Servo Design for High-TPI Hard Disk Drives Using a Delay-Accommodating State Estimator

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ABSTRACT

This paper presents a servo design method for high track-density hard disk drives, in which the plant time delay, mainly due to the processor computation time, is taken into account. The key idea behind the proposed design method is to incorporate the delay model into the output equation of the state-space representation for the plant model; thereby, the delay is accounted for by a standard state observer in a natural manner, with simplified state equations as compared to those for conventional methods. The results from practical application confirm that the proposed method is quite effective in realizing a high-bandwidth servo system in hard disk drives.

1. Introduction

In a hard disk drive (HDD) control system, the state-space controller/observer (or estimator) design is popularly adopted for its advantages such as effective filtering of position and velocity, use of estimation error to handle servo defects, etc. Discrete-time state estimators can be implemented in two forms; one is the so-called "prediction" estimator that estimates the state variable based on the plant output and control of the previous sampling period. The other one is called the current estimator, the name of which comes from the fact that it uses the "current" measurement to estimate the state variables. Prediction estimator equations have the more intuitive form in the sense that it can be obtained as a discretization of the well-known continuous-time state observer. On the other hand, the form of current estimators can be derived by means of a discrete Kalman filter formalism with a priori knowledge on current measurement.

Traditionally, the current estimator has been dominantly used among the HDD servo community. The primary reason is that it is believed to give more reliable estimates for small computational delay [1] . As the track density (represented by tracks-per-inch, or TPI) of disk drives increases, however, the requirement for servo positioning accuracy becomes ever higher. This enforces the servo engineers to push the sampling rate as far as desired data capacity is reserved; and at the same time, more sophisticated control algorithms have to be employed to minimize the track mis-registration (TMR). For these reasons, the computation time delay is no longer negligibly small and should be accounted for at the stage of servo design.

Several techniques have been proposed to incorporate the computational delay into control systems design. One of the most commonly adopted techniques is to use the input-delayed model in the state estimator's prediction stage [1] . It makes the state estimation more reliable, while it still cannot compensate the delay in the state feedback law. An outgrowth of this technique is to modify the feedback law to include the one-sample delayed control signal as in [3] [5] . A similar method is developed in [2] for discrete-equivalent design. A good survey on this subject can be found in [4] .

In this paper, systematic techniques are proposed to accommodate the transport delay (e.g. computation time delay) into the state estimator and eventually into the

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whole control system. The delay considered here is defined to be the time lag between the idealized time of plant output (position) sampling and the time at which the corresponding control becomes effective at the input of our design-oriented plant model. Thus, it may include the analogue-to-digital (and vice-versa) conversion time, demodulation time, computational time, time-lag due to finite bandwidth power amp, etc.

The experimental results based on a commercial hard drive are provided as well as some simulation results. They show that the proposed method effectively helps the stability margin by increasing the phase and gain margin of the system by 3-4° and 0.3dB respectively.

Notation: The sampling time and the delay time will be denoted by T_s and T_d, respectively. A bracket is used to represent a discrete-time signal, e.g. $x[k]$. **R** represents the real line.

2. Modeling of Computation Delay

From the continuous-time point of view, the presence of computation delay T_d can be effectively modeled by putting an ideal transport delay element $e^{-T_d s}$. In most of the literature, it is very common to have this delay element precede the voice-coil motor (VCM) dynamics, which leads to the following input-delayed state-space representation of the plant dynamics:

$$\begin{aligned} \dot{x}(t) &= A_p x(t) + B_p u(t - T_d) \\ y(t) &= C_p x(t) \end{aligned} \quad (1)$$

More generally speaking, the computation delay can be dealt with under the state-space setting in a two different (but, in fact, mutually dual) ways as described in what follows.

2.1 Input Delayed Model

The most popular approach taken in the literature is based on the input-delay model shown in (1). It is important to note that the key logic in putting the delay T_d in the control signal $u(t)$ consists in the assumption that the discrete-time time index of the sampled system coincides with the output sampling instant, i.e., the time instant at which the head position is believed to be sampled.

The discrete version of the input-delayed plant

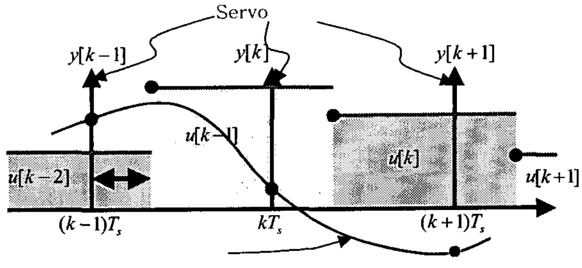


Fig. 1 Timing diagram of discrete-time controllers with computation time delay

model in (1) can be easily calculated in the following form [1] :

$$\begin{aligned} x[k+1] &= A_e x[k] + B_{e0} u[k] + B_{e1} u[k-1] \\ y[k] &= C_e x[k] \;. \end{aligned} \quad (2)$$

Here, (A_e, B_e, C_e) represent the system matrices of the plant model for control design, which may possibly contain some augmented states for bias prediction, etc. Use of this delay-present plant equation (2) in the current estimator formulation yields:

$$\begin{aligned} \hat{x}[k] &= \bar{x}[k] + L\left(y[k] - C_e \bar{x}[k]\right), \\ \bar{x}[k+1] &= A_e x[k] + B_{e0} u[k] + B_{e1} u[k-1]. \end{aligned} \quad (3)$$

The derivation of (2) can be easily understood with reference to Fig. 1.

