Estimation of Thermal Stresses Induced in Polymeric Thin Film Using Boundary Element Methods

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Abstract

The residual thermal stresses at the interface corner between the elastic substrate and the viscoelastic thin film due to cooling from cure temperature down to room temperature have been studied. The polymeric thin film was assumed to be thermorheologically simple. The boundary element method was employed to investigate the nature of stresses on the whole interface. Numerical results show that very large stress gradients are present at the interface corner and such stress singularity might lead to edge cracks or delamination.

1. Introduction

Residual stresses induced into thin film deposited on an elastic substrate can have a major effect on the interface stresses present in such materials. Such stresses are due to the differences between the thermal expansion coefficients of the components. Residual thermal stresses may cause distortion of finished components and premature failure upon external loading.

Residual stresses induced in the viscoelastic thin layer have received much attention. Weitsman(1979) analyzed the mechanical behavior of an epoxy adhesive layer as the adhesive absorbs moisture from the ambient environment. Delale and Erdogan(1981) presented the viscoelastic behavior of an adhesively bonded lap joint.

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Lee(1998) performed the boundary element analysis of the stress singularity for the viscoelastic adhesive layer under transverse tensile strain.

In this study, the residual thermal stresses developed at the interface corner between the elastic substrate and the viscoelastic thin film of a two-dimensional laminate model due to cooling from the cure temperature down to room temperature are investigated. A thermorheologically simple material behavior for the viscoelastic thin film is assumed. The detailed analysis is performed by using the boundary element method.

2. Order of Stress Singularity

The region near the interface corner between perfectly bonded elastic and viscoelastic quarter planes is shown in Fig.1. In the following, a condition of plane strain is considered. A solution of

$$\nabla^4 \Phi(r,\theta:\xi)=0 \tag{1}$$

is to be found such that the normal stress, σ_θ, and shear stress, $\tau_{r\theta}$, vanish along $\theta=\pm(\pi/2)$, further that the displacements and stresses are continuous across the common interface line $\theta=0$. Here r and θ are defined in Fig.1. The solution of this problem is facilitated by the Laplace transform, defined as

$$\Phi^*(r,\theta;p)=\int_0^\infty \Phi(r,\theta;\xi)e^{-p\xi}dt \tag{2}$$

where Φ^* denotes the Laplace transform of Φ and p is the transform parameter. Then eq.(1) can be rewritten using eq.(2) as follows:

$$\nabla^4 \Phi^*(r,\theta:p)=0 \tag{3}$$

Nontrivial solutions that satisfy the plane strain equations of linear elasticity and the boundary conditions can be shown to exist only when s satisfies the following characteristic equation (Bogy 1968):

$$\left[(m_1(p)-m_2(p))\cos^2\left(\frac{s\pi}{2}\right)-m_1(p)(s+1)^2\right]^2+m_3^2(p)\cos^2\left(\frac{s\pi}{2}\right)\sin^2\left(\frac{s\pi}{2}\right)$$
$$-m_2^2(p)(s+1)^2=0 \tag{4}$$

where

$$m_1(p) = 2\left(\frac{1}{\mu_{II}} - \frac{1}{p\mu^*_I}\right)$$

$$m_2(p) = \frac{4}{\mu_{II}}(1-\nu_{II}) - \frac{4}{p\mu^*_I}(1-p\nu^*_I)$$

$$m_3(p) = \frac{4}{\mu_{II}}(1-\nu_{II}) + \frac{4}{p\mu^*_I}(1-p\nu^*_I) \tag{5}$$

Eq.(4) has a form identical with that of two bonded elastic quarter planes if $p\nu^*_I$ and $p\mu^*_I$ are associated with the elastic constants ν_I and μ_I. The calculation of roots of eq.(4) actually can be reduced to two transformed material parameters $\alpha^*(p)$ and $\beta^*(p)$ which are associated with Dundurs' parameters α_D, β_D (Dundurs 1969).

In plane strain, $p\alpha^*(p)$ and $p\beta^*(p)$ are defined as follows:

$$p\alpha^*(p) = \frac{p\mu^*_I(1-\nu_{II}) - \mu_{II}(1-p\nu^*_I)}{p\mu^*_I(1-\nu_{II}) + \mu_{II}(1-p\nu^*_I)}$$

$$p\beta^*(p) = \frac{p\mu^*_I(1-2\nu_{II}) - \mu_{II}(1-2p\nu^*_I)}{2p\mu^*_I(1-\nu_{II}) + 2\mu_{II}(1-p\nu^*_I)} \tag{6}$$

For the problem of two dissimilar bonded elastic quarter planes, it can be easily verified that transformed material parameters $\alpha^*(p)$ and $\beta^*(p)$ are inverted into Dundurs' parameters α_D, β_D. The time dependent behavior of the problems is recovered by inverting eq.(4) into the real time space.

