# AN INTEGRATED DYNAMIC O-D ESTIMATION MODEL FOR URBAN NETWORKS

도시가로망의 동적 O-D 예측을 위한 통합모형개발

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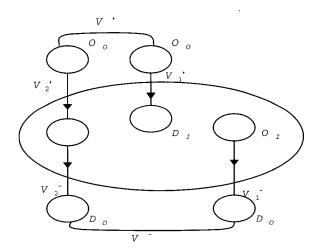
#### I. INTRODUCTION

As the estimation of time-varying O-D distributions at different aggregation levels provides of time-varying O-D distributions at different aggregation levels provides a direct and cost-economic way for understanding urban traffic flow patterns, it has considerable number of methods for O-D estimation has been reported in the literature. Depending on whether a dynamic traffic assignment model (DTA) is needed or not, one may classify all such studies into the following two categories: assignment-based and non-assignment-based methods.

One key feature of the proposed approach is to employ the intersection turning flow data along with the path flow information from a DTA model to increase the observability of a given network system. In this paper, the turning flows are used not only for the subnetwork O-D estimation in a two-stage computational process, but also used to provide an additional set of constraints in identifying path flows from a DTA model. The application of such turning flow information has significantly improved the estimation accuracy.

## II. A CORDONLINE MODEL FOR NETWORK O-D ESTIMATION

For an urban network, a cordonline is defined as a hypothetical closed curve that intersects with a set of link stations, and divides the network into two parts: inside and outside each encircled subnetwork. The set of detector or counting stations on both the cordonline network links provide the time-varying flow information for estimation.



(Figure 1) Cordonline flow

$$V^{+}(k) = V_{1}^{+}(k) + V_{2}^{+}(k)$$
(1)

$$V^{-}(k) = V_{1}^{-}(k) + V_{2}^{-}(k)$$
(2)

 $v_1^+(k)$  and  $v_2^+(k)$  can further be expressed as:

$$V_{1}^{+}(k) = \sum_{i \in \mathcal{O}_{0}} \sum_{j \in \mathcal{D}_{I}} \sum_{m=0}^{M} \rho_{ilj}^{m}(k) b_{ij}(k-m) q_{i}(k-m)$$
(3)

$$V_{2}^{+}(k) = \sum_{i \in O_{j}} \sum_{j \in D_{0}} \sum_{m=0}^{M} \rho_{ilj}^{m}(k) b_{ij}(k-m) q_{i}(k-m)$$
(4)

As all flows in  $V_2^-(k)$  are form flow  $V_2^+(k-m)$  with a time lag m. Thus, the interrelation between  $V_2^-(k)$  and  $V_2^+(k)$  can be expressed as follows:

$$V_2^-(k) = f[V_2^+(k), V_2^+(k-1), ..., V_2^+(k-M)]$$
 (5)

where f is a function. If the cordonline covers a relatively small subnetwork, most trips in  $V_2^-(k)$  shall come from  $V^-(k)$  and  $V_2^+(k-1)$ . Thus, the above equation can be simplified as

$$V_2^-(k) = \alpha(k) V_2^+(k) + [1 - \alpha(k-1)] V_2^+(k-1)$$
(6)

where  $\alpha(k)$  is the fraction of  $V_2^+(k)$  having the second crossing over the cordonline during time interval k. Based on Eqs. (3), (4) and (6), one can construct the following relations between each set of cordonline flows and O-D flows:

$$\alpha(k) V^{+}(k) + [1 - \alpha(k-1)] V^{+}(k-1) - V^{-}(k) 
= \sum_{k \in \mathcal{O}_{i}} \sum_{j \in \mathcal{D}_{i}} \sum_{m=0}^{M} [\alpha(k) \rho_{ij}^{m}(k) + [1 - \alpha(k-1)] \rho_{ij}^{m-1}(k-1)] q_{i}(k-m) b_{ij}(k-m) 
- \sum_{i \in \mathcal{O}_{i}} \sum_{j \in \mathcal{D}_{0}} \sum_{m=0}^{M} \rho_{ij}^{m}(k) q_{i}(k-m) b_{ij}(k-m)$$
(7)

If the fraction parameters  $\{\alpha(k)\}$  and  $\{\rho_{ij}^{m}(k)\}$  are known, one can directly use Eq.(7) to estimate OD parameters.

## 1. Integration of Constraints Form Link And Intersection Turning Flows Given A Reliable DTA Model

Assuming that the estimated O-Ds,  $\{B_r^0(k)\}$ , have been obtained with our proposed non-assignment model, one can assign with an available DTA model to compute the

route-choice matrices  $\{A_n^p(k)\}$ , parameters  $\{p_n^m(k)\}$  and  $\{p_n^m(k)\}$ . The route-choice fractions are used to bridge the O-D flows with their resulting link and turning flows.

