

# 역 비고유치 BEM 을 사용한 소리 굽쇠의 외부 음향 홀로그래픽 재현

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## Exterior Acoustic Holography Reconstruction of a Tuning Fork using Inverse Non-singular BEM

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**Key Words :** BEM, Holography, Tuning Fork, 3 Dimensions, Non-singular, Inverse BEM, Pressure Field.

### Abstract

Non-singular boundary element method (BEM) codes are developed in acoustics application. The BEM code is then used to calculate unknown boundary surface normal displacements and surface pressures from known exterior near field pressures. And then the calculated surface normal displacements and surface pressures are again applied to the BEM in forward in order to calculate reconstructed field pressures. The initial exterior near field pressures are very well agreed with the later reconstructed field pressures. Only the same number of boundary surface nodes (1178) are used for the initial exterior pressures which are initially calculated by Finite Element Method (FEM) and BEM. Pseudo-inverse technique is used for the calculation of the unknown boundary surface normal displacements. The structural object is a tuning fork with 128.4 Hz resonant. The boundary element is a quadratic hexahedral element (eight nodes per element).

### Symbol Notification

$\{\Psi_f\}$  Near/Far Field Pressure Vector

- $\{F\}$  Applied Mechanical Force
- $\{a\}$  Elastic Displacement
- $[K]$  Elastic Stiffness Matrix
- $[M]$  Mass Matrix
- $\omega$  Angular Frequency
- $\rho_f$  Fluid (Air) Density
- $a_n$  Normal Displacement on the Structural Surface
- $k$  Wave Number ( $=\omega/c$ )
- $c$  Sound Speed in Air; 340 [m/sec]
- $ng$  Number of Surface Nodes
- $nt$  Number of Surface Elements
- $\{a\}$  Displacement Vector on the Structural Surface
- $\{\Psi\}$  Pressure Vector on the Structural Surface

### 1. Introduction

Electro-mechanical devices such as motor and engine generate sound with noise. Unless the noise-like sound is non-stationary, it is possibly easier to find where is the origin of the noise. The stationary noise source may be pointed out by an acoustic holographic technique in which an array of microphones measure the sound pressure field in 3 dimensions surrounding an interested noise radiating object in order to geometrically analyze the position of the noise source. The acoustic holographic techniques may be approached by either Spatial Fourier Transformation (SFT) [1,2] or Inverse Boundary Element Method (BEM) [3,4]. In both approaches, spatial sound pressures of a single frequency are measured known complex values and the target of the acoustic holography is to calculate either the unknown surface pressure or the normal velocity of the specimen.

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Then the near or far field sound pressure may be recalculated, so as to reconstruct the original 3 dimensional sound pressure field. This paper presents an inverse non-singular BEM technique for the acoustic holographic analysis. The sound radiating object is a tuning fork. The analysis is done in 3 dimensions. In order to neglect the measurement signal noise of the spatial sound pressure, the forward BEM supplied the 3 dimensional sound field pressures as the original known complex values.

## 2. Numerical methods

### 2.1 Finite Element Method (FEM)

The following equation (1) is the integral formulation of the FEM elastic equations:

$$\{F\} = [K]\{a\} - \omega^2 [M]\{a\} \quad (1)$$

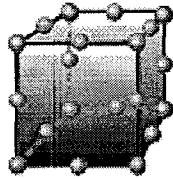


Fig. 1 3D quadratic hexahedral 20 nodes' element

The isoparametric formulation for 3-dimensional structural elements is well documented by Allik H. et. al. [5]. Each 3-dimensional finite element is composed of 20 quadratic nodes and each node has nodal displacement ( $a_x, a_y, a_z$ ) variables. In local coordinates the finite element has 6 surface planes ( $\pm xy, \pm yz, \pm zx$ ) which may be exposed to external air environment. The exposed surface is used as a boundary element which is composed of 8 quadratic nodes.

### 2.2 Boundary Element Method (BEM)

The boundary element solution of sound pressure intensity is very useful to analyze the sound radiation of vibrating devices; intensity, directivity pattern and noise control elements. A tuning fork can be changed in its shape to produce a particular pattern of radiation pattern. For sinusoidal steady-state problems, the Helmholtz equation,  $\nabla^2 \Psi + k^2 \Psi = 0$  represents the fluid mechanics.