2.2 Output-Delayed Model

The transport delay can also be incorporated in the output of a plant model as follows:

$$\begin{aligned} \dot{x}(t) &= A_p x(t) + B_p u(t) \\ y(t) &= C_p x(t - T_d) \end{aligned} \quad (4)$$

The key idea behind this formulation lies in identifying the discrete-time index of the discretized system with the control sampling instant, that is, the instant at which the computed control signal becomes effective to the plant.

It is important to note that the state evolution equation (the first equation of (4)) becomes the standard delay-free one under this output-delay formulation. This property serves as the basis of the proposed delay-accommodating estimator (DAE). The derivation of the DAE will be presented in the next section.

3. Design of DAE's

Two types of estimators can be designed based on the two delay-present model representations presented in the last section.

Type I: Conventionally, the control input is generated in the same way as used in a usual state/observer framework without delay consideration. That is, for a state regulator with regulator gain K,

$$u[k] = -K\hat{x}[k]. \qquad (5)$$

With this controller, however, one cannot prevent one-sample-delayed state $\hat{x}[k]$ from affecting the closed-loop system (see the state prediction equation in (3)), resulting in possible degradation in stability margin. As a partial compensation for this problem, one may use the controller of the following form:

$$u[k] = -\hat{K}\hat{x}[k] - k_1 u[k-1] \qquad (6)$$

where $\hat{K} \in \mathbf{R}^{1\times n}$ and $k_1 \in \mathbf{R}$ are controller gains to be determined below. Note that, in view of (3), our closed-loop system can be rendered to behave as the ideal one without transport delay, provided that

$$B_{e0}u[k] + B_{e1}u[k-1] = -B_e K\hat{x}[k] \qquad (7)$$

for each k. For any choice of gain pair $(\hat{K}, \hat{k}_1)$ in (6), it is generally impossible to satisfy (7) for each k. Nonetheless, (7) can still be approximated in a minimum-norm sense by choosing

$$\hat{K} = B_{e0}^{+}B_e K, \quad \hat{k} = B_{e0}^{+}B_{e1}u[k-1] \qquad (8)$$

where B^{+} is the left pseudo-inverse of a matrix B defined, in this case, by

$$B^{+} \triangleq \left(B^T B\right)^{-1} B^T .$$

Besides the above-proposed design method, one can use the standard LQR formulation for determining the controller gains as done in [5].

Type II: This method (to be called "delay-accommodating estimator (DAE) hereafter) is based on the output-delay model given in (4). To begin with, we define the discrete-time variables used below to be

$$x[k] \triangleq x(kT_s), \quad u[k] \triangleq u(kT_s), \\ y[k] \triangleq C_e x[kT_s - T_d].$$

Note that we let the state variable's time index agree with that of the control signal (see Fig. 1). To obtain the discrete version of (4), also define $x_m[k] \triangleq x(kT_s - T_d)$. Then we have

$$\begin{aligned} x[k] &= e^{T_d A}x_m[k] + \int_{kT_s - T_d}^{kT_s} e^{A(kT_s-\tau)}B_e d\tau\, u[k-1] \\ &= e^{T_d A}x_m[k] + \int_0^{T_d} e^{\lambda A_e}B_e d\lambda\, u[k-1] \end{aligned} \qquad (9)$$

where x follows the delay-free state evolution equation:

$$x[k+1] = A_e x[k] + B_e u[k]. \qquad (10)$$

Now, the following output equation is obtained by using (9):

$$y[k] = C_{ed}x[k] + D_{ed}u[k-1] \qquad (11)$$

where the matrices are defined by

$$C_{ed} \triangleq C_e e^{-T_d A_e}, \quad D_{ed} \triangleq -C_e e^{-T_d A_e}\int_0^{T_d} e^{\lambda A_e}B_e d\lambda$$

Finally, we apply the standard current state estimator equation to the new discrete-time plant given by (10) and (11) to get

$$\begin{aligned} \hat{x}[k] &= \overline{x}[k] + L\left(y[k] - \overline{y}[k]\right) \\ \overline{x}[k+1] &= A_e x[k] + B_e u[k] \end{aligned} \qquad (12)$$

