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고이득 관측기를 이용한 적분 슬라이딩 모드 제어

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Integral sliding Mode Control with High-gain Observer

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Abstract

We consider a single-input-single-output nonlinear system which can be represented in a normal form. The nonlinear system has a modeling uncertainties including the input coefficient uncertainty. A high-gain observer is used to estimate the states variables to reject a modeling uncertainty. A globally bounded output feedback integral sliding mode control is proposed to stabilize the closed loop system. The proposed integral sliding mode control can asymptotically stabilize the closed loop system in the presence of input coefficient uncertainty.

I. Introduction

Since the separation principle does not hold in the nonlinear system, a high-gain observer has been used to reject disturbances due to the imperfect feedback cancellation and modeling uncertainty for a nonlinear system with relative degree higher than one system in the output feedback control[1]. The use of high-gain observer to estimate state variable results in the peaking phenomenon of the state variable. A globally bounded control has been introduced to reduce the peaking phenomenon[2]. Since the globally bounded control was introduced, some works in the various control schemes used the globally bounded control with high gain observer. The works[3] used the globally bounded control in the continuous control scheme. A state feedback controller was designed and analyzed first in the continuos control scheme, and then a Lipschitz property of the continuous controller was used to show that the output feedback controller can recover the state feedback properties. However an asymptotic stabilization was not be achieved due to the presence of a nonvanishing perturbation caused by the estimation error and modeling uncertainty, but an ultimate boundness was achieved. The works[4] used the integral control to achieve the asymptotic stability in the continuous control. The works[5] used a globally bounded control in the discontinuous control scheme such as a sliding mode control[7,8]. The works[5] also only achieved an ultimate boundness in the presence of a nonvanishing perturbation. In particular, an ultimate boundness was achieved in the presence of input coefficient uncertainty. Since the discontinuous controller does not have a Lipschitz property, the design and analysis are different with the continuous one. The work[6] used an integral control with sliding mode control to achieve an asymptotic stability in the presence of input coefficient uncertainty, but limited to the state feedback. We propose a new design scheme using an integral sliding mode control can asymptotically stabilize the closed-loop system with an high-gain observer in the presence of input coefficient uncertainty. We show that integral sliding mode control can reject disturbances due to the input coefficient uncertainty and estimation errors.

II. Problem statement

Consider the single-input single-output nonlinear system

$$\dot{w} = F(w) + G(w)u$$
$$y = h(w) \tag{1}$$

where $w \in R^n$ is the state, u is the control input, y is the measured output. Suppose that F, G, and h are sufficiently smooth function on $U \in R^n$, and $F(0)=0$, $h(0)=0$. Therefore the origin $w=0$ is an equilibrium point of the open loop system. Since we are interested in input-output linearizable nonlinear system, we assume the following assumption on the nonlinear system (1).

Assumption 1

For all $w \in U$,

● The system (1) has an uniform relative degree, i.e.,

$L_G h(w) = \cdots = L_G L_F^{n-2} h(w) = 0$ and $L_G L_F^{n-1} h(w) \neq 0$

● The mapping $x = T(w)$, defined by $x_i = L_F^{i-1} h(w)$, $1 \le i \le n$ and $x = [x_1\ x_2\ \cdots, x_n]^T$ is a proper map.

The uniform relative degree assumption is a necessary and sufficient condition for the mapping $x = T(w)$ to be a local diffeomorphism in the neighborhood of every $w \in U$[7]. The properness of the mapping $x = T(w)$ ensures that it is a diffeomorphism of U onto $T(U)$. The change of variables $x = T(w)$ transforms the system (1) into the following normal form

$$\dot{x} = Ax + B[f(x) + g(x)u]$$
$$y = Cx \tag{2}$$

$$A = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}_{n \times n}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n \times 1},$$

$C = [1\ 0\ \cdots\ 0]_{1 \times n}$.

where $g(x) \neq 0$, $\forall x \in T(U)$. Let $f_o(x)$ and $g_0(x)$ be a known nominal model of $f(x)$ and $g(x)$, respectively. Suppose that $f_o(x)$ and $g_0(x)$ are sufficiently smooth, $f_o(0)=0$, and $g_0(x) \neq 0$ for all $x \in T(U)$. Note that the mapping T could depend on unknown parameter, however the dependence of unknown parameter does not cause the problem, since we are interested in output feedback control. We also assume that the uncertainty of the equation (2) satisfies the following assumption which is a typical matching condition on the modeling uncertainty.

