

The Physical and Mathematical Models for Thin Film Lubrication

P. HUANG and S. X. BAI

College of Mechanical Engineering,
South China University of Technology,
Guangzhou 510640, CHINA

Based on the fact that the lubricant molecular is with a chain structure, the physical and mathematical models for the thin film lubrication are set up after the analysis of relationship of the chain length and the film thickness is carried out. The basic equations of fluid mechanics with the rotation terms are used to derive the equivalent Reynolds equation. The results show that the load carrying capacity has a significant increase while the length effect is considered. Finally, the calculated results are compared with the experimental results and they have the same tendency.

Keywords : Model, Thin Film, Lubrication

1. INTRODUCTION

Many experimental results show that the principles of hydrodynamic lubrication and elastohydrodynamic lubrication no longer fit for thin film lubrication (TFL) when the film thickness is below 100 nanometers^[1]. To reveal these principles further, some physical models have been proposed to explain the forming of TFL.

However, the theoretical analysis of TFL does not develop so fast. Molecular dynamics simulation has been used to analysis TFL earlier. The method has not reach the level of practicality because the problems such as the interaction force of the long molecules and characteristics of subsistent surface have not been solved completely. Of the limited mathematical models of TFL, some proposed by Tichy are concerned with the lubricant viscosity^[2-4]. In recent years, the second order fluid model adding the second order terms in the constitutive equations has been used in TFL owing to its superiority in dealing with non-Newtonian problems^[5]. The micro polar fluid lubrication model is used to consider the lubricant with polar properties^[6,7].

In present paper, the relative sizes of the chain structure of lubricant molecules in the thick and thin film lubrication are analyzed. Then, a physical model of TFL is proposed, in which the lubricant molecular length is considered. It is found that while the film is thin, the lubricant is the same as the polar one. Thus, the high order term relative to rotation is added in the motion equation, to derive a mathematical model of TFL. Compared with the second order fluid model, it doesn't consider the second order term in the stress components, but in the resultant forces.

2. PHYSICAL MODEL OF TFL

According to the material structure theory^[8], the radius of carbon atom is about 0.67 Å, and that of hydrogen atom 0.53 Å. The relative center distance between two carbon atoms is about 1.312 Å. The

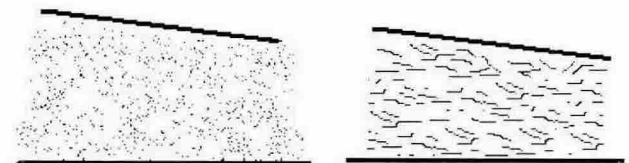
distance of hydrocarbon-bond is about 1.09 Å. Thus, the chain length of the following several pure mineral oils may be obtained in Tab.1.

Table 1 Chain lengths of several pure mineral oils

pure mineral oil	chain length(Å)	chain width (Å)
Decane	15.05	3.24
Dodecane	17.67	3.24
Hexadecane	22.92	3.24

According to Tab.1, the ratios of the length to the width are usually much large. The chain lengths of the above oils are about 1~2 nanometers, but that of lubricating oils in practical use may be longer for additives are often added.

According to the above analysis, a physical model of TFL is proposed as shown in Fig.1(b). When the film is thick as Fig.1(a), the molecules of lubricant can be assumed as the classical model. When the film thickness is close to the lubrication molecular length, the direction of the lubricant chain has to be along with the flow direction, for it is impossible for the lubricant chain to keep in the vertical direction to the surfaces for a long time.



(a) Thick film status ($h \gg l$) (b) Thin film status ($h \leq 30l$)

Fig.1 The thick and thin status of the lubricant molecules

3. MATHEMATICAL MODEL OF TFL

The terms to limit rotation of fluid molecules are added in the momentum equations. Therefore, Reynolds equation of TFL is obtained as follows.

$$\frac{\partial}{\partial x} \left(\frac{\rho f(l, N, h)}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho f(l, N, h)}{12\eta} \frac{\partial p}{\partial y} \right) = \frac{1}{2} \frac{\partial}{\partial x} \{ \rho (U_{in} + U_n) h \} + \frac{1}{2} \frac{\partial}{\partial y} \{ \rho (V_{in} + V_n) h \} - U_n \frac{\partial \rho h}{\partial x} - V_n \frac{\partial \rho h}{\partial y} + \rho W_n - \rho W_{in} \quad (1)$$

where, $f(l, N, h) = h^3 + 12l^2 h - 6Nlh^2 \coth\left(\frac{Nh}{2l}\right)$.

