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# INFLUENCE OF CAPILLARITY AND ELASTICITY ON MICRO-CONTACTS

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One aspect of the stiction problem may be explained by the action of capillary forces in conjunction with surface elasticity. In the present work, the interaction between two elastic half-spaces separated by a small liquid bridge is investigated. By minimizing the total free energy stored in the interface (including elastic energy and surface energy), the equilibrium interface geometry is determined analytically in the case where there is no solid-solid contact. A non-dimensional number, $N_c = 299\frac{\gamma_{LA}^2 \cos^2\theta V_o}{E'^2 H^5}$ is found to govern the structure stability. When $N_c \geq 1$, the two surfaces jump into solid-solid contact and, once this occurs, the contact area will continue to expand until the two surfaces are in full contact.

**Keywords** : Capillarity, Elasticity, Adhesion, Mechanical Stability, Contacts

## 1. INTERFACE CONFIGURATION WITHOUT SOLID-SOLID CONTACT

Consider two identical elastic semi-infinite spaces initially separated by a small gap, H. If the surfaces are perfectly flat and smooth, when a volume of liquid, $V_o$, is introduced into the interface, it will wet a circular area, say, of radius b. A highly curved meniscus is then formed at the liquid-air boundary. The pressure drop across the meniscus, Δp, can be determined from the Laplace-Young equation if the radii of curvature of the meniscus are known. Meanwhile, we know that in the absence of solid-solid contact, the pressure exerted on each surface satisfies

$$p(r) = \begin{cases} -\Delta p, & 0 \leq r \leq b \\ 0, & r > b \end{cases} \tag{1}$$

Assuming there is no plastic deformation, then from elasticity, the deflection of the surface caused by this pressure, u(r), is given by

$$u(r) = \begin{cases} \dfrac{4(1-\nu^2)\Delta p b}{\pi E} \displaystyle\int_0^{\pi/2} \sqrt{1 - \frac{r^2}{b^2}\sin^2\psi}\, d\psi, & r \leq b \\ \dfrac{4(1-\nu^2)\Delta p}{\pi E} \displaystyle\int_0^{\pi/2} \frac{b^2 \cos^2\phi}{r\sqrt{1 - \frac{b^2}{r^2}\sin^2\phi}}\, d\phi, & r \geq b \end{cases} \tag{2}$$

where E and ν are, respectively, the elastic modulus and Poisson ratio of the structure material. The gap h(r) between the deformed surfaces can be represented in the form

$$h(r) = H - 2u(r) \tag{3}$$

The volume $V_o$ is related to the deflection u(r) according to

$$V_o = \int_0^b 2\pi r h(r) dr = \int_0^b 2\pi r (H - 2u(r)) dr \tag{4}$$

Considering surface energy and elastic strain energy contributions, the total free energy $U_T$ can be expressed as

$$U_T = A_{SA}\gamma_{SA} + A_{SL}\gamma_{SL} + A_{LA}\gamma_{LA} + 2\left[\frac{1}{2}\int_0^b 2\pi r p u dr\right] \tag{5}$$

A stable equilibrium configuration is that which minimizes the total free energy for the given liquid volume ($V_o$). By computing the free energy corresponding to all provisional equilibrium configurations, as represented by all combinations of Δp and b consistent with a given liquid volume ($V_o$) and with the deformation relations (2), the actual equilibrium state can be determined. For convenience, we introduce a dimensionless parameter, η, defined by

$$\eta = \frac{4\Delta p b}{E' H} \tag{6}$$

where

$$E' = \frac{E}{1-\nu^2} \tag{7}$$

The parameter η can be used to indicate the extent to which the two surfaces approach each other. In the absence of solid-solid contact, η takes on values between 0 and 1. When η = 1, the surfaces experience point contact. Making use of (5), we can define a dimensionless system free energy as

$$U_T^* = \frac{U_T - 2\pi R^2 \gamma_{SA}}{\sqrt{\pi} E' H^{\frac{3}{2}} V_o^{\frac{1}{2}}} = \frac{\alpha\eta^2}{(1-\beta\eta)^{\frac{1}{2}}} - \frac{2}{(1-\beta\eta)}\sqrt{\frac{\Gamma}{\pi}} \tag{8}$$

where R is the radius of a fixed, arbitrary control volume of height H, α = 0.1061, β = 0.8488, and where Γ is defined as

$$\Gamma = \frac{\gamma_{LA}^2 \cos^2\theta V_o}{E'^2 H^5} \tag{9}$$

Seeking the stationary points of (8) leads to the following condition for system equilibrium:

