Acoustic Properties of Solid Materials: Sound Speed, Transmission Coefficient, and Attenuation

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The speed of sound, transmission coefficient, and attenuation are measured around the center frequency 1 and 2 MHz in solid materials such as bone, sediment, rubber, and Lucite materials. Common and different characteristics of such materials in the sound speed, transmission coefficient, and attenuation are discussed. Ambiguities in estimating such acoustic characteristics are also addressed. Ultrasonic properties of the first and second kind waves are clarified for different materials. Discussions are concentrated on classes of sound speed, broadband ultrasonic attenuation (BUA), and correlations of sound speed and BUA with apparent density. New correlations of inverse sound speed square and BUA with apparent density are suggested.

I. INTRODUCTION

Acoustic characteristics of materials such as sound speed, transmission coefficient, and attenuation attract much attention in their applications [1–4]. They were recently used to diagnose physical properties of bone. In this study, such acoustic characteristics used in diagnosing physical properties of bone are applied to porous materials such as bone itself and sand sediment and to nonrigid materials such as rubber and Lucite.

According to Biot [5], there exist two types of longitudinal waves in the mixture of solid and fluid. One is the fast wave in phase and the other is the slow wave out of phase between solid and fluid materials. Properties of these two waves should be more clarified since they play important roles in understanding acoustic characteristics of various materials.

Sound speed, transmission coefficient, and attenuation are investigated as the functions of frequency and density (or porosity) in water. Common and different acoustic characteristics of different materials are discussed. The separation of the first and second kind waves in measurement is important since they have different characteristics for different materials. Discussions are specially concentrated on the classes of sound speed and on the frequency slopes of attenuation.

II. ACOUSTIC CHARACTERISTICS OF SOLID MATERIALS

The sound speed, transmission coefficient, and attenuation of bone, sediment, rubber, and Lucite materials are measured as the functions of frequency and apparent density (or porosity) in water.

Specimen materials have the following physical properties. The bone specimen is a defatted bovine tibia with the density 750 kg/m^3, the sound speed 1650 m/s, and the thickness 300 mm. The sand sediment slab represents the sand sediment container with the size 42 mm × 100 mm × 100 mm. The slab contains sand sediment with the water porosity of about 0.43, the grain size less than 0.5 mm, the density 2680 kg/m^3 and the sound speed 1614 m/s. Rubber has the density 1112 kg/m^3, the sound speed 1962 m/s, and the size 19.6 mm × 80 mm × 100 mm. Lucite specimens with the density 1200 kg/m^3 and the sound speed 2650 m/s have two different sizes. Thin Lucite has the size 30 mm × 100 mm × 100 mm and thick Lucite has the size 4.8 mm × 100 mm × 100 mm. Thin and thick Lucite are selected to see whether the conventional BUA depends on the thickness of specimen. Furthermore, Lucite specimens with circular cylindrical pores are constructed to adjust the porosity of Lucite material.

A pair of transducers were oriented vertically facing each other for transmission measurement. The distance between the transmitter and the receiver is 70 mm and the distance between the transmitter and the front surface of sample is 10 mm. The transmission coefficient, attenuation, and phase velocity of sediment slab are measured using the pulse transmission technique with one wavelength in broad band frequencies with the center frequency 1 MHz and 2 MHz for bone, sediment, rubber, and Lucite. The advantage of broad band frequency measurement appear in the analysis of acoustic characteristics as a function of frequency. The transducers are connected to a ultrasonic pulser-receiver (Panametrics 5072PR) that are operated in two switching modes of transmitting and receiving. Signals are obtained by a digital storage oscilloscope (LeCroy LT322).

A. Classes of Waves

It is generally known that there are two types of longitudinal transmitting waves in the solid-fluid mixture: the fast wave and the slow wave [5]. In this part, more specific distinctions for two types of waves are tried for different materials. The receiving signals through pure rubber and Lucite correspond to the fast waves. However, the receiving signal through Lucite with circular cylindrical pores possesses the slow wave following the fast wave. The receiving signals through bone and sediment correspond to the mixed waves of the fast and slow waves. The main contribution of each wave depends on the porosity of porous media such as bone and sediment. In the low porosity region, the fast wave is dominant while in the high porosity region, the slow wave is dominant.

