Burn-in When Minimal Repair Costs Vary With Time *

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ABSTRACT

Burn-in is a widely used method to eliminate initial failures. Preventive maintenance policy such as block replacement with minimal repair at failure is often used in field operation. In this paper burn-in and maintenance policy are taken into consideration at the same time. The cost of a minimal repair is assumed to be a non-decreasing function of its age. The problems of determining optimal burn-in times and optimal maintenance policy are considered.

1. Introduction

Let $F(t)$ be a distribution function of a lifetime X. If X has density $f(t)$ on $[0,\infty)$, then its failure rate function $h(t)$ is defined as $h(t) = f(t)/\bar{F}(t)$, where $\bar{F}(t) = 1 - F(t)$ is the survival function of X. Based on the behavior of failure rate, various nonparametric classes of life distributions have been defined in the literature. The following is one definition of a bathtub-shaped failure rate function which we shall use in this article.

Definition. A real-valued failure rate function $h(t)$ is said to be bathtub-shaped failure rate (BTR) with change points t_1 and t_2, if there exist change points $0 \le t_1 \le t_2 < \infty$, such that $h(t)$ is strictly decreasing on $[0,t_1)$, constant on $[t_1,t_2)$ and then strictly increasing on $[t_2,\infty)$.

The time interval $[0,t_1]$ is called the infant mortality period; the interval $[t_1,t_2]$, where $h(t)$

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is flat and attains its minimum value, is called the normal operating life or the useful life; the interval $[t_2, \infty)$ is called the wear-out period.

Burn-in is a method used to eliminate the initial failures of components before they are put into field operation. The burn-in procedure is stopped when a preassigned reliability goal is achieved, e.g. when the mean residual life is long enough. Since burn-in is usually costly, one of the major problem is to decide how long the procedure should continue. The best time to stop the burn-in process for a given criterion to be optimized is called the optimal burn-in time. An introduction to this important area of reliability can be found in Jensen and Petersen (1982). In the literature, certain cost structures have been proposed and the corresponding problem of finding the optimal burn-in time has been considered. See, for example, Clarotti and Spizzichino (1991) and Mi (1994). A survey of recent research in burn-in can be found in Block and Savits (1997).

Mi (1994) consider the following procedure. Consider a fixed burn-in time b and begin to burn-in a new component. If the component fails before burn-in time b, then repair it completely with shop repair cost, then burn-in the repaired component again and so on. If the component survives the burn-in time b, then it is put into field operation. For a burned-in component he consider the block replacement policy with minimal repair. Cha (2000) consider that the failed component is only minimally repaired rather than being completely repaired during a burn-in period. He adopt block replacement policy with minimal repair at failure, assuming that the cost of minimal repair at failure is constant

In this paper, it is assumed that the cost of a minimal repair to the component which fails at age t is is a continuous nondecreasing function of t. Hence, as the component ages it becomes more expensive to perform minimal repair. It is shown that the optimal burn-in time b^* must occur before the change point t_1 of $h(t)$ under the assumption of a bathtub-shaped failure rate function. Explicit solutions for the optimal burn-in time and preventive maintenance policy are given for the Weibull-exponential distribution.

2. Expected Minimal Repair Cost

Consider a fixed burn-in time b and begin to burn-in a new component. If the component fails before burn-in time b, then repair it minimally, and then continue the burn-in procedure for the repaired component. After the fixed burn-in time b, the component is put into field operation. For a burned-in component, block replacement policy with minimal repair at failure is adopted. Under this policy the component is replaced by a burned-in component at planned time kT $(k = 1, 2, \cdots)$, where T is a fixed number, and is minimally repaired at failure between planned replacements.

Let $N_x(y)$ be the random variable denoting the number of minimal repairs performed on the component in $[x, x+y]$. It is well known that $N_x(y)$ has a Poisson distribution with parameter $H_x(y) = H(x+y) - H(x)$ where $H(t) = \int_0^t h(s)ds$. Let $C(t)$ be the cost of minimal repair to the component which fails at time t, where $C(t)$ is a continuous nondecreasing function of t. Now if $N_x(y) = k$, and $t_1, \cdots, t_k$ are the times of the minimal repairs, then the total minimal repair cost in the interval $[x, x+y]$ is $\sum_{i=1}^{k} C(t_i)$. Given $N_x(y) = k$, we know that $\tau_1 = H(t_1), \cdots, \tau_k = H(t_k)$ are distributed as the order statistics of a random sample of size k from the uniform distribution on $[H(x), H(x+y)]$. Hence the expected minimal repair cost in the interval $[x, x+y]$ is

