusses a diagnostics method based on models, and information theory. From an extensive system dynamics bond graph model of a gearbox [1], simulated were various cases germane to this diagnostics approach, including the response of an ideal gearbox, which functions perfectly to designer's specifications, and degraded gearboxes with tooth root cracking. By comparing these cases and constructing a signal flow analogy between the gearbox and a communication channel, Shannon's information theory [2], including theorems, was applied to the gearbox to assess system health, in terms of ability to function.

**Keywords :** Gearbox, Bond Graph, Shannon's Information Theory, Channel Capacity, Information Rate

### 1. INTRODUCTION

Maintenance costs of machines can accrue to purchase prices within a year of operation; downtime losses can far exceed this in minutes [3]. Small reductions in life-cycle maintenance costs can yield substantial savings. Critical are effective machine diagnostic and prognostic methods [4]. To achieve these aims, a fault diagnosis method was developed to predict system condition and identify components about to fail. This method constructs detailed models of a machine, tunes model parameters from machine data, infers from these models the condition of components in the machine, and applies Shannon's information theory to assess machine health in terms of functional capability.

### 2. SHANNON'S INFORMATION THEOREM

We construct an analogy between a gearbox and a communications channel, to diagnose severity of faults by assessing how faults impede flow of information of signals through the machine. To appraise information flow, we treat a machine as a communications channel, Fig. 1, wherein input signal  $x(t)$  containing information is "transmitted" and "received" as output  $y(t)$  over a "machine channel" [5]. Faults add "noise", "...any unwanted component in a received signal [6]", tantamount to the difference  $y(t) - y_i(t)$  between actual received signal  $y(t)$  and the signal  $y_i(t)$  received if the machine had no faults. Output  $y(t)$ , is  $x(t)$  altered by channel dynamics, but with noise  $n(t)$  added. Powerful theorems of Shannon [2], appraise (1) the channel capacity  $C$ , the maximum rate information (in bits per second) can be successfully sent over a channel of bandwidth  $\omega$  and signal to noise (power spectra) ratio  $S/N$ ; and (2) the average rate of information  $R$  that must be sent for a given message. Here  $C$  characterizes the machine channel's condition, and  $R$  characterizes the load on the channel. If  $R < C$ , the information will be received intact, otherwise not [2].

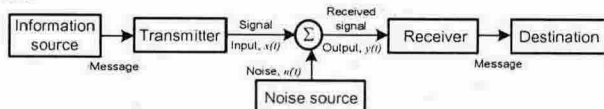


Fig. 1 Shannon-Weaver model [2]

### 3. DIAGNOSTIC METHOD FOR A GEARBOX

#### 3.1 Construction of detailed gearbox model

Important to this method are models which describe the

machine's behavior, including functional condition. To test/diagnose a system, there should exist a direct correspondence between elements of the model and components/items in the physical system. The gearbox bond graph model of Fig. 3, based on [1] and [7], models the Fig. 2 apparatus of Dalpiaz [8].

The gearing system is composed of two identical single stage gear units and two shafts (Fig. 2). Contained in the systems model are physics of a gear system, including tooth-to-tooth contact for spur gears [1], tooth bending (stiffness and inertia), Hertzian contact stiffness for teeth, slip and rolling typical of spur gear contacts, and gear system effects such as shaft and bearing dynamics among others. The two ellipses having bond graph structures shown in [1] represent dynamics of meshing gears.