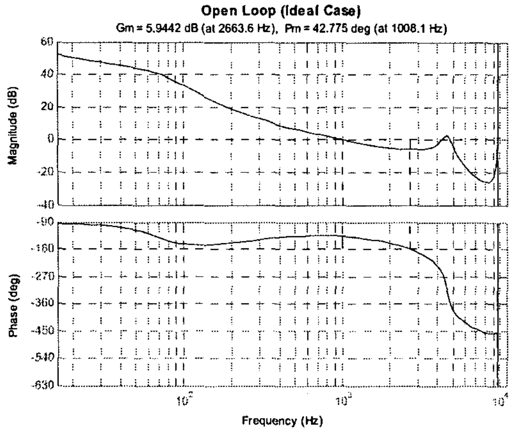
where the variable $y[k]$ can be pre-calculated in the stage of state prediction according to

$$\overline{y}[k+1] = C_{ed}\overline{x}[k+1] + D_{ed}u[k]$$

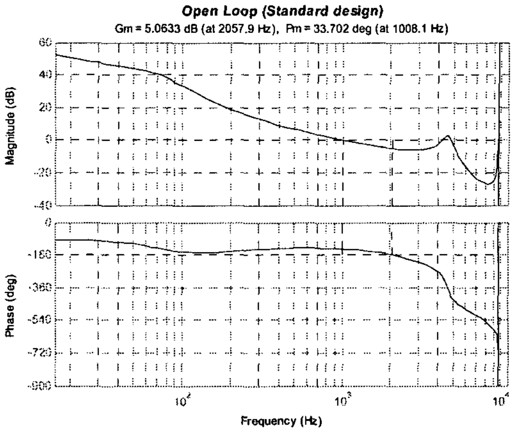
Since we have the ideal state equation as shown in (10), the delay-free state feedback law can be implemented by simply using the state estimate in (12). In this way, accommodation of time delay is automatically achieved. An additional advantage of this DAE is that the estimator state equation is simplified compared to the estimator in Type I. This feature is quite helpful in practical implementation, where control multi-rate control strategy may be utilized.

3. Simulations and Experiments

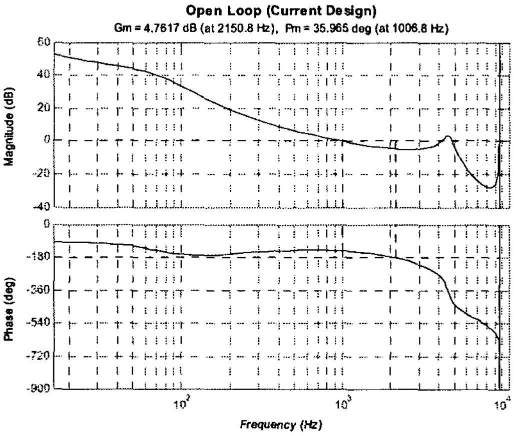
First, the impact of using the DAE is examined through some computer simulation; State-space controllers are designed according to the following four cases (resulting open loops are presented in Fig. 2): (a) an ideal actuator model (no delay) with standard controller



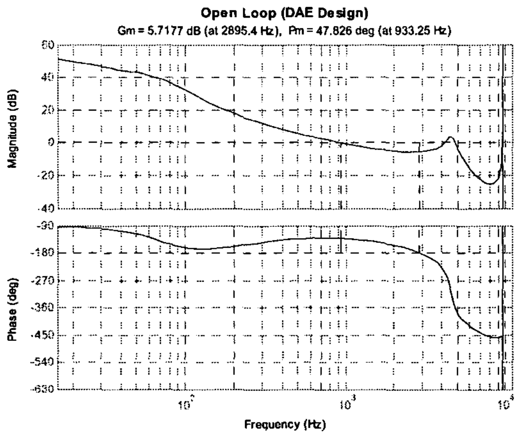
(a) ideal plant + standard control



(b) delay plant + standard control



(c) delay plant + estimator with delay model



(d) plant with delay + DAE

Fig. 2 Open-loop bode plots of the servo control systems for DAE and non-DAE designs.

design (b) delay-present actuator model with standard controller design, (c) delay-present actuator model with controller considering the delay only at state-prediction stage (see Section 2.1), (d) delay-present actuator with the proposed DAE design.

As seen in Fig. 2-(b), disregarding the computation delay results in a significant degradation of phase margin (from 43deg to 34deg). On the other hand, incorporating the delay model into the prediction equations as in (3) slightly recovers the phase margin to 36deg at the cost of noticeable reduction of gain margin. Finally, application of DAE assuming exact knowledge of the delay time T_d leads to an impressive (over-) recovery of stability margins (gain and "first" phase margin) as shown in Fig. 2-(d). It can be seen that the DAE also reduces the open-loop crossover frequency under the same feedback gains. It should be noted, however, that in the realistic case, neither the exact knowledge of T_d is available, nor the delay dynamics are exactly the "transport" one. Nonetheless, the simulation results clearly demonstrate the usefulness of DAE design in recovering stability margins of the closed-loop system.