$\Psi$  is the acoustic pressure with time variation,  $e^{j\omega t}$ . In order to solve the Helmholtz equation in an infinite air media, a solution to the equation must not only satisfy structural surface boundary condition (BC),  $\partial \Psi / \partial n = \rho_f \omega^2 a_n$  but also the radiation condition at

infinity,  $\lim_{|r| \rightarrow \infty} \int_S (\partial \Psi / \partial r + jk\Psi)^2 dS = 0$ .  $\partial / \partial n$  represents differentiation along the outward normal to the boundary.

The Helmholtz integral equation derived from Green's second theorem provides such a solution for radiating pressure waves;

$$\int_S \left\{ \Psi(q) \frac{\partial G_k(p,q)}{\partial n_q} - G_k(p,q) \frac{\partial \Psi(q)}{\partial n_q} \right\} dS_q = \beta(p) \Psi(p) \quad (2)$$

$$\text{where } G_k(p,q) = e^{-jk r} / 4\pi r, \quad r = |p - q|$$

$p$  is any point in either the interior or the exterior and  $q$  is the surface point of integration.  $\beta(p)$  is the exterior solid angle at  $p$ .

The acoustic pressure for the  $i^{th}$  global node,  $\Psi(p_i)$ , is expressed in discrete form [6]: ( $1 \leq i \leq ng$ )

$$\beta(p_i) \Psi(p_i) = \int_S \left\{ \Psi(q) \frac{\partial G_k(p_i,q)}{\partial n_q} - G_k(p_i,q) \frac{\partial \Psi(q)}{\partial n_q} \right\} dS_q \quad (3a)$$

$$= \sum_{m=1}^{nt} \int_{S_m} \left\{ \Psi(q) \frac{\partial G(p_i,q)}{\partial n_q} - G(p_i,q) \frac{\partial \Psi(q)}{\partial n_q} \right\} dS_q \quad q \in S_m \quad (3b)$$

$$= \sum_{m=1}^{nt} \int_{S_m} \left\{ \sum_{j=1}^8 N_j(q) \Psi_{m,j} \frac{\partial G(p_i,q)}{\partial n_q} - G(p_i,q) \sum_{j=1}^8 N_j(q) \frac{\partial \Psi_{m,j}}{\partial n_q} \right\} dS_q \quad (3c)$$

$$= \sum_{m=1}^{nt} \sum_{j=1}^8 \left\{ \int_{S_m} N_j(q) \frac{\partial G(p_i,q)}{\partial n_q} dS_q \right\} \Psi_{m,j} - \rho_f \omega^2 \sum_{m=1}^{nt} \sum_{j=1}^8 \left\{ \int_{S_m} N_j(q) G(p_i,q) h_q dS_q \right\} p_{m,j} \quad (3d)$$

$$= \sum_{m=1}^{nt} \sum_{j=1}^8 A^i_{m,j} \Psi_{m,j} - \rho_f \omega^2 \sum_{m=1}^{nt} \sum_{j=1}^8 B^j_{m,j} a_{m,j} \quad (3e)$$

where  $nt$  is the total number of surface elements and  $a_{m,j}$  are three dimensional displacements.

Equation (3b) is derived from equation (3a) by discretizing integral surface. And equation (3c) is derived from equation (3b) since an acoustic pressure on an integral surface is interpolated from adjacent 8 quadratic nodal acoustic pressures corresponding the integral surface. Then equation (3d) is derived from equation (3c) by swapping integral notations with summing notations. Finally the parentheses of equation (3d) is expressed by upper capital notations for simplicity.

When equation (3e) is globally assembled, the discrete Helmholtz equation can be represented as

$$[A] - \beta [I] \{\Psi\} = + \rho_f \omega^2 [B] \{a\} \quad (4)$$

where  $[A]$  and  $[B]$  are square matrices of  $(ng \text{ by } ng)$  size.

ng is the total number of surface nodes.

