Block Designs for Comparisons within Two Groups of Inbred Lines in Diallel Crosses

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Abstract

Sometimes we have two groups of inbred lines and there are only interest in gca comparisons within group(p_1) and group(p_2) and not between two groups. For example, suppose there two Lab, each of the 2 Labs have obtained the best lines. For this purpose we now give a method of constructing block designs for diallel cross experiments and we will explain how to calculate efficiency. Then we show the efficiencies in the table.

Keywords : diallel crosses, general combining ability, comparisons within two groups of inbred lines .

1. Method of Construction

Although we are interested in $\widehat{g_i}-\widehat{g_j}$, $(i<j=1,2,\cdots,p_1)$ and $\widehat{g_i}-\widehat{g_j}$, $(i<j=p_1+1,\ p_1+2,\cdots,p_1+p_2)$ (that is, within group comparisons), for high efficiency for these comparisons, crosses between two groups are better, We now present a method of constructing block designs for comparisons within two groups of inbred lines. For this purpose, we construct block designs using Latin square design.

Let p_1, p_2 be the number of inbred lines in each group, and consider $p_1=c_1p_2+c_2$ ($c_1c_2>0$ integer). First, p_2 inbred lines are set out in an $p_2\times p_2$ array such that each inbred line occurs once in each row and once in each column of the array. We represent the array as L. For $p_1>p_2$, we take any (p_1-p_2) rows of p_2 rows belonging to L and augment them to columns of L such that the number of columns of the augmented L is p_1. In order to make cross between two groups, we superimpose p_1 inbred lines belonging to 1st group on each row of the augmented L in turn. Then making cross on the superimposed and augmented L, we construct a block design D with parameters.

$$p=p_1+p_2,\ b=p_2,\ k=p_1,\ r_c=1,\ n_c=p_1p_2,\ \lambda_1=0,\ \lambda_2=1$$

where $\lambda_i(\,i=1,2)$ denote the number of replicated times of cross between lines i and j belonging to ith group. We explain method of construction using following example.

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Example. For $p=8, p_1=5, p_2=3$, we have two groups of lines as following:

Group 1 : 1, 2, 3, 4, 5

Group 2 : 6, 7, 8

we can construct the following rectangular arrays in order to make cross between two group.

$$\begin{bmatrix} 6 & 7 & 8 & 6 & 7 \\ 7 & 8 & 6 & 7 & 8 \\ 8 & 6 & 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

In our method, block design is obtained by making cross between cross between the two groups.

Block 1: (1,6), (2,7), (3,8), (4,6), (5,7)

Block 2: (1,7), (2,8), (3,6), (4,7), (5,8)

Block 3: (1,8), (2,6), (3,7), (4,8), (5,6)

For the purpose of estimating gca effects within groups of inbred lines, we find information matrix C of design D. First, from our method of construction of D, it is easily verified that $g_{ii}(i=1,2,\cdots,p)$ of G is given by

$$g_{ii}=\begin{cases} p_2, & \text{if } i=1,2,\cdots,p_1 \\ p_1, & \text{if } i=p_1+1, p_1+2,\cdots,p_1+p_2 \end{cases}$$

Also, for non-diagonal elements of G, if two lines i and j belong to the same group, g_{ii} $(i<j=1,2,\cdots,p)=0$, otherwise $g_{ii}=0$. Next, find the elements of NN'. Note that for finding the number of within-block concurrences of two lines, the lines are taken as the contents of a block. First, we can see that the elements of NN' are p_2 when two lines i and j belong to 1st group, and p_1 when both lines i and j belong to the different group. Finally, find the elements of NN' when two lines i and j belong to 2nd group. In here, consider only 2nd group elements in which $p_1=c_1p_2+c_2$. Let d and λ^* be diagonal and non-diagonal element of NN' when two lines i and j belong to 2nd group. For $c_2=0$, it can be verified that the d and λ^* are given by

$$d=\lambda^*=p_2c_1^2$$

For $c_1=1$, let α be a inbred line belonging to 2nd group, Since all p_2 inbred lines belonging to 2nd group are replicated c_1 times in each block of p_2 blocks and α appears in only one block among p_2 blocks, we can see that the d and λ^* are given by