Each box is mounted to the foundation with compliances  $C_{\text{boxL}}$ ,  $C_{\text{boxR}}$  and dampings  $R_{\text{boxL}}$ ,  $R_{\text{boxR}}$ . All Inertance elements are rotational mass moments of inertia associated with shafts ( $I_{p1}$ ,  $I_{p2}$ , and  $I_{w1}$ ), gears ( $I_{pL}$ ,  $I_{pR}$ ,  $I_{wL}$ , and  $I_{wR}$ ) or the gearbox housing ( $I_{\text{boxL}}$  and  $I_{\text{boxR}}$ ). Likewise, all compliances are torsional compliances of shafts ( $C_{p1}$ ,  $C_{p2}$ ,  $C_{p3}$ , and  $C_{w1}$ , and  $C_{w2}$ ) and gearbox housing, and dampings ( $R_{br1}$ ,  $R_{br2}$ ,  $R_{br3}$ , and  $R_{br4}$ ) from bearings. The power bonds extending from the middle of the left and right sides of the 1-junction in the gearbox housings and foundation section form triangular structures which model the reaction torques applied to the boxes by the shafts, bearings, and gears [9].

#### 3.2 Tuning the parameters of gearbox model

Experimental results by Dalpiaz, *et al.* [10] are compared with results from the bond graph model in Fig. 3 to tune the model's parameters. Tuning parameters from data permits complete monitoring of the dynamic system. As a fault progresses, measured signals change, and for simulations to match measurements, parameters in the model must change. Only by changing certain parameters in the model, can the complex signals measured by the sensors be reproduced by the models. By tuning a model, and then following the progressive changes of parameters, faults can be sensed and tracked since parameters have direct correspondence with specific components (and faults).

Fig. 4 shows tuning of a parameter. If a crack appears in the root of a gear tooth, the compliance of that tooth must increase. The initial compliance can be analytically estimated from design data, by assuming a gear tooth as a tapered cantilever beam, or from sensor data, such as the Fig. 4 (a) power spectra. Fig. 4 (b) through (d) show power spectral

densities corresponding to increased tooth compliance. As compliance increases, sidebands in the power spectral density increase in number and amplitude. In current diagnostic practice, increases in number and amplitude of such sidebands may indicate a fault condition [10], including a cracked tooth. Fig. 4 (e) and (f) from [10], show power spectra from faulty gears with small and large cracks respectively.

**3.3 Information theory for detection of faults in a gearbox**

The channel capacity was calculated for the gear testing machine with a cracked tooth (with varying compliances), using equation (1), derived from Shannon [2]:

$$C = \int_0^{\omega} \log_2(S^*/N) d\omega \tag{1}$$

Power spectral densities  $S^*$  and  $N$  are the magnitude squared of the Fourier transforms for output signal  $y(t)$  and noise  $n(t)$  respectively. These arose from simulations of models tuned from sensor data.

Channel capacity vs. tooth compliance is shown in Fig. 4 (g). The upper limit of equation (1), bandwidth frequency  $\omega = 5 \times 10^4$  Hz was set inverse to the numerical time step, in accordance with Shannon's sampling theorem [2]. The dashed line in Fig. 4 (g) pertains to a 45% noise (or error) tolerance estimated by rate of information

$$R = \omega \log_2(S_i/N_i) \tag{2}$$

with  $S_i/N_i=0.45$ . Equation (2) involves  $S_i$ , the average power of the desired signal  $y_i(t)$  to be received,  $N_i$ , the maximum allowed RMS error, and  $\omega$ , the bandwidth of  $y_i(t)$  [2]. Collectively equation (2) describes the information load imposed on the channel in terms of error, speed, and job (signal) complexity.

In Fig. 4 (g), the channel capacity monotonically decreases with increasing severity of fault. Below the dashed line,  $R < C$  is violated and the system fails in a functional sense. This tendency suggests channel capacity and information rate can be an index for estimation of overall health of machinery.