B. Speed of Sound

The time difference of the transmitted signals between through specimen and through pure water is used to calculate the experimental sound speed of specimen with respect to that of water.

There is ambiguity in measuring the sound speed. There exist two types of waves as explained above: the fast and slow waves. The sound speed of the fast wave decreases as the water porosity increases while the phase velocity of the slow wave increases as the water porosity increases as illustrated in Figure 1. Theoretical calculations use the Modified Biot-Attenborough (MBA) model [6]. There are also two types of sound speeds: phase velocity and group velocity. In addition, the shape of sound wave may be distorted in transmitting a dispersive media due to attenuation, scattering, dispersion, etc. Therefore, when the sound speed is measured, the type of the sound speed should be clarified.

New correlations of sound speed with apparent density are here suggested. The correlation of apparent density versus the sound speed was normally used before as shown in Figure 2. However, the new correlation of apparent density versus the inverse sound speed square (ρ vs. $1/c^2$) is suggested in the osteoporosis diagnosis as shown in Figure 3. The slope of the latter correlation is related to the elastic modulus of the material. In fact, the latter correlation shows a slightly better result than the former correlation.

C. Transmission Coefficient

The Fourier transformed amplitude of the transmitted temporal signal through specimen is used to determine the experimental transmission coefficient with respect to that through water. Experimental power spectra and transmission coefficients of bone, sediment, rubber, and Lucite are analyzed as a function of frequency.

Equations for the acoustic pressure transmission coefficient and reflection coefficient can be obtained for the three medium propagation in which a plane acoustic wave encounters porous medium delimited by a plane normal to the incident wave. Applying the boundary conditions, the transmission coefficient T becomes

$$T = \frac{2}{(1 + z_1/z_3)\cos(k_b d) + i(z_b/z_3 + z_1/z_b)\sin(k_b d)} \quad (1)$$

with the characteristic impedances $z_1 = \rho_1 c_1$, $z_b = \rho_b c_b$, and $z_3 = \rho_3 c_3$. Note that the subscript b in the propagation constant k_b and impedance z_b for the total wave should be replaced with the subscript bf for the fast wave and the subscript bs for the slow wave.

The transmission coefficient explicitly depends on specimen thickness according to the above equation. The effect of specimen thickness in the magnitude of transmission coefficient can be large in the case of small attenuation. For example, if the specimen thickness is the multiple of the half wave length of the sound wave it represents the complete transmission. The experimental transmission coefficient of Lucite is given as a function of frequency in Figure 4, which illustrates the effect of thickness.

There is also ambiguity in analyzing the transmission coefficient. The transmission coefficient of the fast wave decreases as the water porosity increases but the transmission coefficient of the slow wave increases as the water porosity increases as seen in Figure 5. Theoretical calculations use the MBA model [6]. The overall transmission coefficient in the low porosity (or high density) region is roughly dominated by the fast wave, but the overall transmission coefficient in the high porosity (low density) region is dominated by the slow wave.

The transmission coefficients for pure rubber and Lucite are mainly determined by the fast waves but the transmission coefficients for sediment and bone are mainly determined by the mixed waves of the fast and slow waves. Therefore, when the transmission coefficient is discussed, the mixed ratio between the fast wave and the slow wave should be clarified for porous media such as sediment and bone. Furthermore, the used frequency and specimen thickness are required since the transmission coefficient depends on the frequency and thickness.

D. Attenuation

The Fourier transformed amplitude of the transmitted temporal signal through specimen is also used to determine the experimental attenuation with respect to that through water.

The linearity analysis in the frequency slope of attenuation is one of important methods in osteoporosis. The porous materials such as bone and sediment have linear frequency slopes in attenuation. However, the nonporous materials such as rubber and Lucite have nonlinear frequency slopes in attenuation.