When the impedance matrices of equation (4), [A] and [B], are computed, two types of singularity arise [7]. One is that the Green's function of the equation,  $G_k(p_i, q)$ , becomes infinite as q approaches to  $p_i$ . This problem is solved by mapping such rectangular local coordinates into triangular local coordinates and again into polar local coordinates [8]. The other is that at certain wave number the matrices become ill-conditioned. These wave number are corresponding to eigenvalues of the interior Dirichlet problem [9]. One approach to overcome the matrix singularity is that [A] and [B] of equation (4) are modified to provide a unique solution for the entire frequency range [10-13]. The modified matrix equation referred to as the modified Helmholtz gradient formulation (HGF) [13] is obtained by adding a multiple of an extra integral equation to equation (4).

$$([A] - \beta[I] \oplus \alpha[C])\{\Psi\} = +\rho_f \omega^2 ([B] \oplus \alpha[D])\{a\} \quad (5)$$

where

$$\alpha = \frac{\sqrt{-1}}{k \cdot (\text{Number of surface element adjacent a surface node})}$$

[C] and [D] are rectangular matrices of (nt by ng) size.  $\oplus$  symbol indicates that the rows of [C],[D] corresponding to surface elements adjacent a surface node are added to the row of [A],[B] corresponding to the surface node, that is,

$$\left. \begin{aligned} \sum_{i=1}^{ng} \sum_{j=1}^{ng} A(i, j) &= \sum_{i=1}^{ng} \sum_{j=1}^{ng} A(i, j) + \sum_{i=1}^{ng} \sum_{j=1}^{ng} \left( \sum_{m=1}^{S(i)} \alpha C(m, j) \right) \\ \sum_{i=1}^{ng} \sum_{j=1}^{ng} B(i, j) &= \sum_{i=1}^{ng} \sum_{j=1}^{ng} B(i, j) + \sum_{i=1}^{ng} \sum_{j=1}^{ng} \left( \sum_{m=1}^{S(i)} \alpha D(m, j) \right) \end{aligned} \right\} \quad (6)$$

where S(i) is the number of surface element adjacent a surface node. The derivation of the extra matrices [C], [D] are well described by Francis D.T.I. [13]. Equation (6) may be reduced in its formulation using superscript  $\oplus$  for convenience;

$$A^\oplus \{\Psi\} = +\rho_f \omega^2 B^\oplus \{a\} \quad (7)$$

where  $[A] - \beta[I] \oplus \alpha[C] \equiv A^\oplus$ ,  $[B] \oplus \alpha[D] \equiv B^\oplus$

Equation (7) can be written as

$$\{\Psi\} = +\rho_f \omega^2 (A^\oplus)^{-1} B^\oplus \{a\} \quad (8)$$

Since the present acoustic vibrator produces displacement data  $\{a\}$  at a natural frequency, the surface pressure  $\{\Psi\}$  of the tuning fork is calculated from

equation (8). Once  $\{a\}$  and  $\{\Psi\}$  are known, the acoustic pressure in the far field is determined by  $\beta(p)=1$  of equation (2) for given values of surface nodal pressure and surface nodal displacement;

$$\Psi(p_i) = \sum_{m=1}^{nt} \sum_{j=1}^8 A^{i,m,j} \Psi_{m,j} - \rho_f \omega^2 \sum_{m=1}^{nt} \sum_{j=1}^8 B^{i,m,j} a_{m,j} \quad (9)$$

### 2.3 Pseudo Inverse BEM

Previous forward BEM solves unknown near/far field acoustic pressures once the surface displacement vector and the surface pressure scalar of the vibrating tuning fork are known. Equation (8) calculates the surface pressures from the given 3 dimensional surface displacement vectors which are supplied by the FEM equation (1). Therefore the finally calculated near field acoustic pressures derived by equation (9) may be used as if measured sound pressures for the acoustic holographic approach. These calculated near field acoustic pressures are notified as an original input sound pressures. The number of the original input sound pressures are taken as the same as the number of unknown surface pressures, that is, the number of the surface nodes (ng).

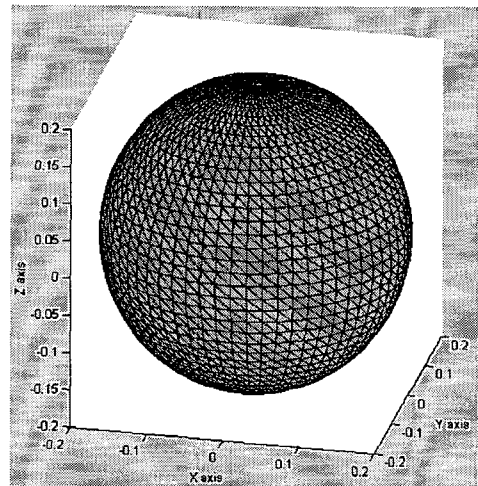


Fig. 2 Three dimensional near field original sound pressure positions.

