# 저간섭 CDMA 시스템을 위한 3 상확산코드에 관한 연구

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# The study of a class of the ternary spreading codes for Interference-cancelled CDMA system

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## Abstract

In this paper, a class of multi-level spreading codes, i.e., ternary ZCD(zero-correlation duration) spreading codes are introduced. Novel ternary ZCD code sets are generated by carrying out a chip-shift operation of the ternary preferred pairs (TPP) with a ZCD of $(0.75N + 1)$ chips. Ternary ZCD codes have superior family size than the binary ZCD codes and they are effective for approximately synchronized (AS) CDMA

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## 1. Introduction

For any two spreading codes of period $N$ ; $S_N^{(x)} = (s_0^{(x)}, \cdots, s_{N-1}^{(x)})$ and $S_N^{(y)} = (s_0^{(y)}, \cdots, s_{N-1}^{(y)})$ , the periodic correlation function for a shift $\tau$ is defined as

$$\theta_{x,y}(\tau) = \sum_{n=0}^{N-1} s_n^{(x)} s_{(n+\tau, \bmod N)}^{(y)} \qquad (1)$$

This function becomes the autocorrelation function (ACF) when $x = y$ and the crosscorrelation function (CCF) when $x \neq y$ . Since the maximum magnitude of periodic ACF sidelobes ($\theta_{as}$) and the maximum magnitude of periodic CCF ($\theta_c$) are bounded by theoretical limits [1], spreading codes with both zero $\theta_{as}$ and zero $\theta_c$ cannot be constructed. However, it is possible to construct spreading codes with both zero $\theta_{as}$ and zero $\theta_c$ at the local duration around $\tau = 0$. This local duration is defined as the zero-correlation duration (ZCD). This ZCD property enables approximate synchronization without multiple access interference (MAI) at the uplink of CDMA systems, i.e., approximately synchronized CDMA (AS-CDMA) systems [2]. AS-CDMA was proposed to eliminate MAI at the uplink of CDMA[2,3]. One of the most important problems of spreading code theory of AS-CDMA is to construct code sets with the larger family sizes and the wider ZCD. Especially, to eliminate MAI of CDMA, wider ZCD is preferred. The ZCD property in the binary codes has been reported for some documents [4-8]. However, the maximum ZCD of binary codes is bounded in $(0.5N + 1)$ chips for the period $N$ [4,9], and the larger family sizes with ZCD is still required to increase the system capacity of CDMA. In this paper, we introduce a

new class of ternary spreading codes with wide ZCD [10-11]. At first, we present a generalized construction method for ternary preferred pairs (TPP) that have the ZCD of $(0.75N + 1)$ chips and periods of $4 \times 2^n (n = 1,2,3 \cdots)$. Secondly, using the chip-shift operation to the TPP, we generate new ternary ZCD codes that have sufficient ZCD and family sizes for AS-CDMA systems.

## 2. Construction method of ternary ZCD codes

### Step (I): Method for constructing TPP

The TPP of period $N = 4 \times 2^n (n = 1,2,3 \cdots)$ with ZCD of $(0.75N + 1)$ is denoted as $\{C_N^{(a)}, C_N^{(b)}\}$, where $C_N^{(a)} = (c_0^{(a)}, c_1^{(a)}, \cdots, c_{N-1}^{(a)})$, and $C_N^{(b)} = (c_0^{(b)}, c_1^{(b)}, \cdots, c_{N-1}^{(b)})$.

The initial basic matrix to construct $\{C_N^{(a)}, C_N^{(b)}\}$ is defined as

$$GA = \begin{bmatrix} + & z & + & z & + & z & - & z \\ + & z & + & z & - & z & + & z \\ + & z & - & z & + & z & + & z \\ - & z & + & z & + & z & + & z \end{bmatrix} \text{ or } GB = \begin{bmatrix} + & + & z & z & + & - & z & z \\ + & + & z & z & - & + & z & z \\ + & - & z & z & + & + & z & z \\ - & + & z & z & + & + & z & z \end{bmatrix} \quad (2)$$

where $+$, $-$, and $z$ denote $1$, $-1$, and zero, respectively.

Any row of $\pm GA$ or $\pm GB$ is denoted as $C_8^{(a)} = (c_0^{(a)}, \cdots, c_7^{(a)})$ $= (e_0, z, e_1, z, e_2, z, e_3, z)$ or $(e_0, e_1, z, z, e_2, e_3, z, z)$.