$$\eta^2(1-0.6366\eta)^2(1-0.8488\eta)=\frac{64\Gamma}{\pi} \tag{10}$$

The solution space of eqn. (10) is plotted in Fig. 1, where $\eta_{eq}$ represents the value of $\eta$ satisfying eqn. (10) for a given value of $\Gamma$. Comparing the dimensionless free energy curves in Fig. (2), we can see that when the nondimensional parameter $\Gamma$ takes different values, the character of the solution of eqn. (10) changes. When $\Gamma < 0.98\times10^{-3}$, there is only one equilibrium value of $\eta$ in the range from 0 to 1, corresponding to a local energy minimum, which is also a global energy minimum (Fig. 2). When $0.98\times10^{-3} \le \Gamma < 3.34\times10^{-3}$, there are two values of $\eta_{eq}$, the smaller one corresponding to a local energy minimum and the larger one to a local energy maximum. When $\Gamma = 3.34\times10^{-3}$ (@ $\eta = 0.5577$), the two solutions merge into one, yielding an inflection point on the energy curve (Fig. 2). A global minimum appears at $\eta = 1$, which indicates a state with solid-solid contact. At the inflection point, the equilibrium is unstable. Whenever there is a disturbance, the system will "slide down" to the point of global minimum free energy ($\eta = 1$) meaning that the solid surfaces jump into contact. When, $\Gamma > 3.34\times10^{-3}$ there is no solution of eqn. (10) and thus no stationary point appears on the energy curve. In this case, the interface is predicted to go immediately from its initial configuration to a state of solid-solid contact. For convenience, we can define a non-dimensional number, $N_c$, to specify the critical condition for the onset of solid-solid contact.

$$N_c=\frac{\Gamma}{3.34\times10^{-3}}=299\Gamma \tag{11}$$

Hence, when $N_c \ge 1$ solid-solid contact occurs. It is seen that $N_c$ is a measure of the relative strength of capillary forces to the elastic restoring forces.

## 2. INTERFACE CONFIGURATION WITH SOLID-SOLID CONTACT

If the interface can reach an equilibrium configuration after solid-solid contact appears, we can do a similar analysis as in Section 1. But now besides $\Delta p$ and b, another parameter, the radius of contact region, $a$, is required to describe the interface configuration. In the contact region, the pressure is unknown, while the surface deflection is equal to H/2. Outside the contact region (i.e., $r < a$), the deflection is unknown, while the pressure is known. In the wetted annulus (i.e., $a < r \le b$) the pressure is uniform and equals the negative of the pressure drop across meniscus, $\Delta p$. Outside of the wetted area (i.e., $r > b$) the pressure is zero. To obtain the unknown surface deflection, u(r), outside of the contact region (i.e., $r > a$) along with the unknown pressure distribution, p(r), within the contact region (i.e., $r \le a$), we use pressure-based flexibility influence coefficients. Again the gap between the deformed surfaces, h(r), satisfies eqns. (3) and (4). We can still use $\eta$ as the indicator of the degree of surface interaction. After solid-solid contact occurs, $\eta$ is greater than 1. For given values of $\eta$, $\Delta p$ and b, $a$ can be determined by conserving the liquid volume and preventing surface interpenetration. The corresponding interface free energy can then be computed. This calculation shows that the free energy decreases monotonically with increasing $\eta$. Therefore no equilibrium state can be achieved after the occurrence of solid-solid contact.

## 3. CONCLUSIONS

The interaction between two elastic half-spaces separated by a fixed volume of liquid is investigated. It is found that the interface stability is governed by a nondimensional number, $N_c$, given by

$$N_c=299\frac{\gamma_{LA}^2\cos^2\theta\, V_o}{E'^2H^5} \tag{12}$$

When $N_c < 1$, the system will attain equilibrium without solid-solid contact. On the other hand, when $N_c \ge 1$, the interface experiences solid-solid contact. Once point contact occurs at the origin, the contact region will continue to expand until the two surfaces are in full contact.

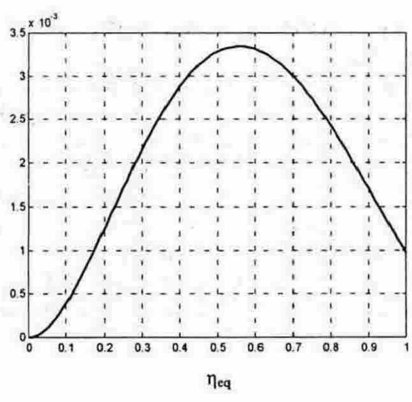


Fig. 1. The solution space of eqn. (9)

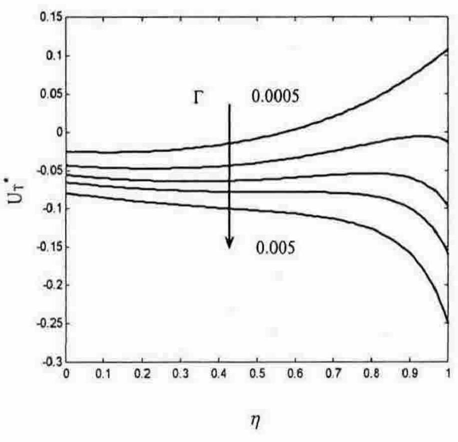


Fig. 2. Energy curves before solid-solid contact occurs.