Next, the proposed design method was applied to an 80GB HDD (Samsung Electronics, Co. Ltd). The delay-accommodating estimator of type II was implemented and compared with the conventional state feedback controller, in which the computation delay is considered only in the state prediction model. (case (c) in the first paragraph of this section)

The comparison was first made for an open-loop bandwidth of 950Hz. And, in order to see how well the controllers perform for boosted bandwidth, the bandwidth was increased then to 1.1kHz. As common measure of system's stability margin with respect to uncertainties, the gain/phase margin and the peak amplitude of the error sensitivity function (ESF) were compared for each case. The resulting on-track PES statistics are also measured to compare each controller's performance in terms of TMR capability. It should be noted however that the adaptive feedforward controller for the rejection of a particular set of repeatable run-outs were disabled for fair comparison of controllers. The experimental results are summarized in Table 1.

As predicted in the simulation, the use of DAE significantly recovers the both the phase and gain margin as compared to the conventional controller partially accounting for time delay. It is important to note that this is the case for almost the same open-loop crossover frequency. The corresponding open-loop bode plots are presented Fig. 3. The phase margin is shown to be increased near the cross over frequency. On the other hand, in the high-frequency region, some boost of gain could be observed.

The reduction of phase margin translates to the decrease of the error sensitivity peaking as shown in Fig. 4. This effectively attenuates the amplification ratio of disk-mode related disturbances, resulting in the reduction of standard deviation of the non repeatable PES as shown in Table 1. The experimental results confirm that the DAE is practically useful in achieving high-bandwidth servo systems required for high-TPI HDD's.

Table 1 Summary of experimental results

	950 Hz		1100 Hz	
	Current	DAE	Current	DAE
f_{gc} (Hz)	959	989	1102	1084
PM (deg)	30.9	35.4	30.6	34.0
GM (dB)	4.8	5.0	3.9	4.5
ESF Peak	2.5	2.4	2.8	2.7
RRO	9.1	9.1	6.7	6.6
NRRO	10.8	9.8	8.9	8.7
PES STD	14.1	13.4	11.1	10.9

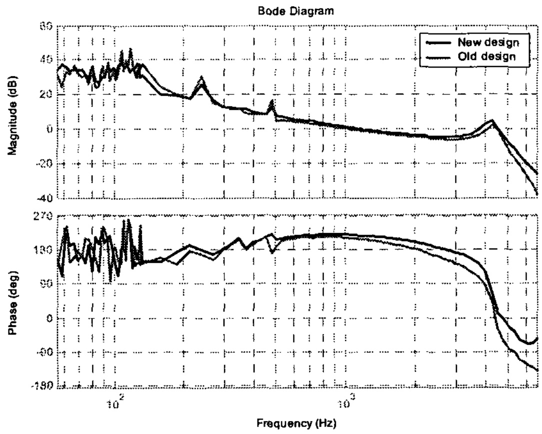


Fig. 3 Comparison of open-loop transfer functions.

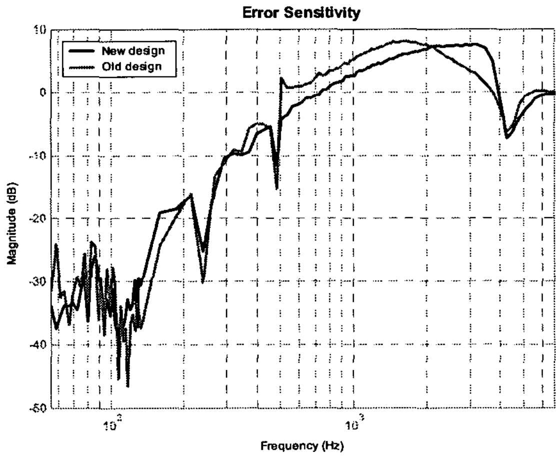


Fig. 4 Comparison of error sensitivity functions.

4. Conclusion

This paper considered an efficient method to compensate for the presence of computational delay in the digital servo control system for hard disk drives. The best advantage of the proposed control method is that it improves the nominal stability margin without heavily modifying the existing state-space control system; thereby leading to an increased bandwidth, or, better error rejection capability. For a viable TMR in the TPI of more than 100kTPI, it is believed that every single nonidealities that have been neglected, should be brought to servo engineers' attention. The computational delay should be one of the examples. The DAE method is in fact shown to be very effective also in track-seeking controllers, although the results are not presented in this paper. A possible side-effect that has been known so far may be a slight increase in the resulting open loop gain in high-frequency region.

Acknowledgement

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