$C_8^{(b)} = (c_0^{(b)}, \cdots, c_7^{(b)})$ $= (v_0, z, v_1, z, v_2, z, v_3, z)$ or $(v_0, v_1, z, z, v_2, v_3, z, z)$ ) is generated from $C_8^{(a)}$, where $v_q = (-1)^q e_q$ $(q = 0,1,2,3)$. A pair of $\{C_8^{(a)}, C_8^{(b)}\}$ has ZCD of $(0.75 \times 8 + 1)$ chips and is defined as the initial ternary pair. By using $\{C_8^{(a)}, C_8^{(b)}\}$, the TPP with longer period are constructed recursively as the following extension method. Using any given TPP of $\{C_m^{(a)}, C_m^{(b)}\}$ with period $m = 4 \times 2^i$ $(i = 1,2,3 \cdots)$, the extension matrix $DA$ or $DB$ with period $2m$ is generated as Eqn (3), where $DA$ is resulted from $\pm GA$ and $DB$ is resulted from $\pm GB$ respectively.

$$DA = \begin{bmatrix} X & Z & Y & Z & X & Z & -Y & Z \\ X & Z & Y & Z & -X & Z & Y & Z \\ X & Z & -Y & Z & X & Z & Y & Z \\ -X & Z & Y & Z & X & Z & Y & Z \end{bmatrix} \text{ or }$$

$$DB = \begin{bmatrix} V & W & Z & Z & V & -W & Z & Z \\ V & W & Z & Z & -V & W & Z & Z \\ V & -W & Z & Z & V & W & Z & Z \\ -V & W & Z & Z & V & W & Z & Z \end{bmatrix} \quad (3)$$

where $Z = \frac{m}{4}$ zeros, $X = (c_0^{(a)}, \cdots, c_{\frac{m}{8}-1}^{(a)}, c_{\frac{2m}{8}}^{(a)}, \cdots, c_{\frac{3m}{8}-1}^{(a)})$, $Y = (c_{\frac{4m}{8}}^{(a)}, \cdots, c_{\frac{5m}{8}-1}^{(a)}, c_{\frac{6m}{8}}^{(a)}, \cdots, c_{\frac{7m}{8}-1}^{(a)})$, $V = (c_0^{(a)}, \cdots, c_{\frac{m}{4}-1}^{(a)})$, and $W = (c_{\frac{2m}{4}}^{(a)}, \cdots, c_{\frac{3m}{4}-1}^{(a)})$.

Any row of $\pm DA$ or $\pm DB$ is

$C_{2m}^{(a)} = (c_0^{(a)}, c_1^{(a)}, c_2^{(a)}, \cdots, c_{2m-1}^{(a)})$ with period $2m$. $C_{2m}^{(b)} = (c_0^{(b)}, c_1^{(b)}, c_2^{(b)}, \cdots, c_{2m-1}^{(b)})$ is generated from $C_{2m}^{(a)}$, where $c_q^{(b)} = (-1)^q c_q^{(a)}$ $(q = 0,1,\cdots,2m-1)$. $\{C_{2m}^{(a)}, C_{2m}^{(b)}\}$ is a TPP with ZCD of $(0.75 \times 2m + 1)$ chips. Thus, for the period of $N = 4 \times 2^n (n = 1,2,3 \cdots)$, $\{C_N^{(a)}, C_N^{(b)}\}$ that have ZCD of $(0.75N + 1)$ chips can be constructed.

### Example(i):

A TPP with a period $4 \times 2^2 = 16$ and ZCD=13 becomes

$$\begin{cases} C_{16}^{(a)} = (+ + zz + - zz + + zz - + zz) \\ C_{16}^{(b)} = (+ - zz + + zz + - zz - - zz) \end{cases} \text{ or }$$

$$\begin{cases} C_{16}^{(a)} = (+ + + - zzzz + + - + zzzz) \\ C_{16}^{(b)} = (+ - + + zzzz + - - - zzzz) \end{cases}$$

where $+$, $z$, and $-$ denote $1$, $0$, and $-1$, respectively.

### Step (II): Construction of sets of ternary ZCD code:

A set of $M$ ternary ZCD codes means a ternary code set that has ZCD $\leq (0.75N + 1)$ chips and a family size of $M$. This set can be constructed by the chip-shift operation using $\{C_N^{(a)}, C_N^{(b)}\}$. Let $T^l$ be the chip-shift operator, which shifts a code cyclically to the left by $l$ chips, a set of $M$ ternary ZCD codes of period $N$ can be generated from $\{C_N^{(a)}, C_N^{(b)}\}$ as

$\{ C_N^{(a)}, C_N^{(b)}, T^{\Delta}[C_N^{(a)}], T^{\Delta}[C_N^{(b)}], T^{2\Delta}[C_N^{(a)}], T^{2\Delta}[C_N^{(b)}], \cdots$
$, T^{(k-1)\Delta}[C_N^{(a)}], T^{(k-1)\Delta}[C_N^{(b)}], T^{k\Delta}[C_N^{(a)}], T^{k\Delta}[C_N^{(b)}] \}$ (4)

where $\Delta$ is a chip-shift increment and $k$ the maximum number of chip-shifts for a code.