Now the next step is to inversely find the unknown surface pressure or surface displacement from the known near filed sound pressure. Equation (3) as well as the following equations (3~9) are modified, so that the displacement vector is changed to the normal displacement scalar. It ensures that  $A^\oplus$  and  $B^\oplus$  have the same matrix sizes as ng by ng.

$$\{\Psi\} = +\rho_f \omega^2 (A^\oplus)^{-1} B^\oplus \{a_n\} \quad (10)$$

Equation (9) can be reformulated as

$$A_1 \{\Psi\} + B_1 \{a_n\} = \{\Psi_f\} \quad (11)$$

And if equation (10) is added into equation (11),

$$\left( \rho_f \omega^2 A_1 (A^\oplus)^{-1} B^\oplus + B_1 \right) \{a_n\} = G \{a_n\} = \{\Psi_f\} \quad (12)$$

Equation (12) is solved by pseudo inverse matrix technique which is derived by singular value decomposition (SVD) since the coefficient matrices of equation (12) have complex values. If the SVD of G matrix is  $U\Lambda W^H$ , then

$$\{a_n\} = W\Lambda^{-1}U^H \{\Psi_f\} \quad (13)$$

The size of G matrix is  $n_g$  by  $n_g$ . If more than  $n_g$  near filed sound pressures are supplied, then the number of rows in G matrix is more than the number of columns in G matrix. Even though G becomes rectangular matrix, equation (13) can be still solved because the SVD provide singular values from the highest order.

### 3. Results

#### 3.1 Tuning Fork FEM Application

The FEM is applied to the analysis of the tuning fork. Figure 3 shows 3 dimensional tuning fork FEM elements and Table 1 shows the material properties of the air and the steel.

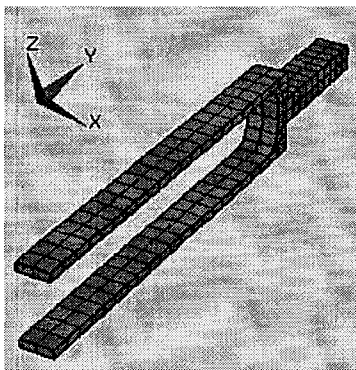


Fig. 3 Three dimensional tuning fork FEM elements. Length=152.4 [mm], Width=25.4 [mm], Material=Steel(4130)

Table 1 Material Properties

	Density ( $\rho$ ) [Kg/m <sup>3</sup> ]	Young's Modulus (E) [N/m <sup>2</sup> ]	Poisson Ratio ( $\nu$ )
Air	1.22	1.411E5	-
Steel	7822.9	2.0684E11	0.30

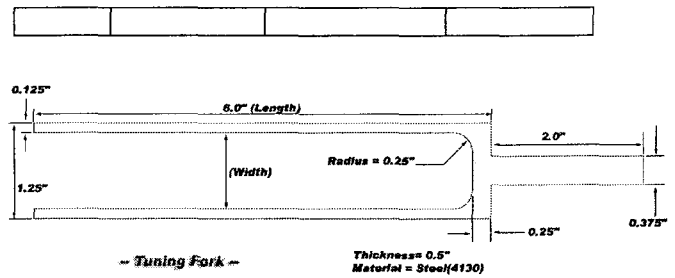


Fig. 4 Tuning fork dimensions

The tuning fork has the first modal frequency at 128.4 Hz. Figure 5 shows the 3 dimensional modal shape at the first mode.

