The 3D Shape Optimal Design of Transformer Tank Shield by Using Parameterized Design Sensitivity Analysis

Yingying Yao, Jae Seop Ryu*, Chang Seop Koh*, and Dexin Xie

School of EE, Shenyang University of Technology, Shenyang, Liaoning 110023, P.R.China. yaoying62@hotmail.com
* School of ECE, Chungbuk National University, Cheongju, Chungbuk, 361-763, Korea. kohcs@chungbuk.ac.kr

***Abstract*—A 3D shape optimization algorithm integrates the geometric parameterization, 3D F.E. performance analysis, steepest descent method with design sensitivity and mesh relocation method. The design sensitivity of the surface nodal points is also systematically converted into that of the design variables for the application to parameterized optimization. The proposed algorithm is applied to the optimum design of tank shield model of transformer and the effectiveness is proved.**

***Index Terms*—shape optimization, mesh regeneration, design sensitivity, deformation theory, eddy current.**

I. Introduction

For the 3D shape optimal design, the deterministic method combined with the design sensitivity analysis is thought to be, even more, a good choice [1,2,4]. In the design sensitivity analysis, the moving points are the nodes on design surface. Sometimes, the design variables are geometric dimensions. It is necessary to parameterize the design surface and find the relationships between the design variables and moving points. Another important part of the shape optimal design is renewing the finite element mesh as the design variables change. In this process the mesh distortion must be minimized in order to get an accurate finite element analysis results, and the newly generated mesh should maintain the same topology with the previous mesh[3].

In this paper, a parameterized design sensitivity formula for the 3D eddy current problem is derived in complex form using finite element and adjoint variable methods. A 3D mesh regeneration method, based on the deformation of the elastic body under stress, is also presented. Using the method, a topologically constant 3D mesh with relatively good quality is obtained. Finally a 3D shape optimal design algorithm is developed by integrating the finite element performance analysis, the steepest descent method with design sensitivity and the mesh relocation method. The developed algorithm is applied to the optimal design of the tank shield model of transformer.

II. Design Sensitivity for 3D Eddy Current Problem

The governing equations for the 3D electromagnetic system with time harmonic excitation is:

$$[\dot{S}][\dot{X}]=[\dot{Q}]. \tag{1}$$

With the help of adjoint variable $[\lambda]$, the design sensitivity of the objective function with respect to the design variables can be calculated as follows[4]:

$$\frac{dF}{d[p]^T}=\frac{\partial F}{\partial[p]^T}+2\mathrm{Re}\left([\lambda]^T\frac{\partial}{\partial[p]^T}\left[[\dot{Q}]-[\dot{S}][\tilde{\dot{X}}]\right]\right) \tag{2}$$

$$[\dot{S}]^T[\lambda]=\frac{\partial F}{\partial\dot{X}} \tag{3}$$

where F is the objective function, $[p]$ is the design variable vector composed of the surface nodal points, $\dot{X}$ is the state variable, respectively. $[\tilde{\dot{X}}]$ is the solution of (1). It can be seen that the adjoint variable $[\lambda]$ is independent of the number of design variables.

The design variables are renewed, using the computed design sensitivity, as follows:

$$[p]_{new}=[p]_{old}-\alpha F(p)\frac{dF}{d[p]^T}\Big/\left|\frac{dF}{d[p]^T}\right|^2 \tag{4}$$

where α is the relaxation factor.

III. Parameterization of the Tank Shield

In the design sensitivity formula, it is assumed that the design variables are surface nodal points. When the shape is parameterized, the design variables are not the surface nodal points themselves but the parameters. The design sensitivity for the design variables, hence, should be computed by using those for the surface nodal points. The tank shield of transformer, in this paper, is parameterized in the following two ways. The first parameterization is achieved, as shown in Fig. 1, using the linear functions. In this case, the coordinate of the nodal point on the design surface is given as:

$$x_i=L-x_k+\frac{L_{k+1}-L_k}{y_{k+1}^0-y_k^0}(y_i-y_k^0),\quad \text{when}\ \ y_{k-1}^0\le y_i\le y_k^0 \tag{5}$$

where (x_i, y_i) is the coordinates of node i and L, L_k, y_k^0 are explained in Fig.1. The design variables are defined as the vertexes $L_k (k=1,2,3,4)$.

Another parameterization is done, as shown in Fig. 2, using the step functions. In this case, the coordinate of the nodal point on the design surface is expressed as follows:

$$x_i = L - L_k, \qquad \text{when } y_{k-1}^0 \le y_i \le y_k^0 \tag{6}$$

where (x_i, y_i) is the coordinates of node i, and L, L_k, y_k^0 are shown in Fig.2. The thickness of the each step, $L_k (k=1,2,3,4)$, are taken as the design variables.

