Reliability Evaluation on Multi-State Flow Network

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ABSTRACT

We consider a multi-state flow network consisted of undirected links and focus on how to find efficiently the union of minimal paths transmitting a required flow when minimal paths are known.

Key Words : Multi-state link, Expanded minimal path, Expanded composite path, Maximum capacity flow

1. INTRODUCTION

In real fields, multi-state flow networks are considered more practically and reasonably than binary-state flow networks. According to graph theory, a flow network is modeled as a graph $G(V, E)$ which V and E represent a node set and a link set, respectively. In multi-state flow network, links have multi-states, and different capacities are assigned to each state of links. Therefore, a multi-state flow network is the network considering both connectivity and an amount of flow transmitted from source to terminal. Also, maximum capacity flow is considered when a flow is transmitted.

Many researchers have considered performance index or reliability as measures for evaluating the performance of multi-state flow networks when minimal paths or minimal cuts are known. Performance index is the expected value of source to terminal capacity divided by maximum source to terminal capacity, and network reliability is defined as the probability of transmitting the required amount of flow successfully from source node to terminal node.

Ref [10] suggest the method to evaluate performance index on multi-state flow network and use the expanded minimal paths (*emp*) representing all permutation of link states with non-zero capacity in each minimal path. But [9] presents the counter example that the method of [10] are incorrect in some cases. Ref [5], [6], [7], [12] and [13] use minimal paths to evaluate network reliability, and ref [2], [4], [8] and [13] use minimal cuts. Among these papers, [8], [9] and [12] consider the multi-state link capacities as well as node failures.

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Ref [13] suggest the algorithms which find all minimal paths vectors and all minimal cut vectors transmitting the required flow d, refered to as d-MPs and d-MCs, but [3] and [7] point out that the algorithm of [13] has many superfluous steps in finding all d-MCs and d-MPs, respectively, because the algorithm have to transform the original network to series-parallel network when the original network is not series-parallel network. Ref [7] use the flow conservation law to present a more efficient algorithm which can apply a directed multi-state flow network.

In this paper, we consider a multi-state flow network consisted of undirected links and propose a method for evaluating network reliability. Section 2 gives notations and assumptions, and an efficient algorithm is described in Section 3. Section 4 gives a numerical example to illustrate the method.

2. NOTATIONS AND ASSUMPTIONS

Notations

P	*mp*
C	*cp*
$\mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}$	link state vector of its corresponding path
$P_{\mathbf{x}}$	expanded *mp* with $\mathbf{x}$
$C_{\mathbf{z}}$	current expanded *cp* with $\mathbf{z}$
$W(C)$	MCF of the (sub)network induced by C
W_{ALL}	$= W(\{$ all links with their maximum states in the network $\})$
W_{min}	a required flow transmitted from source node to terminal node
$\lvert\cdot\rvert$	number of elements of $\cdot$
$\mathbf{u} = \mathbf{v}$	$u_i = v_i$ for all i and $\lvert\mathbf{u}\rvert = \lvert\mathbf{v}\rvert$

Assumptions

1. The nodes are perfect and each has no capacity limit.
2. The links are s-independent and have multi-states with known probabilities.
3. All links are undirected and each link flow is bounded by the link capacity.
4. No flow can be transmitted through a failed link.
5. The network is good iff a specified amount of flow can be transmitted from the source node to the terminal node.
6. The *mp* of the network, considering connectivity only, is known.

3. ALGORITHM

In multi-state network, we need information which link is functioning in which state. To obtain this information, at initialization, our method generates *emp* for representing all permutation of link states with non-zero capacity in each minimal path. For example, let (A, B) be a *mp* and link A and B have two states and three states containing state 0, respectively. Then, the *emp* of (A, B) are obtained as (A_1, B_1) and (A_1, B_2). Our algorithm focus on how to find efficiently the expanded composite paths (*ecp*), union of expanded paths (*ep*) consisted of *emp* and subpaths of *emp* (*esp*), transmitting a required flow. To do this, we present methods which make a comparison of *ep* given in 3.1. The algorithm given in 3.2 is basically following [5].

3.1 Comparison of expanded paths

Let $G_{\mathbf{x}}$ and $G'_{\mathbf{y}}$ be *emp* or *esp*. Two $G_{\mathbf{x}}$ and $G'_{\mathbf{y}}$, are *equal* when $G = G'$ and $\mathbf{x} = \mathbf{y}$, and the union of $G_{\mathbf{x}}$ and $G'_{\mathbf{y}}$, $G_{\mathbf{x}} \cup G'_{\mathbf{y}}$, are obtained by $G \cup G'$ with the link state vector which consists of the link state of uncommon links and the larger state of common links in G and G'. Also, the difference of $G_{\mathbf{x}}$ and $G'_{\mathbf{y}}$, $G_{\mathbf{x}} - G'_{\mathbf{y}}$, is a *esp* of $G_{\mathbf{x}}$ on $G'_{\mathbf{y}}$, and is consisted of the expanded links on $G_{\mathbf{x}}$ except the same expanded links in both $G_{\mathbf{x}}$ and $G'_{\mathbf{y}}$. For example, $(A_1, B_2) \cup (A_2, C_2)$ and $(A_1, B_2) - (A_1, C_2)$ are (A_2, B_2, C_2) and (B_2), respectively. Also, $(A_1, B_2) - (A_1, C_2)$ and $(A_1, B_2) - (A_1, D_2)$ are *equal.*