Assumption 2 For all $x \in T(U)$, there is a scalar Lipschitz function $\rho(x)$ such that

$$|f(x) - f_o(x)| \le \rho(x)$$
$$|g(x)/g_0(x) - 1| < k_g < 1 \tag{3}$$

where k_g is nonnegative constant.

Our goal is the design of output feedback controller to stabilize the nonlinear system given by the equation (2) over the domain $T(U) = D$.

III. Observer and sliding surface design

Since we are interested in an output feedback control, we use the following high-gain observer to estimate the state variable x,

$$\dot{\hat{x}}_i = \hat{x}_{i+1} + \frac{a_i}{\varepsilon^i}(y - \hat{x}_1), \quad i = 1, \cdots, n-1$$
$$\dot{\hat{x}}_n = \frac{a_n}{\varepsilon^n}(y - \hat{x}_1) + f_0(\hat{x}) + g_0(\hat{x})u \tag{4}$$

where $\hat{x}_i$ is the estimate of the state variables x_i and ε is a positive constant to be specified. The positive constant a_i are chosen such that the roots of the following equation are in the open left half plane.

$$s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n = 0$$

We rewrite the observer equation (4) into the compact form

$$\dot{\hat{x}} = A\hat{x} + B[f_0(\hat{x}) + g_0(\hat{x})u] + D(\varepsilon)LC(x - \hat{x}) \tag{5}$$

where $L = [a_1, \cdots, a_n]^T$, and $D(\varepsilon) = diag[1/\varepsilon\ 1/\varepsilon^2 \cdots 1/\varepsilon^n]$.

We choose the following sliding surface

$$S(\hat{x}, \sigma) = M\hat{x} + \sigma \tag{6}$$

where $M = [m_1, \cdots, m_{n-1}, 1]$ and m_i is chosen such that

$$\overline{A} = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ -m_1 & \cdots & \cdots & \cdots & -m_{n-1} \end{bmatrix}_{(n-1)\times(n-1)}$$

is Hurwitz matrix,

$\dot{\sigma} = -M(A\hat{x} + BK\hat{x}) - MD(\varepsilon)LC(x - \hat{x}) - r(\hat{x})$ where K is chosen such that $A + BK$ is a Hurwitz matrix, and

$$r(\hat{x}) = -\frac{1}{1-k_g}(\rho(\hat{x}) + k_g|f_0(\hat{x})| + k_g|K\hat{x}|)SGN(\hat{x}^TPB)$$

where $SGN(\cdot)$ denotes the signum function and P is a positive definite matrix such that $P(A+BK)+(A+BK)^TP=-I$. Note that $\dot{\sigma}=-M(A\hat{x}+BK\hat{x})$ was used for the state feedback case in the presence of input coefficient uncertainty[9]. The sliding surface (6) contains the estimate of state variable as well as an integrator output σ. The reason for the choice of the sliding surface will be clear as the stability analysis of the closed-loop system is progressed later on. Let $e_i = x_i - \hat{x}_i$ be the estimation error, and define the scaled variables $\zeta_i = (1/\varepsilon^{n-i})e_i$. The closed-loop equations (2) and (5) can be rewritten as

$$\begin{aligned} \dot{x} &= Ax + B[f(x) + g(x)u] \\ \varepsilon\dot{\zeta} &= (A-LC)\zeta + \varepsilon B[f(x) - f_0(\hat{x}) + \{g(x) - g_0(\hat{x})\}u] \end{aligned} \quad (7)$$

Note that $(A-LC)$ is a Hurwitz matrix. Let $V(x) = x^TPx$. Define

$$\begin{aligned} \Omega_r &= \{x \in R^n \mid V(x) \le v_r\} \subset D \\ \Omega_{\zeta_r} &= \{\zeta \in R^n \mid \|\zeta\| < c_\zeta/\varepsilon^{n-1}\} \\ \Omega &= \Omega_r \times \Omega_{\zeta_r} \end{aligned}$$

where v_r is a positive constant such that $v_r > \frac{1}{\lambda_{\min}(P)}(2\lambda_{\max}(P)(1+k_g)\delta_1\|PB\|)^2$ and δ_1 and c_ζ are arbitrary positive numbers. Note that $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum eigenvalue and maximum eigenvalue of the arguments, respectively. The set Ω is taken as the region of interest. We use a globally bounded control function as a control input. We will specify the control input u to make a globally bounded control later on. The following lemma states that the fast variables ζ decays very rapidly during a short time period with a globally bounded control. The proof of the lemma is the same as the proof of Lemma 1 in [7], hence it is omitted.