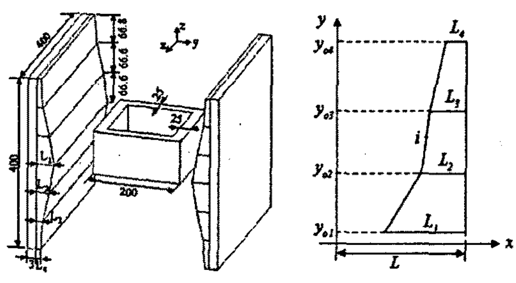
The relation between the design variables and the coordinate of the nodal points on the design surface can be writen in following matrix form from (5) and (6):

$$[p] = [p_0] + [\phi][C] \tag{7}$$

where $[p]$ is the vector composed of the coordinate of the nodal points on the design surface, the coefficient matrix $[\phi]$ is $(ns \times 4)$ if the number of the nodal point on the design surface is ns, and $[C]$ is the design variable vector.

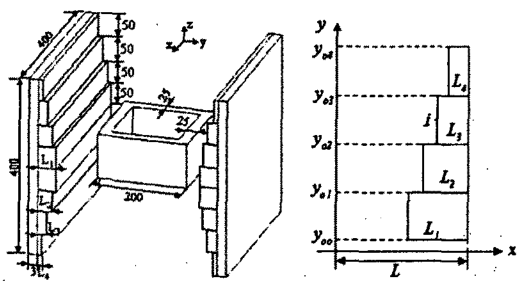
The design sensitivity for the design variable can be computed using (2) and (7) as follows:

$$\frac{dF}{d[C]} = \frac{dF}{d[p]}\frac{\partial[p]}{\partial[C]} = \frac{dF}{d[p]}[\phi] \tag{8}$$



(a) (b)

Fig. 1 Parameterization of the tank shield using linear functions.
(a) overall view, (b) parameters.



(a) (b)

Fig. 2 Parameterization of the tank shield using step functions.
(a) overall view. (b) parameters.

IV. Mesh Relocation using Structural Deforming Analysis

In the shape optimization process, the finite element mesh is needed to be modified or regenerated according to the shape change. If the deformation theory of the elastic body under stress is applied, the consistent and interrelated properties make the original mesh change proportionally and continuously, and a smooth geometric contour of the shape can be obtained[3,4].

The gaverning equation of elastic deformation analysis is used for the mesh relocation during the optimal shape design of electromagnetic device as

$$[\mathbf{K}]\{\Delta x\} = \{\mathbf{f}_s\} \tag{9}$$

where $[\mathbf{K}]$ is the global stffiness matrix for stress analysis, $\{\Delta x\}$ is the nodal displacement, that is the amount of relocation of the nodal coordinates $\{\mathbf{x}\}$, and $\{\mathbf{f}_s\}$ is a fictious load force to control the mesh density appropriately. The perturbation of the boundary can be simply considered as a displacement at the boundary. With no additional external forces and a given displacements at the boundary, (9) can be used to find the displacements of the whole nodes. Rewrite equation (9) as follows in segmented form:

$$\begin{bmatrix} K_{bb} & K_{bd} \\ K_{db} & K_{dd} \end{bmatrix} \begin{Bmatrix} \Delta x_b \\ \Delta x_d \end{Bmatrix} = \begin{bmatrix} f_b \\ 0 \end{bmatrix} \tag{10}$$

where $\{\Delta x_b\}$ is the known perturbation of nodes on the boundary, $\{\Delta x_d\}$ is the unkown nodal displacement vector for the interior nodes, and $\{f_b\}$ is the fictitious boundary force acting on the boundary. The unknown interior nodal displacement vector can be obtained from the following equation:

$$[K_{dd}]\{\Delta x_d\} = -[K_{db}]\{\Delta x_b\} \tag{11}$$

In order to evaluate $\{\Delta x_d\}$, it is necessary to suppress all the degrees of freedom that represent the fixed shape contour of a domain in the finite element analysis. Since this structural analysis is merely used to get a proper relocation of the interior nodes from the displacement of the surface nodes, no emphasis is placed on simulating actual deformation of a physical structure. In this reason the material parameters related to (9) could be free to choose. In order to limit the computation efforts for mesh regeneration, only a part of the electromagnetic analysis region can be defined as the structural analysis.