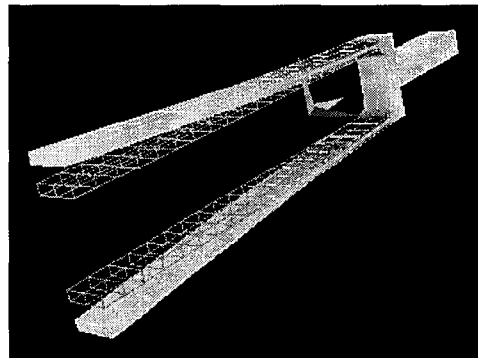


Fig. 5 Modal shape of tuning fork (Color=Von Mises Stress) at 128.4 Hz (1<sup>st</sup> mode)

#### 3.2 Tuning Fork Forward BEM Application

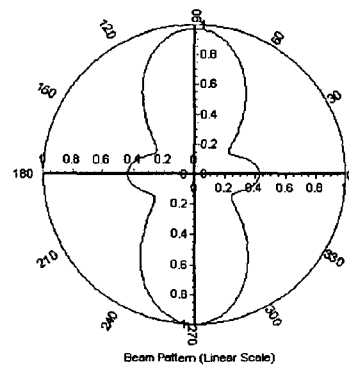


Fig. 6 Beam pattern of tuning fork in 2 dimensions

The surface pressure of the tuning fork is calculated by equation (8) from the given surface displacement provided by the FEM eigenvectors. Then the so-called original near field acoustic pressures in 3 dimensions are calculated by equation (9) at the same first modal frequency, 128.4 Hz. Figure 6 shows the directivity pattern of the tuning fork in 2 dimensions' view. And Figure 7 shows the directivity pattern of the tuning fork in 3 dimensions.

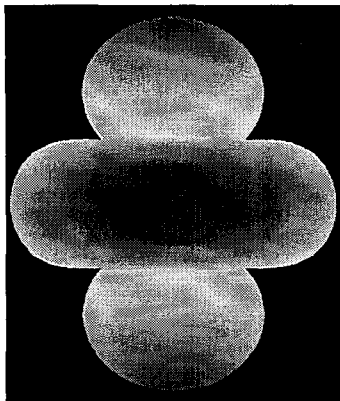


Fig. 7 Beam pattern of tuning fork in 3 dimensions

### 3.3 Tuning Fork Pseudo Inverse BEM Application

Only normal surface displacements are considered in the pseudo inverse BEM. The normal surface displacement of the tuning fork is calculated by equation (13) from the supplied exterior near field acoustic pressures which are complex values. Then the surface pressure of the tuning fork is calculated by equation (10). Figure 8 (a) and (b) show the real and the imaginary surface pressures of the tuning fork respectively. The blue continuous lines indicate the original surface pressure of the tuning fork derived by equation (8) while the red dotted lines indicate the recalculated surface pressure of the tuning fork derived by equation (10).

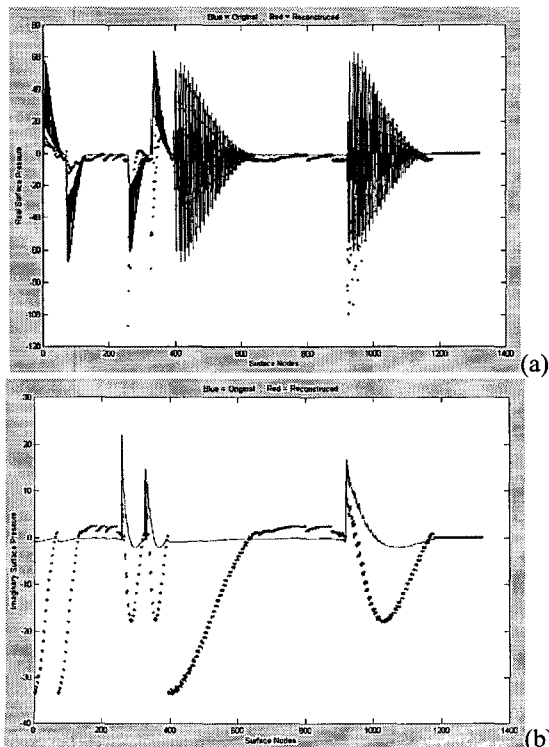


Fig. 8 The real (a) and the imaginary (b) surface

pressures of the tuning fork. The blue continuous lines = Original surface pressure, The red dotted lines = Recalculated surface pressure.

And figure 9 shows the 3 dimensional deformed shape of the tuning fork drawn from the recalculated normal surface displacement. It should be noted that the overall deformed shape of the tuning fork is quite similar to that of the originally displaced tuning fork, that is, the two ends of the bars are significantly deformed in +Z and -Z axes directions. The main difference between figure 9 and figure 5 is that the inner surfaces of the tuning fork ends have much smaller displacements than the outer surfaces of the tuning fork ends.