$$\begin{aligned}
& E_{N_x(y)}(E(C(t_1) + \cdots + C(t_k)|N_x(y) = k)) \\
&= E_{N_x(y)}\left(kE(C(H^{-1}(\tau))|N_x(y) = k)\right) \\
&= E_{N_x(y)}\left(\frac{k}{H(x+y) - H(x)} \int_{H(x)}^{H(x+y)} C(H^{-1}(t))dt\right) \\
&= \left(\frac{1}{H(x+y) - H(x)} \int_{H(x)}^{H(x+y)} C(H^{-1}(t))dt\right) E_{N_x(y)}(k) \\
&= \int_{H(x)}^{H(x+y)} C(H^{-1}(t))dt \\
&= \int_{x}^{x+y} C(t)h(t)dt. \qquad (1)
\end{aligned}$$

3. Optimal Burn-in

Let $C_1(t)$ and $C_2(t)$ denote the costs of a minimal repair which fails at time t during burn-in period and in field operation, respectively. We assume that $C_1(t) \le C_2(t)$ for all $t \ge 0$, then this means that the cost of a minimal repair during a burn-in procedure is lower than that of a minimal repair in field operation. Then from (1) the expected minimal repair costs in the interval $[0, b]$ and $[b, b+T]$ are given by

$$\int_0^b C_1(t)h(t)dt \text{ and } \int_b^{b+T} C_2(t)h(t)dt,$$

respectively. Let $c_0(b)$ be the cost for burn-in, where $c_0(b)$ is a continuous nondecreasing function of b. Hence the long-run average cost $C(b, T)$ is given by

$$C(b,T) = \frac{1}{T}\Big(c_0(b) + \int_0^b C_1(t)h(t)dt + c_r + \int_b^{b+T} C_2(t)h(t)dt\Big), \qquad (2)$$

where c_r the cost of a replacement.

Theorem 1. Suppose the failure rate function $h(t)$ is differentiable and BTR with change points t_1 and t_2. If $C_2(t)h(t)$ is not eventually constant, then the optimal burn-in time b^* and the corresponding optimal age $T^* = T^*_{b^*}$ satisfy

$$0 \le b^* \le t_1 \text{ and } T^* = T^*_{b^*} > 0.$$

Corollary 2. Suppose the failure rate function $h(t)$ is differentiable and BTR with change points t_1 and t_2. If $C_2(t)h(t)$ is non-decreasing and not eventually constant, then the optimal burn-in time b^* is 0 and the corresponding optimal age T^* exists uniquely.

4. Examples

Suppose that the failure rate of a component has a Weibull-exponential distribution i.e.

$$h(t) = \begin{cases} \alpha\beta(\frac{t}{\alpha})^{\beta-1} & 0 \le t \le t_1 \\ \alpha\beta(\frac{t_1}{\alpha})^{\beta-1} & t \ge t_1, \end{cases}$$

where $0 < \beta < 1$ is the shape parameter, α is the scale parameter and t_1 is the change point. The failure rate function is strictly decreasing on $[0, t_1]$ and stays at a constant for $t \ge t_1$. We take $c_0(b) = c_0 b$, $C_i(t) = c_i \exp(aH(t))$ for $i = 1, 2$ with $c_1 \le c_2$. Table 1 presents the

Table 1 Optimal burn-in time b^* and maintenance policy T^* for Weibull-exponential distribution

	t_1					
	1		2		3	
c_r	b^*	T^*	b^*	T^*	b^*	T^*
3.0	0.0098	2.9849	0.0142	3.7578	0.0165	4.3012
4.0	0.0080	3.2052	0.0118	4.0117	0.0141	4.5702
5.0	0.0066	3.3980	0.0100	4.2360	0.0120	4.8101
6.0	0.0057	3.5696	0.0086	4.4376	0.0105	5.0268
7.0	0.0049	3.7249	0.0076	4.6207	0.0090	5.2253
8.0	0.0043	3.8669	0.0066	4.7892	0.0081	5.4079
9.0	0.0038	3.9977	0.0058	4.9451	0.0072	5.5778
10.0	0.0034	4.1193	0.0054	5.0901	0.0066	5.7363

optimal burn-in time b^* and maintenance policy T^* when $\alpha = 1$, $\beta = 2/3$, $a = 0.5$, $c_0 = 0.5$, $c_1 = 1$, $c_2 = 2$ and various choice of c_r and t_1.

From Table 1 we notice that the optimal burn-in time b^* becomes shorter and T^* gets greater as c_r increases. We also notice that b^* and T^* get greater as t_1 increases.

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