$\Delta$ and $k$ should satisfy $\left| (k+1)\Delta \right| \leq \left| \frac{3N}{8} + 1 \right|$, where $\Delta$ is a positive and $k$ a nonnegative integer. $M$ and ZCD of the generated code become

$$M = 2(k+1) \quad \text{and} \quad ZCD = |2\Delta - 1| \quad (5)$$

The generated codes have the normalized ACF peak value of 0.5. The family sizes of the proposed ternary ZCD codes are listed in Table 1.

Fig. 1 shows the family size against ZCD of the proposed codes with a period of 128. It is clear that proposed codes have larger family sizes than that of binary ZCD code pairs with ZCD [4] or QS(OG-r) codes generated from orthogonal Gold codes [5]. Here, large family sizes produce a larger CDMA user number and large ZCDs produce longer cell radius of AS-CDMA systems, respectively.

## 3. SYSTEM APPLICATION EXAMPLE

The system capacity of AS-CDMA system is estimated. Required ZCD of an AS-CDMA system is determined from cell radius $R$ and chip rate of the spreading code $Rc$ as

$$ZCD = (4R / c) \cdot Rc + 1 \quad (6)$$

where c is the light speed. For example, when $R$=150$m$ and $Rc$=3Mcps, the required

ZCD is $((4 \cdot 150)/(3 \cdot 10^8)) \cdot 3 \cdot 10^6 + 1 = 17$ chips.

An example of the AS-CDMA system specification using the proposed codes is listed in Table 2. It is noted that the uplink of AS-CDMA system with 215 users is available, where bit rate is 16 kbps and $R = 155 \ [m]$.

## 3. Conclusion

New ternary ZCD codes that have superior family size than that of binary ZCD codes were constructed. The proposed ternary ZCD codes can be usefully employed in AS-CDMA systems with sufficient system performance without creating. MAI.

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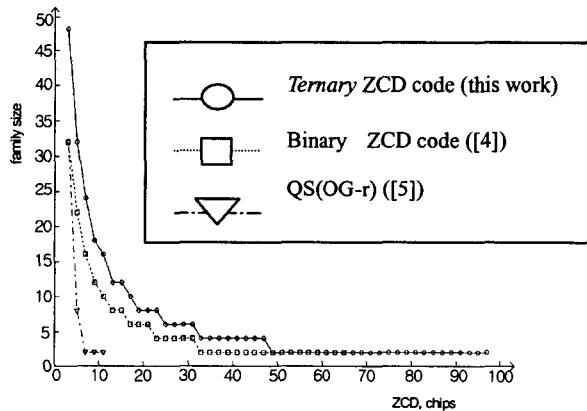
Fig. 1. Family size against ZCD of the *ternary* ZCD codes of period 128

Table 1: Family sizes of the *ternary* ZCD codes for N = 32, 64,128 and 256

| ZCD \ N | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 25 | 27 | 29 | 31 | ... | 95 | ... | 193 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 32 | 12 | 8 | 6 | 4 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | - | - | - | - | - | - | - |
| 64 | 24 | 16 | 12 | 8 | 8 | 6 | 6 | 6 | 4 | 4 | 4 | 2 | 2 | 2 | 2 | ... | - | - | - |
| 128 | 48 | 32 | 24 | 18 | 16 | 12 | 12 | 10 | 8 | 8 | 8 | 6 | 6 | 6 | 6 | ... | 2 | - | - |
| 256 | 96 | 64 | 48 | 38 | 32 | 26 | 24 | 20 | 18 | 16 | 16 | 14 | 12 | 12 | 12 | ... | 4 | ... | 2 |

Table 2: An example of the AS-CDMA system specification

| | System A | System B |
|---|---|---|
| Access, Duplex | CDMA/FDMA, TDD | |
| Spreading code | Ternary ZCD codes (*N*=128) | |
| | *M*=12, ZCD=15 | *M* =24, ZCD=7 |
| Cell radius $R$ | 350 m | 155 m |
| Available user number /cell | 104 | 215 |
| Bit rate $Rb$ | 16 k bit/s | 16 k bit/s |
| Chip rate $Rc$ | 3.0 M chip/s | 2.9 M chip/s |
| Information modulation | BPSK | |

$$L = (ZCD-1)/2 \,,\ R_C = N/MFt \,,\ \delta = L/R_C \,,\ R = (c \cdot \delta)/2$$
$$User\,number = (M \cdot BW)/R_C \quad,\quad Rb = (Rc \cdot rT)/(N+2L)$$

$L$ : guard chip, $M$ : family size, ZCD : zero correlation duration
$\delta$ : maximum propagation delay time, $BW$ : 26 MHz (2.4 GHz ISM band )
$MFt$ : $N$ chip-processing time of matched filter
Period of spreading code : Tx : $N+2L$ (=$L+N+L$) , Rx :$N$
$rT$ : uplink share of TDD, i.e., 0.75 [4]