V. Numerical Shape Optimization Examples

By integrating the finite element performance analysis, the steepest descent method with design sensitivity and the mesh relocation method, a novel 3D shape optimal design algorithm is developed, and summarized in Fig.3 The developed algorithm is applied to the optimal design of the tank shield model of transformer. The transformer tank shield models, shown in Fig.1 and Fig. 2, are the benchmark model proposed by the Investigation Committee of the IEE of Japan for reducing the volume of shielding plate and for constraining the maximum eddy current density J_{em} at the tank within a specified value $J_{ems}(0.24\times10^6\,A/m^2)$ in order to avoid the local over heating[2]. The tank plate is made of conducting steel, whose conductivity and relative permeability are $0.75\times10^7 S/m$ and 1000, respectively, while the shielding plate is made of non-conducting grain-orinted silicon steel of which the relative permeabilities are 3000 and 30 for the easy and hard axes, respectively. The exciting current has 5484 AT (12A(max), 457 turns, 60Hz).

The objective function and constrains are defined as follows:

$$F=\begin{cases} F_V = V\,[m^3] & \text{while } J_{em} < J_{ems} \\ F_J = (J_{em} - J_{em0})^2\,[A/m^2] & \text{while } J_{em} \ge J_{ems} \end{cases}$$

$$0 < L_1, L_2, L_3, L_4 < 0.01\ [m]$$

where J_{em} and J_{ems} are the computed and maximum allowable values of the maximum eddy current densities in the tank plate, respectively, and J_{em0} is set to be less then J_{ems}.

The tank shield shape is parametrerized in two ways, as shown in Fig. 1 and Fig. 2, using the linear functions and step functions, respectively. In both parameterizations, the design variables are taken as the dimensions L_1, L_2, L_3, L_4, and the design sensitivities are calculated as follows:

$$\frac{dF}{d[L]} = \beta\frac{dF_J}{d[L]} + \gamma\frac{dF_V}{d[L]}$$

where the coefficients (β,γ) are set to (0.7,0.3) when J_{em} is larger than J_{ems}, and (0.3,0.7) when J_{em} is less then J_{ems}, respectively.

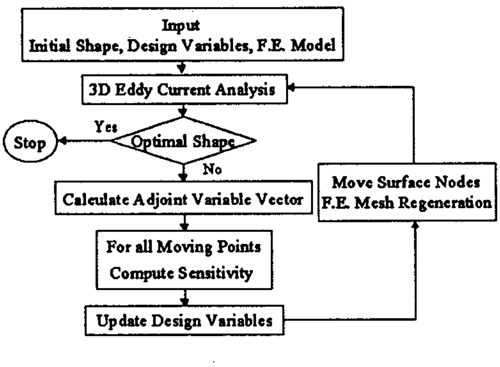


Fig.3 The optimization system based on design sensitivity analysis

When the tank shield shape is parameterized using the linear functions, the optimum result is obtained after 20 iterations. Fig.4 shows the variations of the maximum eddy current density at the tank and the volume of the tank shield during the optimization process. The initial and optimized dimensions of the tank shield are compared in Table I. It can be seen that the maximum eddy current density at the tank is kept less than the specified value while the volume of the shielding plate is much reduced. Fig. 5 compare the distributions of the magnetic flux density at the symmetric plane, and the distributions of the eddy current at the tank, respectively, for the initial and optimized shapes. The relocated meshes for the shielding plate during the optimization process are shown in Fig. 6.

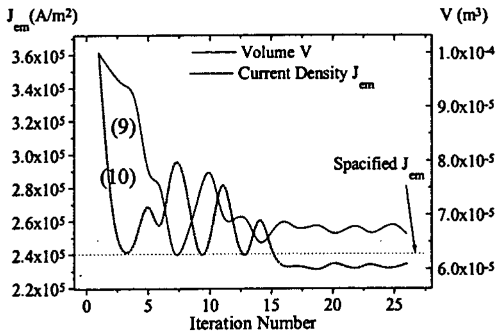
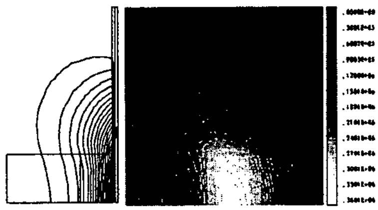


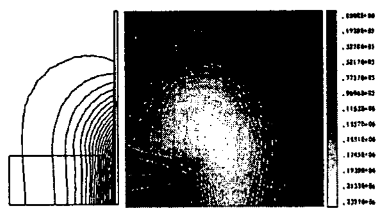
Fig.4 The variations of the maximum eddy current density and the volume when the shape is parameterized using the linear function..

TABLE I
RESULTS OF THE SHIELD OPTIMIZATION

	L_1	L_2	L_3	L_4	V	J_{em}
	(mm)				(m³)	(A/m²)
Initial	2.50	2.50	2.50	2.50	1.000E-4	3.64026E5
Optimal	4.14	2.16	0.57	0.30	0.659E-4	2.35463E5



(a) with the initial shape of shielding plate,



(b) with the optimized shielding plate.