$$d=(c_1+1)^2+(p_2-c_2)c_1^2$$

$$\lambda^*=2c_1(c_1+1)+c_1^2(p_2-2)$$

Next, we find d and λ^* for $c_2\geq 2$. Similarly for $c_2=1$, all p_2 inbred lines are replicated c_1 times in each block of p_2 blocks, and c_2 inbred lines belonging to 2nd group are repeated in each block of p_2 blocks. We now restrict attention to the case in which the design constructed by the c_2 lines appearing in each block is a balanced incomplete block design with parameters

$$v=b=p_2,\ r=k=c_2,\ \lambda=\frac{c_2(c_2-1)}{p_2-1}$$

Let α,β be two lines belonging to 2nd group and $\lambda_{\alpha,\beta}$ be the numbers of concurrences of the lines α and β in the balanced incomplete block design. For determining d and λ^*, it is helpful to consider the following 3 cases separately for α,β in the balanced incomplete block design.

i) α,β occur together,

ii) $\alpha(\beta)$ occurs with a line other than $\beta(\alpha)$,

iii) α,β don't occur in any block among p_2 blocks.

For (i), α,β occur together in λ blocks and α,β are repeated (c_1+1) times, respectively. So, contribution to $\lambda_{\alpha,\beta}$ from λ blocks is $\lambda(c_1+1)^2$.

For (ii), appear to $c_2-\lambda$ blocks without $\beta(\alpha)$ and $\alpha(\beta)$ is repeated (c_1+1) times in these blocks. So, contribution to $\lambda_{\alpha,\beta}$ from $c_2-\lambda$ blocks is $2(c_2-\lambda)c_1(c_1+1)$.

Finally, for (iii), Since the number of remaining blocks is given by

$$p_2-\lambda-(c_2-\lambda)-(c_2-\lambda)$$

and both α and β in these blocks are repeated c_1 times. So, it can be seen that contribution to $\lambda_{(\alpha,\beta)}$ from remaining blocks is given by

$$c_1^2(p_2-2c_2+\lambda)$$

Thus,

$$d=(c_1+1)^2+(p_2-c_2)c_1^2,$$

$$\lambda^*=\lambda(c_1+1)^2+2(c_2-\lambda)c_1(c_1+1)+c_1^2(p_2-2c_2+\lambda)$$

For the purpose of calculating efficiency of block design D, we now explain how to calculate the variance of a contrast among gca parameters within two groups of inbred lines.

Theorem 1. Let σ_1^2,σ_2^2 be the variance of a contrast among gca parameters within each group of inbred lines.

Then,

$$\sigma_1^2=Var_1(\widehat{g_i}-\widehat{g_j})=\frac{2\sigma^2}{p^2},\quad i,j\langle 1,2,\cdots,p_1$$

$$\sigma_2^2=Var_2(g_i-g_j)=\frac{2p_1\sigma^2}{p_2\lambda^*},\quad i,j\langle p_1+1,p_1+2,\cdots,p_1+p_2$$

Proof. For block design D,

$$C=\begin{bmatrix} p_2I_{p_1} & J_{p_1p_2} \\ J_{p_2p_1} & p_1I_{p_2}\end{bmatrix}-\frac{1}{p_1}\begin{bmatrix} p_2I_{p_1} & p_1J_{p_1p_2} \\ p_1J_{p_2p_1} & B\end{bmatrix}$$

$$=\frac{1}{p_1}\begin{bmatrix} p_1p_2I_{p_1}-p_2J_{p_1} & 0 \\ 0 & p_1^2I_{p_2}-B\end{bmatrix}$$

where, $B=\begin{bmatrix} d & \lambda^* & \cdots & \lambda^* \\ \lambda^* & d & \cdots & \lambda^* \\ \vdots & \vdots & \ddots & \vdots \\ \lambda^* & \lambda^* & \cdots & d \end{bmatrix}$

Using d and λ^*, C is written by

$$C = \frac{1}{p_1}\begin{bmatrix} p_1p_2I_{p_1} - p_2J_{p_1} & 0 \\ 0 & \lambda^*(p_2I_{p_2} - J_{p_2}) \end{bmatrix}$$

$$= \begin{bmatrix} p_1p_2(I_{p_1} - \frac{1}{p_1}J_{p_1}) & 0 \\ 0 & \lambda^* p_2(I_{p_2} - \frac{1}{p_2}J_{p_2}) \end{bmatrix}$$