With all such information, one can construct the following set of constraints based of the flow counts  $\{Z_l(k)\}_{on link l}$ :

$$Z_{l}(k) = \sum_{m} \sum_{r} \sum_{p} B_{r}(k-m) A_{r}^{p}(k-m) \delta(p,l) p_{rl}^{m}(k)$$
(8)

Note that Eq. (10) is identical to Eq. (1) but with different notation, and it is the core equation of all assignment-based O-D estimation models. To take full advantage of available information, we propose, in addition to Eq. (10), to construct the following set of new constraints from intersection turning flow data:

$$T_{d}(k) = \sum_{m} \sum_{r} \sum_{p} B_{r}(k-m) A_{r}^{p}(k-m) \delta^{*}(p,d) p_{rd}^{*m}(k)$$
(9)

To compress the notation, we redefine the following vectors:

$$Z(k) = (Z_1(k),...,Z_I(k),...)^T$$

Then, Eqs. (8) and (9) be restated in a more compact form as:

$$Z(k) = \sum_{m=0}^{M} C^{m}(k)B(k-m)$$
(10)

$$T(k) = \sum_{m=0}^{M} C^{*m}(k)B(k-m)$$
 (11)

where M is the number of lag intervals.

Eqs. (10) and (11) are the two sets of assignment-based constraints which serve as the measurement equations for model estimation. For instance, one can restate Eqs. (10) and (11) into the following forms:

$$Z(k) - \sum_{m=1}^{M} C^{m}(k)B(k-m) = C^{0}(k)B(k) + \varepsilon(k)$$
 (12)

$$T(k) - \sum_{m=1}^{M} C^{*m}(k)B(k-m) = C^{0}(k)B(k) + \gamma(k)$$
(13)

where  $\varepsilon(k)$  and  $\gamma(k)$  are two error terms. The left-side terms of Eqs. (12) and (13) can be computed with measurable flows (e.g.  $\{Z(k)\}$  and  $\{T(k)\}$ ) and previously estimated O-Ds (e.g. B(k-m), m=1,...,M). The vector B(k) in the right terms is the system parameter that needs to be estimated.

To improve the estimation accuracy, one can also employ the estimated O-Ds from the

previously discussed two-stage approach to set up the following additional constraints:

$$B^{0}(k) = B(k) + \eta(k)$$
 (14)

where  $\eta(k)$  is a vector of error terms associated with the estimated O-Ds  $\{B^0(k)\}$ . Eq. (14) serve as a measurement equation along with Eqs. (12) and (13) for model estimation.

Similar to those studies in the literature, we apply the Kalman-filtering approach on system Eqs.(12-14) to derive the O-D matrix. To do so, one needs to assume that the time-varying O-Ds follow an auto regression process as follows:

$$B(k) = \alpha_1 B(k-1) + \alpha_2 (k-2) + \dots + \alpha_p B(k-p) + \mu(k)$$
 (15)

where  $\mu(k)$  is a vector of error terms and p is a prespecified constant.

Applying the Kalman-filtering procedures on those measurement and state transmission equations (i.e. Eqs. (12)-(15)), one can easily obtain the following recursive solution for O-D parameters  $\{B(k)\}$ :

$$B(k) = B(k-1) + G_1[Z(k) - \sum_{m=1}^{M} C^m(k)B(k-m) - C^0(k)\widetilde{B}(k)]$$

+ 
$$G_2[T(k) - \sum_{m=1}^{M} C^{*m}(k)B(k-m) - C^{*0}(k)\widetilde{B}(k)v]$$

$$+G_3[B^0(k)-\widetilde{B}(k)] \tag{16}$$

where  $\widetilde{B}(k)$  is the predicted O-Ds with state transmission equation that is computed as:

$$\widetilde{B}(k) = \alpha_1 B(k-1) + \alpha_2 (k-2) + \dots + \alpha_p B(k-p)$$
 (17)

and  $G = (G_1, G_2, G_3)$  is the gain matrix which can be computed as:

$$(G_1, G_2, G_3) = U(k) \begin{pmatrix} c \\ C^{*0}(k) \\ I \end{pmatrix}^T$$

$$\begin{bmatrix} C^{0}(k) \\ C^{*0}(k) \\ I \end{bmatrix} U(k) \begin{bmatrix} C^{0}(k) \\ C^{*0}(k) \\ I \end{bmatrix}^{T} + \begin{bmatrix} V_{\varepsilon}(k) & 0 & 0 \\ 0 & V_{\gamma}(k) & 0 \\ 0 & 0 & V_{\eta}(k) \end{bmatrix}^{-1}$$
(18)

I in Eq. (18) is the identical matrix. U(k) is the covariance matrix of  $\mu(k)$ , and  $V_{\varepsilon}(k)$ ,  $V_{\gamma}(k)$ ,  $V_{\eta}(k)$  are covariance matrices of  $\varepsilon(k)$ ,  $\gamma(k)$  and  $\eta(k)$ , respectively.