Lemma 1 *Consider the closed-loop system (7) and suppose that the control input u is globally bounded. Then, for all* $(x(0), \zeta(0)) \in \Omega_0 \subset \Omega$ *, there exist* ε_1 *and* $T_1 = T_1(\varepsilon) \le T_3$ *such that for all* $0 < \varepsilon < \varepsilon_1$, $\|\zeta\| < k_\zeta\varepsilon$ *for all* $t \in [T_1, T_4)$ *where* k_ζ *is some positive constant,* T_3 *is a finite time, and* $T_4 > T_3$ *is the first time* $x(t)$ *exits from the compact set* Ω.

Proof: see [7]

3.2. Globally bounded controller design

We will design globally bounded control as we opposed it in the previous section. Consider the function

$$\tilde{u}(\hat{x}) = \frac{1}{g_0(\hat{x})}[-f_0(\hat{x}) + K\hat{x} + r(\hat{x}) - \delta_1 SGN(S)] \quad (8)$$

We take a control input u as $\tilde{u}$, saturated outside set Ω_r. In particular, let

$$\tilde{u}_1 = \frac{1}{g_0(\hat{x})}[-f_0(\hat{x}) + K\hat{x} + r(\hat{x})], \quad \tilde{u}_2 = -\frac{\delta_1}{g_0(\hat{x})},$$

$s_i = \max_{\hat{x}\in\Omega_r}|\tilde{u}_i(\hat{x})|$, and take the control input

$$u = s_1 sat(\tilde{u}_1(\hat{x})/s_1) + s_2 sat(\tilde{u}_2(\hat{x})/s_2)SGN(S) \quad (9)$$

where $sat(\cdot)$ is the saturation function. One can verify that $u(\hat{x})$ is a globally bounded control input.

Lemma 2 *Consider the closed-loop system (7) with control input* u *defined by (9). Then*

- *the sliding mode condition* $S\dot{S} \le -\delta_2|S|$ *is satisfied as long as* $\|\zeta\| < k_\zeta\varepsilon$ *where* δ_2 *is some positive constant.*
- $\|\zeta\| < k_\zeta\varepsilon$ *for all* $t \ge T_1$.

Proof: The proof of this lemma has two parts. One part is to prove that the sliding mode condition is satisfied with the control input (9). The second part is to prove that ζ is $O(\varepsilon)$ for all $t \ge T_1$. The first part can be proved using the equation (9) and the fact that $u = \tilde{u}$ for $\hat{x} \in \Omega_r$ which is provided by $\|\zeta\| < k_\zeta\varepsilon$. Using the derivative of S along the trajectories of the equation (7), it can be seen that

$$\begin{aligned} S\dot{S} &= S[f_0(\hat{x}) + g_0(\hat{x})u - K\hat{x} - r(\hat{x})] \\ &= S[-\delta_1 SGN(S)] \\ &\le -\delta_2 SGN(S) \end{aligned}$$

where $\delta_2 < \delta_1$ is some positive constant. Lemma 1 implies that $\|\zeta\| < k_\zeta\varepsilon$ as long as the state variable $x \in \Omega_r$ for all time. Therefore we will show that $x \in \Omega_r$ for all time. Using $\hat{x} = x - \overline{D}(\varepsilon)\zeta$, the derivative of $V(x) = x^TPx$ along the trajectories of the equation (7) is given by

$$\begin{aligned} V(x) = \; & x^T P(Ax+BKx-BK\overline{D}(\varepsilon)\zeta) \\ & +(Ax+BKx-BK\overline{D}(\varepsilon)\zeta)^T Px \\ & +2x^T PB[f(x)-f(\hat{x})+f(\hat{x})-f_0(\hat{x}) \\ & +\frac{g(x)-g_0(\hat{x})}{g_0(\hat{x})}(-f_0(\hat{x})+K\hat{x}) \\ & +\frac{g(x)}{g_0(\hat{x})}r(\hat{x})-\frac{g(x)}{g_0(\hat{x})}\delta_1 SGN(S)] \end{aligned} \quad (10)$$

where $\overline{D}(\varepsilon)=diag[\varepsilon^{n-1},\varepsilon^{n-2},\cdots,1]$.
Using

$$r(\hat{x})=-\frac{1}{1-k_g}(\rho(\hat{x})+k_g|f_0(\hat{x})|+k_g|K\hat{x}|)SGN(\hat{x}^T PB),$$