Let all links in G be in G'. Then, $G_{\mathbf{x}}$ is said to be a ***subset of*** $G'_{\mathbf{y}}$ if all elements of $\mathbf{y} - \mathbf{x}$ for common links are not negative, and it is denoted by $G_{\mathbf{x}} \subset G'_{\mathbf{y}}$. Also, $G_{\mathbf{x}}$ is said to be a ***proper subset of*** $G'_{\mathbf{y}}$ if all elements of $\mathbf{y} - \mathbf{x}$ are not negative for common links as $G \subset G'$ and $G \neq G'$, or at least one positive and 0's as $G = G'$. For example, both (A_1) and (A_1, B_1) are ***subsets of*** (A_1, B_1), (A_1) is a ***proper subset*** of (A_1, B_1) but (A_1, B_1) is not.

3.2 Algorithm

At initialization, we expand all *mp* that each of links in a *mp* obtains all permutations of link states with non-zero capacity, and the *emp* which all elements in a link state vector are 1's are considered as candidates added to current *ecp*. Set the *emp*'s in the set of additive failure *emp* (AFEMP) and others in the set of non-additive failure *emp* (NAFEMP), and set FEMP by {*emp*'s in AFEMP : *emp*'s in NAFESP}. The *ecp* transmitting a required flow is refered to as success *ecp* (*secp*).

Let $P_{\mathbf{x}}$ be the *ep* which gives the maximal increase on MCF among AFEMP, $C_{\mathbf{z}}$ be a current *ecp*, and ELGEMP be a set of *ep*'s preventing the generation of a *secp* containing the obtained *secp*. Set $C_{\mathbf{z}} = \emptyset$ and ELGEMP $= \emptyset$. If $W_{ALL} < W_{min}$, *STOP*. Otherwise, find $P_{\mathbf{x}}$ in AFEMP.

Case 1. $W(C_{\mathbf{z}} \cup P_{\mathbf{x}}) \geq W_{min}$

Record $C_{\mathbf{z}} \cup P_{\mathbf{x}}$ as a *secp*, and search for next *secp* with $C_{\mathbf{z}}$ and FEMP = FEMP $- \{P_{\mathbf{x}}\}$. Remove $P'_{\mathbf{y}}$ in NAFEMP if $P_{\mathbf{x}} \subset P'_{\mathbf{y}}$ and set ELGEMP = ELGEMP $\cup P_{\mathbf{x}}$.

Case 2. $W(C_{\mathbf{z}} \cup P_{\mathbf{x}}) < W_{min}$

Update $C_{\mathbf{z}} = C_{\mathbf{z}} \cup P_{\mathbf{x}}$ and FEMP = FEMP $- \{P_{\mathbf{x}}\}$. For efficient searching *secp*, check that there are *equal ep*'s in FEMP, and remove the *ep*'s except one, and some proper subsets among *ep*'s in FEMP are considered as candidates added to $C_{\mathbf{z}}$.

Case 3. There is no choice :

Retreat to the step where the last *ep* was added to generate, $C_{\mathbf{z}}$, at which time, $C_{\mathbf{z}} = C'_{\mathbf{z}'} \cup$ (last *ep*) for some $C'_{\mathbf{z}'}$. Remove $P'_{\mathbf{y}}$ in NAFEMP if $P_{\mathbf{x}} \subset P'_{\mathbf{y}}$ and set ELGEMP = ELGEMP$\cup$ $P_{\mathbf{x}}$.

4. EXAMPLE

We consider the multi-state flow network with undirected links. All links have three states, and their capacities and state probabilities are given in Figure 4.1. Let $W_{min} = 8$.

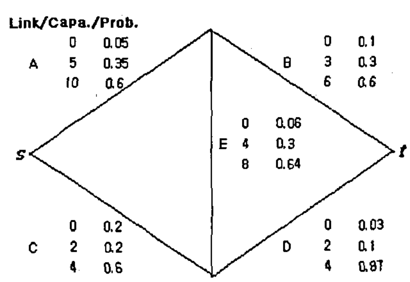

Figure 4.1: Bridge Network

In this network, we have 4 minimal paths: (A, B), (A, E, D), (C, D) and (C, E, B). As the minimal paths are expanded, the number of *ep* corresponding to (A, B), (A, E, D), (C, D) and (C, E, B) have 4, 4, 8, 8, respectively. We present one part of the whole process. Let the current *ecp*, $C_{\mathbf{z}}$, be (A_1, B_2), and FEMP and ELGEMP corresponding the *ecp* be $\{(C_1, D_1), (A_2), (C_1, E_1), (E_1, D_1) : (C_1, D_2), (C_2, D_1), (C_2, D_2), (E_1, D_2), (E_2, D_1), (E_2, D_2),$

$(A_2, E_1, D_1), (A_2, E_1, D_2), (A_2, E_2, D_1), (A_2, E_2, D_2), (C_1, E_2), (C_2, E_1), (C_2, E_2)\}$ and $\emptyset$, respectively. Since $W((A_1, B_2)) < 8$, Case 2 is considered.