Figure 10 (a) and (b) show the real and the imaginary near field sound pressures of the tuning fork respectively. The blue continuous lines indicate the original near field pressure of the tuning fork while the red dotted lines indicate the reconstructed near field pressure of the tuning fork. Both original and reconstructed near field pressures are almost perfectly agreed each other.

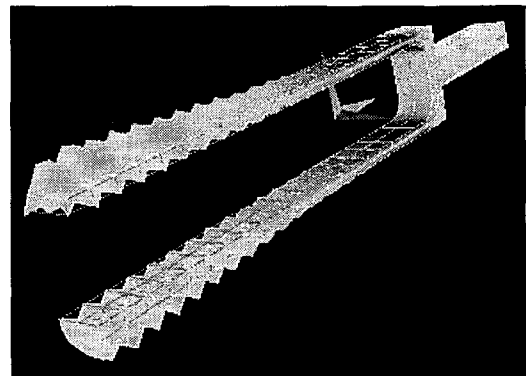
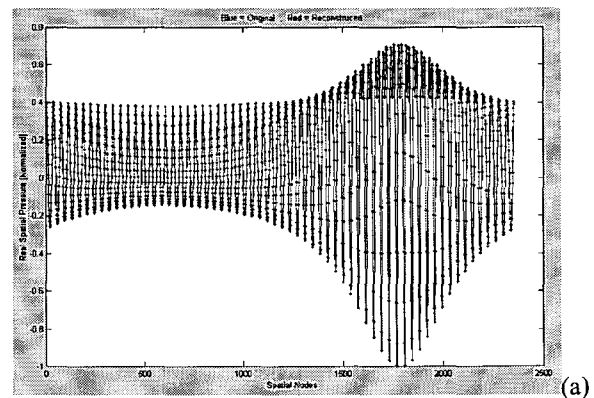


Fig. 9 Three dimensional deformed shape of the tuning fork drawn from the recalculated normal surface displacement. (Color=Von Mises Stress)



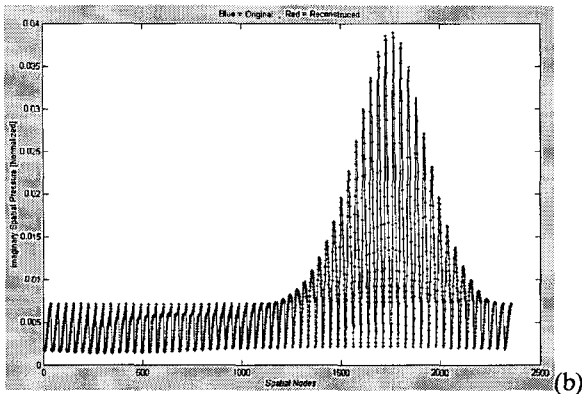


Fig. 10 The real (a) and the imaginary (b) near field acoustic pressures of the tuning fork. The blue continuous lines = Original near field pressure, The red dotted lines = Reconstructed near field pressure.

Figure 11 shows the reconstructed directivity pattern of the tuning fork in 3 dimensions.

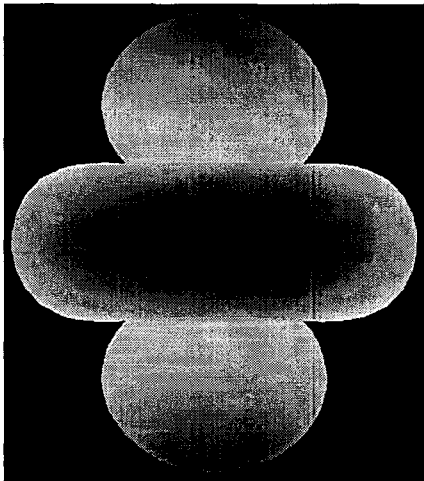


Fig. 11 Reconstructed beam pattern of tuning fork in 3 dimensions

#### 4. Conclusion

Non-singular BEM codes are developed in acoustics application. The BEM code is then used to calculate unknown boundary surface normal displacements and surface pressures from known exterior near field pressures. And then the calculated surface normal displacements and surface pressures are again applied to the BEM in forward in order to calculate reconstructed field pressures. The initial exterior near field pressures are very well agreed with the later reconstructed field pressures.

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