From the above C, we can see that C^- is given by

$$C^- = p_1\begin{bmatrix} \left(p_1p_2(I_{p_1} - \frac{1}{p_1}J_{p_1})\right)^- & 0 \\ 0 & \left(\lambda^* p_2(I_{p_2} - \frac{1}{p_2}J_{p_2})\right)^- \end{bmatrix}$$

$$= \frac{1}{p_2}\begin{bmatrix} I_{p_1} & 0 \\ 0 & \frac{p_1}{\lambda^*}I_{p_2} \end{bmatrix}$$

Hence the Theorem.

2. Efficiency of Block Design

In this section, we now calculate efficiency of block design obtained using our method of construction and give efficiencies of block designs for $p \le 24$. Let e_1^*, e_2^* be the efficiency in each group of D, respectively. Then the efficiencies e_1^* and e_2^* are following:

$$e_1^* = \frac{2\sigma^2}{p-2} / Var(\hat{g}_i - \hat{g}_j) = \frac{2\sigma^2}{p-2} / \frac{2\sigma^2}{p_2} = \frac{p_2}{p-2}$$

$$e_2^* = \frac{2\sigma^2}{p-2} / Var(\hat{g}_i - \hat{g}_j) = \frac{2\sigma^2}{p-2} / \frac{2p_1\sigma^2}{p_2\lambda^*} = \frac{p_2\lambda^*}{p_1(p-2)}$$

Also, from the above e_1^* and e_2^*, we can see that the efficiencies e_1 and e_2 using equation of Singh and Hinkelmann(1998) are given by

$$e_1(Adjusted\ e_1^*) = \left(\frac{p(p-1)}{2} / n_c\right)e_1^* = \frac{p(p-1)}{2p_1p_2}e_1^* = \frac{(p_1+p_2)(p_1+p_2-1)}{2p_1(p_1+p_2-2)}$$

$$e_2(Adjusted\ e_2^*) = \left(\frac{p(p-1)}{2} / n_c\right)e_2^* = \frac{p(p-1)}{2p_1p_2}e_2^* = \frac{p\lambda^*}{2p_1^2}\left(\frac{p-1}{p-2}\right)$$

Table 1. Efficiencies of block designs

p_1	p_2	b	k	e_1	e_2
3	2	2	3	1	1
3	3	3	3	1	1
4	2	2	4	0.938	1
4	3	3	4	1	1
4	4	4	4	1	1
5	2	2	5	0.84	1
5	3	3	5	0.933	1
5	4	4	5	1	1
5	5	5	5	1	1
6	2	2	6	0.778	1
6	3	3	6	0.857	1
6	5	5	6	1	1
6	6	6	6	1	1
7	2	2	7	0.735	1
7	3	3	7	0.804	1
7	4	4	7	0.873	1
7	5	5	7	0.943	1
7	6	6	7	1	1
7	7	7	7	1	1
8	2	2	8	0.703	1
8	3	3	8	0.764	1
8	4	4	8	0.825	1
8	5	5	8	0.886	1
8	6	6	8	0.948	1
8	7	7	8	1	1
8	8	8	8	1	1
9	2	2	9	0.679	1
9	3	3	9	0.733	1
9	4	4	9	0.788	1
9	5	5	9	0.843	1
9	6	6	9	0.897	1
9	8	8	9	1	1
9	9	9	9	1	1
10	2	2	10	0.66	1
10	3	3	10	0.709	1
10	5	5	10	0.808	1
10	6	6	10	0.857	1
10	7	7	10	0.907	1
10	9	9	10	1	1
10	10	10	10	1	1
11	2	2	11	0.645	1
11	3	3	11	0.689	1
11	4	4	11	0.734	1
11	5	5	11	0.779	1
11	6	6	11	0.824	1
11	7	7	11	0.869	1

p_1	p_2	b	k	e_1	e_2
12	2	2	12	0.632	1
12	3	3	12	0.673	1
12	4	4	12	0.714	1
12	6	6	12	0.797	1
12	11	11	12	1	1
12	12	12	12	1	1

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