Case 2.

We update the current *ecp*, C_z, with (C_1, D_1) which gives maximal increase on MCF among all *ep* in AFEMP. Then, $C_z = (A_1, B_2, C_1, D_1)(= (A_1, B_2) \cup (C_1, D_1))$ and $W(C_z)$ is 7. Also, FEMP = FEMP $- \{(C_1, D_1)\}$. Check that there are *equal ep*'s in FEMP, and remove the *ep*'s except one. Then, one (E_1) and one (E_2) are removed from AFEMP and NAFEMP, respectively. For updating FEMP, we consider that the proper subsets among *ep* in FEMP are candidates added to C_z. Then, AFEMP $= \{(A_2), (E_1), (D_2), (C_2)\}$ and NAFEMP = NAFEMP $- \{(D_2), (C_2)\}$ are obtained.

Case 1.

The MCF of the union of C_z and the *ep*, (A_2), is larger than W_{min}. Thus, record the union, (A_2, B_2, C_1, D_1), as a *secp*. Delete *ep* which contain (A_2) from NAFEMP and update ELGEMP = ELGEMP $\cup \{(A_2)\} = \{(A_2)\}$. Using ELGEMP, we can prevent the generation of a new C_z containing a *secp*.

Case 3.

Search for next *secp* with current C_z and *ep* in AFEMP. As the MCF of the union of the current *ecp* and any one *ep* in AFEMP is 7, and the MCF is less than W_{min}, we update the current *ecp* with E_1 to find next *secp*.

We omit the remaining procedure. In the following we obtain three more *secp*, $(A_1, B_2, C_2, D_1, E_1)$, (A_1, B_2, C_2, D_2) and (A_2, B_2, D_1, E_1). All 4 *secp* is also minimal success *ecp* (*msecp*).

By using the reliability evaluation method of [1] with all *msecp*, the network reliability, R, is obtained as:

$$\begin{aligned} R &= p_{A_2}p_{B_2}p_{C_1}p_{D_1} + p_{A_2}p_{B_2}(1 - p_{C_1})p_{D_1}p_{E_1} \\ &+ (p_{A_1} - p_{A_2})p_{B_2}p_{C_2}p_{D_1}p_{E_1} + (p_{A_1} - p_{A_2})p_{B_2}p_{C_2}p_{D_2}(1 - p_{E_1}). \end{aligned}$$

The probability p_{L_j} means $P\{$ state of link $L \geq$ state j of link $L\}$ where $L = A, B, C, D, E$ and $j = 1, 2$. Then, the reliability is 0.46647 according to the probabilities in Figure 4.1.

REFERENCES

[1] Aven, T. (1985), "Reliability evaluation of multistate systems with multimode components", IEEE Transactions on Reliability, **34**, 473-479.

[2] Jane, C.C., Lin, J.S. and Yuan, J. (1993), "Reliability Evaluation of a limited-flow network in terms of minimal cuts", IEEE Transactions on Reliability, **42**, 354-361.

[3] Hudson, J.C. and Kapur, K.C. (1983), "Reliability analysis for multistate systems with multistate components", IIE Transactions, **15**, 127-135.

[4] Hudson, J.C. and Kapur, K.C. (1985), "Reliability bounds for multistate systems with multistate components", Operations Research, **33**, 153-160.

[5] Lee, S.M. and Park, D.H. (2001). "An efficient method for evaluation of network reliability with variable link-capacities", IEEE Transactions on Reliability, **50**, 374-379.

[6] Lin, J.S., Jane, C.C and Yuan, J. (1995), "On reliability evaluation of a capacitated-flow network in terms of minimal pathsets", Networks, **25**, 131-138.

[7] Lin, Y.K. (2001), "A simple algorithm for reliability evaluation of a stochastic-flow network with node failure". Computers & Operations Research, **28**, 1277-1285.

[8] Lin, Y.K. (2002), "Using minimal cuts to evaluate the system reliability of a stochastic-flow network with failures at nodes and arcs". Reliability Engineering & System Safety, **75**, 41-46.

[9] Schanzer, R. (1995). "Comment on : Reliability modeling and performance of variable link-capacity networks", IEEE Transactions on Reliability, **44**, 620-621.

[10] Varshney, P.K., Joshi, A.R. and Chang, P.L. (1994) "Reliability modeling and performance evaluation of variable link capacity networks", IEEE Transactions on Reliability, **43**, 378-382.

[11] Yeh, W.C. (2001), "A simple algorithm to search for all *d*-MCs of a limited-flow network", Reliability Engineering & System Safety, **71**, 15-19.

[12] Yeh, W.C. (2001), "A simple algorithm to search for all *d*-MPs with unreliable nodes", Reliability Engineering & System Safety, **73**, 49-54.

[13] Xue, J. (1985), "On multistate system analysis", IEEE Transactions on Reliability, **34**, 329-337.