4. RESULTS AND DISCUSSION

Fig.2 shows the relationship of velocity and film thickness while the load w_0 is fixed. In the traditional lubrication theory, there is a linear relation between film thickness and velocity when the load does not change, as shown by the curve with the characteristic length $l=0$ in Fig.2. Having the lubricant characteristic length been considered (curves with $l=3 \times 10^{-9}$ m and 3×10^{-8} m in Fig.2), the relationship agrees with the traditional lubrication theory in the thick film regime only. However, in thin film regime, a significant change has occurred. The linear relationship no longer exists. And, there is a declining trend for the effect of the film thickness with its decrease. This tendency is the same as the experimental results in the references [9].

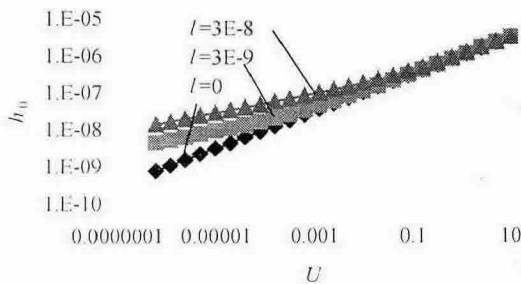


Fig.2 The min-mum film thickness via velocity

In Fig.3, the dimensionless load W may be obtained by integrating the dimensionless pressure P over the dimensionless coordinate X . Accordingly, the dimensionless load W_0 is equal to 9.45×10^{-3} in the case of thick film lubrication. It is to analyze the effect of the characteristic length and the coupling coefficient on the load carrying capacity with decrease of the film thickness.

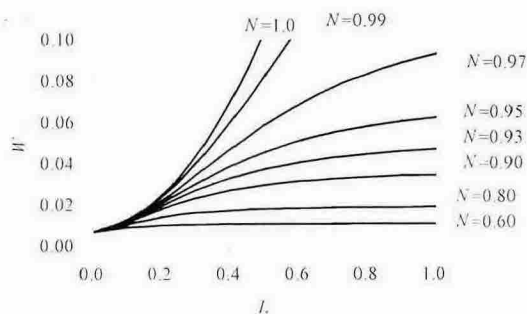


Fig.3 The non-dimensional load and characteristic length via coupling coefficients

When L is equal to 0, it is the case of thick film, that is, the lubricant length may be ignored. When L is equal to 1, the film thickness decreases to as thin as the characteristic length. Fig.3 shows that: (1) the load increases with increase of the characteristic length; (2) the smaller coupling coefficient has no significant effect on the load while the larger coupling coefficient, however, not only has significant effect on load but also makes load increase quickly and the characteristic length keeps increasing. It means that there is a close relationship between the characteristic length and the coupling coefficient in effect on the load carrying capacity.

5. CONCLUSIONS

- (1) The TFL model has been set up with consideration of the lubricant length close to the film thickness dimension, which is mainly determined by the lubricant molecular length. According to the numerical results, the range of TFL should be within $h < 20 \times l$.
- (2) The numerical analysis shows that within the above range, the effect of velocity on film thickness decreases in the thin film regime, which results that the film thickness of the TFL model is little larger than that of the traditional lubrication model.
- (3) The main parameters affecting the properties of TFL are the characteristic length and the coupling coefficient. The larger characteristic length and the coupling coefficient, the more significant the effect on the film thickness.

REFERENCES

- [1] Huang, P., et. al. "NGY-2 Interferometer for Nanometer Lubrication Film Thickness Measurement", J. of Tribology, 1995, 14(2), 175-179
- [2] Tichy, J., "A Surface Layer Model for Thin Film Lubrication", Tribology Transaction, 1995, 38(3), 577-582
- [3] Tichy, J., "A Porous Media Model for Thin Film Lubrication," J. of Tribology, 1995, 117.(1), 16-21
- [4] Jang, S., Tichy, J., "Rheological Model for Thin Film EHL Contacts," J. of Tribology, 1995, 117(1), 22-28
- [5] Huang, P., Li, Z.H., et. al. "Study on Hydrodynamic Lubrication with Second-order Fluid(I)-Basic equations," Science in China, Series A, Vol. 44 Supplement, 1-7
- [6] Singh, C., and Sinha, P., "The three-dimensional Reynolds equation for micropolar-fluid-lubricated bearing," Wear, 1982, 76 (2), 199~209
- [7] Lukaszewicz, G., "Micropolar Fluids—Theory and Applications," Birkhauser, Boston 1999
- [8] Xu, G.X., "Mater Structure," Beijing: People's Education Press, 1978:135~149
- [9] Luo, J.B., Wen, S.Z., and Huang, P., "The Relation and Transition Between EHL and Thin Film Lubrication," J of