#### 2. Algorithm

A step-by-step description of the estimation algorithm for the proposed integrated model is presented below:

Step 0 : Perform the two-stage O-D estimation and compute O-D flows  $\{B_r^0(k)\}$  estimated with non-assignment approaches:

Step 1: Assign the estimated O-D  $\{B_r^0(k)\}$  flows with the available DTA model, and compute the assignment matrix  $A_r^p(k)$ 

Step 2: Construct constraints shown as Eq. ((12)-(14)) and establish additional constraints if sampled or partial O-Ds are available:

Step 3: Compute the coefficient matrix  $C^{m}(k)$ ,  $C^{*m}(k)$  in Eq. (12) and (13):

Step 4: Compute the gain matrix G with Eq. (18);

Step 5: Predict  $\{B_r^0(k)\}$  with state transmission Eq. (17)

Step 6: Compute the O-D estimate with Eq. (16)

## III. AN ILLUSTRATIVE EXAMPLE

## 1. Example Network Design

origin nodes: 1, 2

destination nodes: 5, 6, 7, 8

8 O-D parameters:  $b_{\rm l,5}$  ,  $b_{\rm l,6}$  ,  $b_{\rm l,7}$  ,  $b_{\rm l,8}$  ,  $b_{\rm 2,5}$  ,  $b_{\rm 2,6}$  ,  $b_{\rm 2,7}$  ,  $b_{\rm 2,8}$ 

intermediate nodes: 3, 4

pretimed signalized intersections: 3, 4, 5, 6

two entry streams  $g_1$ ,  $g_2$  at nodes 1, 2

four exit streams  $\mathcal{Y}_5$ ,  $\mathcal{Y}_6$ ,  $\mathcal{Y}_7$ ,  $\mathcal{Y}_8$  at nodes 5,6,7,8 35 sets of different entry volumes and turning fractions, where  $k=1,2,\cdots,35$ 

each time interval: 10 min.

The information of turning flows, route-choice splits and actual O-D data were identified from the simulation output data.

Simulation tool: NETSIM.

#### 2. Experimental Design

Scenario-1

Entry flows from nodes 1, 2

Exit flows from nodes 5, 6, 7, 8

Non-assignment based approach based on entry and exit flows

Scenario-2

Entry flows from nodes 1, 2

Exit flows from nodes 5, 6, 7, 8

Cordonline flows from two cordonlines  $l_1$ ,  $l_2$ Non-assignment based approach with a cordonline model

Scenario-3

Entry flows from nodes 1, 2

Exit flows from nodes 5, 6, 7, 8

Cordonline flows from two cordonlines  $l_1$ ,  $l_2$ 

Flows from all links

Route-choice splits from each O-D pair and its feasible paths

Turning flows at intersection 3, 4, 5, 6

Integrated estimation method with intersection turning flow data

Using the rooted-mean-squared (RMS) errors as the evaluation criterion: the comparison results between those scenarios are reported in Table 1. It clearly indicated that if a reliable DTA is available, the proposed combined approach can provide an effective and accurate of dynamic network O-Ds

Table 1. Comparison of RMS among Different Scenarios

	Scenario-1	Scenario-2	Scenario-3
$b_{1,5}$	0.0459	0.0373	0.0219
$b_{1,6}$	0.0318	0.0292	0.0157
$b_{1,7}$	0.0283	0.0284	0.0106
$b_{1,8}$	0.0341	0.0195	0.0163
b 2,5	0.0406	0.0242	0.0156
b 2,6	0.0276	0.0203	0.0091
h 2 2	0.0384	0.0272	0.0122
L 7 0	0.0338	0.0179	0.0078
overall	0.0351	0.0201	0.0137

#### IV. CONCLUSIONS

This paper presents an effective method for dynamic O-D distributions in urban networks. The proposed method not only has the strengths of both categories of dynamic O-D estimation models in the literature, but also is capable of taking advantage of intersection turning fraction data.

The construction of an additional set of constraints with available or estimated intersection turning flows has substantially improved the estimation results. It thus is not only a core of our proposed combined estimation method, but also an improvement to those studied relying on a DTA model for exploring the dynamic O-D issue.

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