**4 SUMMARY AND CONCLUSION**

To summarize, our method

- Models the system
- Measures response, with common sensors (most likely vibration)

- Tunes system parameters from data and tracks changes of parameter(s)
- Assesses severity of faults and predicts condition, using Shannon's information theory

**5. ACKNOWLEDGEMENTS**

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**6. REFERENCES**

[1] Kim, J. and Bryant, M. D., "A Bond Graph Model of Gear Tooth Contacts and Effects of Vibration on Tooth Surface Failures", *The Advancing Frontier of Engineering Tribology, STLE*, pp. 163-175, 1999.  
 [2] Shannon, C. E. and Weaver, W., "The mathematical Theory of Communication", The University of Illinois Press, Illinois, 1948.  
 [3] Manufacturing Studies Board, "The Competitive Edge", National Academy Press, 1991.  
 [4] Clark, R. N., Frank, P. M., and Patton, R. J., "Fault Diagnosis in Dynamic Systems: Theory and Application", Prentice-Hall, UK, 1989.  
 [5] Bryant, M. D., "Application of Shannon's Communication Theory to Degradation Assessment of Systems", *Proceeding of the ASME Congress*, Vol. 72, No. 9, pp. 1192-1201, 1998.  
 [6] P. J. Fish, "Electronic Noise and Low Noise Design", NY: McGraw-Hill, 1994.  
 [7] Choi, J and Bryant, M. D., "Combining Lumped Parameter Bond Graphs with Finite Element Shafts in a Gearbox Model", CMES, in press.  
 [8] Dalpiaz, G., "Detection and Modeling of Fatigue Cracks in Gears", *Condition Monitoring-Proceedings of the 3rd International Conference*, pp. 73-82, 1991.  
 [9] Hrovat, D. and Tobler, W. E., "Bond graph modeling of automotive power trains", *Journal of the Franklin Institute*, Vol. 328, No.5/6, pp. 623-662, Great Britain, 1991.  
 [10] Dalpiaz, G., Rivola, A., and Rubini, R., "Effectiveness and Sensitivity of Vibration Processing Techniques for Local Fault Detection in Gears", *Mechanical Systems and Signal Processing*, Vol. 14, pp. 387-412, 2000.

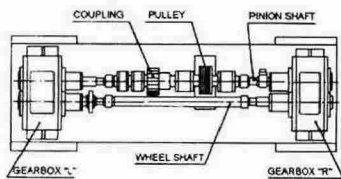


Fig. 2 Gear testing machine [8]

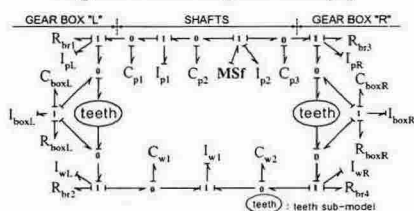


Fig. 3 Bond graph model of gear testing machine in Fig. 2

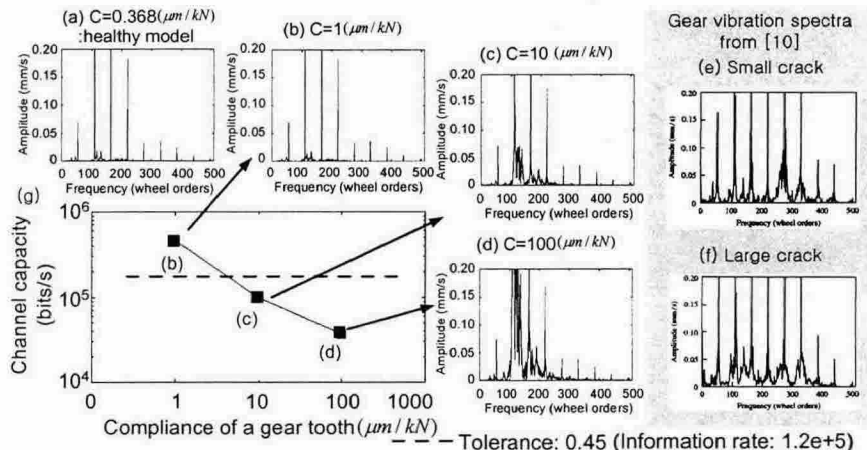


Fig. 4 Power spectra and channel capacities, for a cracked tooth