The problem in the measurement of attenuation as a function of frequency, which is normally called broadband ultrasonic attenuation (BUA), is that BUA in bone specimen does not take care of the effect coming from the transmission coefficient of bone. Basically, the conventional BUA is the insertion loss rather than the attenuation, which is experimentally defined by equation (2).

BUA depends on frequency, specimen thickness, transmission coefficient, etc. The effect of transmission coefficient in BUA measurement is large in small attenuation while it is small in large attenuation. The effect of specimen thickness in BUA measurement is also large in small attenuation while it is small in large attenuation. Thus, BUA is more closer to the conventional attenuation as a function of frequency only in the case of large transmission coefficient and large attenuation of specimen.

In order to remove the contribution of transmission coefficient or the dependence of specimen thickness, attenuation measurement should be performed by the comparison of transmitted pressure fields through two samples with different thickness. The attenuation coefficient α in dB/m as the imaginary part of the propagation constant is computed in the SI units by

$$\alpha = -[20(D_m - D_1)^{-1} / \ln(10)] Re \ln[p_m/p_1] \qquad (2)$$

where D_m is the thickness of a thick sample, D_1 is the thickness of a thin sample, p_m is the transmitted pressure amplitude through the thick sample, and p_1 is the transmitted pressure amplitude through the thin sample [3]. In this calculation, the attenuation does not depend on the transmission coefficient or the thickness of specimen.

The distinction between insertion loss and attenuation comes from the different reference signal compared with the tested signal: the signal through water or through thin reference specimen. If the reference signal through thin reference specimen is used, the form of the attenuation coefficient $\alpha = af^n$ is suggested For example, the attenuation coefficient for pure sediment material $\alpha = \alpha_0 f^n = 3 \times 10^{-5} f$ with the frequency f and the index $n = 1$ is used according to Hamilton [4]. If the reference signal through water is used, the formula for the attenuation $\alpha = af^n + b$ is suggested. The first term determines the frequency dependent attenuation associated with the pure attenuation through specimen and the second term determines the frequency independent attenuation mainly associated with the transmission coefficient. If a specimen is thicker, the discrepancy between insertion loss and attenuation is lower.

It is also necessary to distinguish the attenuation coefficients for the fast wave and slow wave since they show different frequency dependence. In the mixed situation of two waves, their interference patterns are important in determining the attenuation. The linearity observed in the data of apparent density vs. BUA is only plausible in the low porosity sample containing mainly for the fast wave or in the high porosity sample mainly for the slow wave. Since attenuation coefficients for rubber and Lucite are roughly proportional to $f^{0.5}$, the linearity in the data of apparent density vs. BUA can not be applied to rubber and Lucite unlike bone and sediment.

The correlation of BUA with apparent density for bone is shown in Figure 6 when the reference signal is water. The new correlation of BUA with apparent density for bone is suggested if the different reference signal is used. The new correlation can show a better result than the old one since there is no dependence of the specimen thickness and the transmission coefficient.

III. CONCLUSIONS

The speed of sound, transmission coefficient, and attenuation are measured as the functions of frequency and density (or porosity) around the center frequency 1 and 2 MHz in bone, sediment, rubber, and Lucite materials.

Common and different characteristics of such materials in the sound speed, transmission coefficient, and attenuation are discussed. Ambiguities in analyzing such acoustic characteristics, which are mainly attributed from opposite properties of the fast and slow waves and nonlinear dependences of the attenuation data, are addressed.

The separate analysis of the first kind (fast) wave and the second kind (slow) wave in the measurement of acoustic characteristics is important for different materials. They usually have the opposite acoustic characteristics. The transmitting signals through pure rubber and Lucite materials are almost the fast wave while signals through bone and sediment are the mixed ones of the fast and slow waves.

Even though the phase velocity and the group velocity are almost the same in the ultrasound frequency range, the distortion of the transmitting signal due to the material nonlinearity makes some errors in analyzing the speed of sound. The non-porous materials such as rubber and Lucite have nonlinear frequency slopes in attenuation but the porous materials such as bone and sediment have linear frequency slopes in attenuation.