$\hat{x}=x-\overline{D}(\varepsilon)\zeta$, it can be verified that

$$\begin{aligned} \dot{V}(x) \le \; & -\frac{V(x)}{\lambda_{max}(P)}+2(1+k_g)\delta_1\|PB\|\sqrt{\frac{V(x)}{\lambda_{min}(P)}} \\ & +O(\varepsilon) \end{aligned} \quad (11)$$

for sufficiently small ε. Note that k_1 and k_2 are independent with ε. Therefore $\dot{V}(x)\le 0$ for $V(x)>c_1$ where $c_1=\frac{1}{\lambda_{min}(P)}(2\lambda_{max}(P)(1+k_g)\delta_1\|PB\|)^2$,

Since $v_r > c_1$, x can not leave the set Ω_r.

Lemma 2 implies that there is a finite time to reach the sliding manifold $S=0$ and $S=0$ holds thereafter. We can reach the following conclusion after performing the Lyapunov analysis for the closed-loop system in the sliding manifold.

Theorem 1 *Consider the closed-loop system(7) with the control input (9). Suppose that Assumption 1 and 2 are satisfied. Then for all* $(x(0),\zeta(0))\in\Omega_0$, *there is* $\varepsilon_2>0$ *such that for all* $0<\varepsilon<\varepsilon_2$ *such that* $\lim_{t\to\infty}(x,\zeta)=0$ *and* σ *is bounded.*

Proof: Since the sliding mode condition is satisfied, the control input u can be replaced by

$$u_{eq}(\hat{x},\sigma)=\frac{1}{g_0(\hat{x})}[-f_0(\hat{x})+K\hat{x}+r(\hat{x})]$$

in the sliding manifold which is the same as the control input u defined in the equation (9) with $\delta_1=0$. Let $W(x,\zeta)=x^T Px+\zeta^T P_1\zeta$ where P_1 is a positive definite matrix such that $P(A-LC)+(A-LC)^T P=-I$. The derivative of $W(x,\zeta)$ along the trajectories of the equation (7) is given by

$$\begin{aligned} \dot{W}(x,\zeta) = \; & \dot{V}(x)|_{\delta_1=0}-\frac{1}{\varepsilon}\|\zeta\|^2+2\zeta^T PB \\ & [f(x)-f(\hat{x})+f(\hat{x})-f_0(\hat{x}) \\ & +\frac{g(x)-g_0(\hat{x})}{g_0(\hat{x})}(-f_0(\hat{x}))+K\hat{x}+r(\hat{x}))] \end{aligned}$$

where $\dot{V}(x)|_{\delta_1=0}$ denotes $\dot{V}(x)$ with $\delta_1=0$ in the equation (10). Using the inequality (11) with $\delta_1=0$, it can be verified that

$$\begin{aligned} \dot{W}(x,\zeta) \le \; & -\|x\|^2+2k_1\|x\|\|\zeta\|+2k_2\|\zeta\|^2-\frac{1}{\varepsilon}\|\zeta\|^2 \\ & +2k_3\|\zeta\|^2+2k_4\|\zeta\|\|x\| \\ = \; & -\|x\|^2+2k_5\|x\|\|\zeta\|+2k_6\|\zeta\|^2-\frac{1}{\varepsilon}\|\zeta\|^2 \\ = \; & -[\|x\|\|\zeta\|]\begin{bmatrix} 1 & k_5 \\ k_5 & \frac{1}{\varepsilon}-2k_6 \end{bmatrix}\begin{bmatrix} \|x\| \\ \|\zeta\| \end{bmatrix} \end{aligned}$$

for some positive constant k_3, k_4, k_5, and k_6. Note that $k_3 \sim k_6$ are independent with εLet $P_2=\begin{bmatrix} 1 & k_5 \\ k_5 & \frac{1}{\varepsilon}-2k_6 \end{bmatrix}$. P_2 is a positive definite matrix for sufficiently small ε. Thus implies that $\lim_{t\to\infty}(x,\zeta)=0$. Since $\hat{x}$ is bounded and $S\dot{S}\le -\delta_2|S|$, σ is bounded for all time.

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