Fig.5 Distributions of the magnetic flux line and the eddy current density.

When the tank shield is parameterized using the step functions, the correspondence results are shown in Fig.7, Fig.8, Fig.9 and Table II.

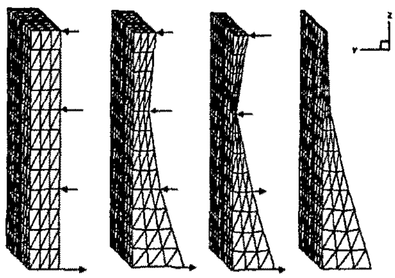


Fig.6. The relocated meshes for the shielding plate during the optimization process, where the arrows indicate the moving directions and amounts computed from design sensitivity.

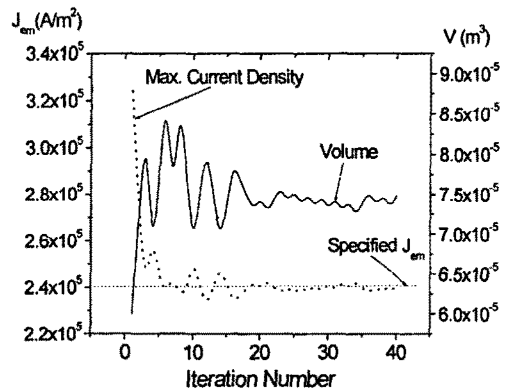


Fig.7 The variations of the maximum eddy current density and the volume when the shielding plate is parameterized using the step functions.

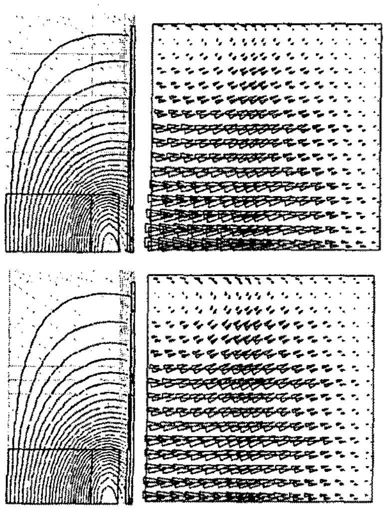
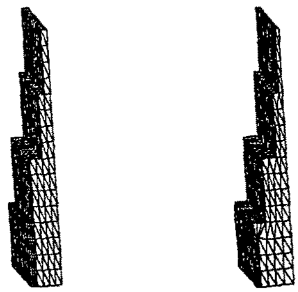


Fig.8 Distributions of the magnetic flux and eddy current density. (a) with the initial shape of shielding plate, (b) with the optimized shielding plate.

TABLE II

DIMENSIONS OF THE SHIELDING PLATE WHEN PARAMETERIZED USING STEP FUNCTIONS

	L_1 (mm)	L_2 (mm)	L_3 (mm)	L_4 (mm)	V (m^3)	J_{em} (A/m^2)
Initial shape	2.50	2.00	1.00	0.50	0.600E-4	3.23975E5
Optimal shape	3.44	2.29	1.27	0.49	0.749E-4	2.39120E5



(a) (b)

Fig.9 The relocated meshes for the shielding plate when it is parameterized using the step functions. (a) initial shape, (b) optimized shape.

V. CONCLUSIONS

A new 3D shape optimization algorithm is developed for the electromagnetic devices carrying the eddy current. In the algorithm, the 3D finite element analysis, steepest descent method with design sensitivity, and mesh relocation method are combined. Through the numerical applications to the tank shield of transformer, the strategy using the adjoint variable and design sensitivity is proved to be very effective for the 3D shape optimization with small computational efforts.

REFERENCES

[1] S.Wang and J.Kang, (2000), "Shape optimization of BLDC motor using 3D finite element method", IEEE Trans. on Magn., vol.36, No.4, pp.1119-1123.

[2] N.Takahashi, M.Horii, (2000), "Optimal design of tank shield model of transformer", IEEE Trans. on Magn., vol.36, No.4, pp.1089-1093.

[3] Konrad Weeber and S.R.H. Hoole,(1992), "A structural mapping technique for geometric parameterization in the optimization of magnetic devices", International Journal for Numerical Methods in Engineering, vol.33, pp.2145-2179.

[4] Yingying Yao, Jae Seop Ryu, Chang Seop Koh and Dexin Xie, "3D Shape Optimization of Electromagnetic Device Includeing Eddy Currents by Using Design Sensitivity Analysis", CEFC'2002, Italy