The conventional frequency slope and density slope of the attenuation used in bone analysis depend on the physical properties of materials and the effect of specimen thickness. The transmission coefficient effect should be considered in a material with the low transmission coefficient. The conventional technique of BUA measurement is valid in materials with the high transmission coefficient. The linearity in the data of apparent density vs. BUA is also valid in the high porosity material through which the slow wave mainly propagates.

New correlations of inverse sound speed square and broadband ultrasonic attenuation (BUA) with apparent density are suggested in order to improve the credibility for physical properties of specimens.

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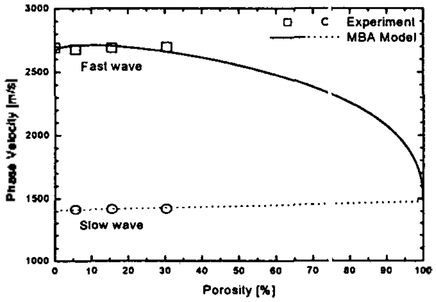

Figure 1. Sound speed as a functior of porosity for thick Lucite.

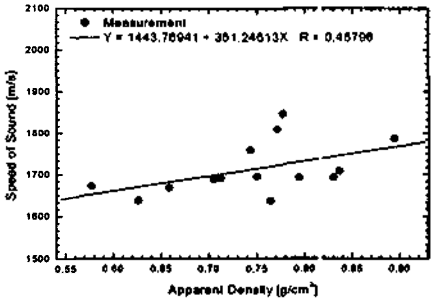

Figure 2. Correlation of sound speed with apparent density for bone.

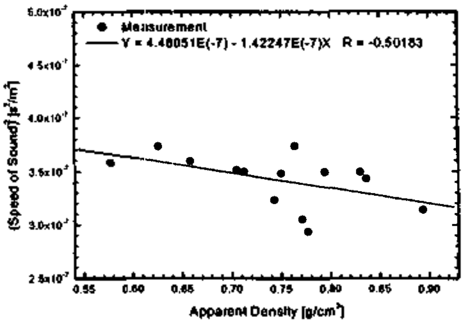

Figure 3. Correlation of inverse sound speed square with apparent density for bone.

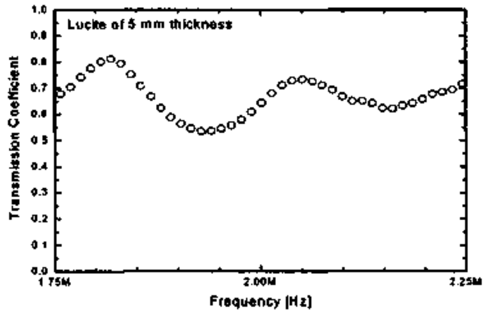

Figure 4. Experimental transmission coefficient as a function of frequency for thin Lucite.

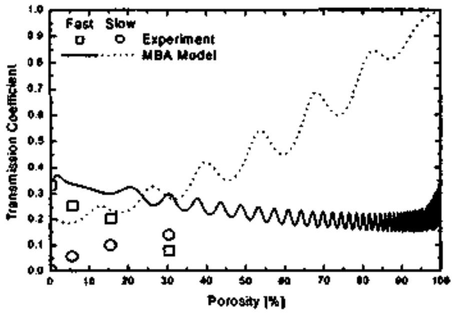

Figure 5. Transmission coefficient as a function of porosity for thick Lucite.

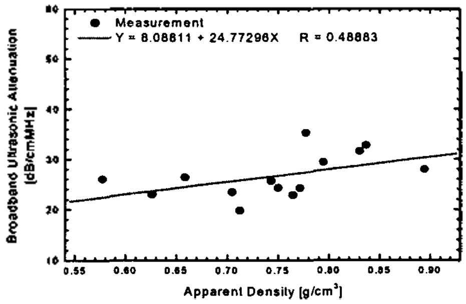

Figure 6. Correlation of BUA with apparent density for bone.