3. Boundary Element Analysis of Interface Stresses

Fig.2 shows the two-dimensional plane strain model for analysis of the micro stresses at the interface corner due to cooling from cure temperature down to room temperature. The analysis model of Fig.2(b) is divided into two sub domains, one viscoelastic zone and one elastic zone; in each zone the material is isotropic and homogeneous. This problem can be treated using a viscoelastic boundary integral formulation for viscoelastic zone and a separate elastic boundary integral formulation for the elastic zone. These boundary integral equations formulated for every homogeneous zone plus the displacement continuity and traction equilibrium conditions over the common interface produce a system that can be solved once the external boundary conditions are considered.

The uniform temperature changes $T(t)H(t)$ in each zone of analysis model are equivalent to increasing the tractions by $\gamma^e T(t) n_k$ and $\gamma^v \Theta(t) n_k$ where

$$\gamma^e = 3K^e \alpha_e \qquad (7)$$
$$\gamma^v = 3K^v \alpha_o$$

and $H(t)$ is the Heaviside unit step function. Here, 'e' and 'v' represent the elastic and viscoelastic zones, respectively, K is the bulk modulus, n_k are the components of the unit outward normal to the boundary surface, and α is the coefficient of thermal expansion.

The "pseudo-temperature" $\Theta(t)$ and α_o are defined by

$$\Theta(\xi) = \frac{1}{\alpha_o} \int_{T_o}^{T(\xi)} \alpha_v(T')dT' \qquad (8)$$
$$\alpha_o = \alpha_m(T_o)$$

where α_v is the coefficient of the thermal expansion of the viscoelastic film.

Assuming that no body forces exist, the boundary integral equations for the model under uniform temperature change can be written as follows:

For the elastic zone

$$c^e_{ij}(\mathbf{y}) u^e_j(\mathbf{y},\xi) + \int_{S^f} u^e_j(\mathbf{y}',\xi) T^e_{ij}(\mathbf{y},\mathbf{y}') dS^e(\mathbf{y}') \qquad (9)$$
$$= \int_{S^f} t^e_j(\mathbf{y}',\xi) U^e_{ij}(\mathbf{y},\mathbf{y}') dS^e(\mathbf{y}') + \int_{S^f} \gamma^e T(\xi) n_j U^e_{ij}(\mathbf{y},\mathbf{y}') dS^e(\mathbf{y}')$$

For the viscoelastic zone

$$c^v_{ij}(\mathbf{y}) u^v_j(\mathbf{y},\xi) \qquad (10)$$
$$+ \int_{S^m} \left[u^v_j(\mathbf{y}',\xi) T^v_{ij}(\mathbf{y},\mathbf{y}';0+) + \int_0^{\xi} u^v_j(\mathbf{y}',\xi-\xi') \frac{\partial T^v_{ij}(\mathbf{y},\mathbf{y}';\xi')}{\partial \xi'} d\xi' \right] dS^v(\mathbf{y}')$$
$$= \int_{S^m} \left[t^v_j(\mathbf{y}',\xi) U^v_{ij}(\mathbf{y},\mathbf{y}';0+) + \int_0^{\xi} t^v_j(\mathbf{y}',\xi-\xi') \frac{\partial U^v_{ij}(\mathbf{y},\mathbf{y}';\xi')}{\partial \xi'} d\xi' \right] dS^v(\mathbf{y}')$$
$$+ \int_{S^m} \left[\gamma^v \Theta(\xi) n_j U^v_{ij}(\mathbf{y},\mathbf{y}';0+) + \int_0^{\xi} \gamma^v \Theta(\xi-\xi') n_j \frac{\partial U^v_{ij}(\mathbf{y},\mathbf{y}';\xi')}{\partial \xi'} d\xi' \right] dS^v(\mathbf{y}')$$

where u_j and t_j denote the displacement vector and the traction vector in reduced time space, respectively, and S is the boundary of the given domain. c_{ij} is dependent only upon the local geometry of the boundary. For y on a smooth surface, the free term c_{ij} is simply a diagonal matrix $0.5\delta_{ij}$. U_{ij} and T_{ij} represent the fundamental solutions and

$$\xi = \int_0^t \frac{1}{a_T[T(\lambda)]} d\lambda \tag{11}$$

The shift function a_T is a basic property of the material and must, in general, be determined experimentally.

Closed form integrations of eqs. (9) and (10) are not, in general, possible and therefore numerical quadrature must be used. Approximations are required in both time and space. Eqs.(9) and (10) can be solved in a step by step fashion in time by using the modified Simpson's rule for time integrals and employing the standard boundary element method for the surface integrals(Lee and Westmann 1995).

The properties for the viscoleastic material are from experimental data(Weitsman 1979):

$$\begin{aligned} E^v(\xi) &= \frac{3.2\times 10^3}{1+0.0336\xi^{0.19}} MPa \qquad (\xi : \text{min}) \\ K^v(\xi) &= K_o = 3.556\times 10^3 MPa \\ a_T &= \exp\left[\frac{6480}{T} - 21.82\right] \\ \alpha_v &= 0.5\times 10^{-4} \quad {}^oC^{-1} \end{aligned} \tag{12}$$

Fig.3 shows the variation of the stress singularity factor. It is shown that the stress singularity factor is relaxed with time.

4. Conclusions

The singular stresses at the interface corner between the elastic substrate and the viscoelastic thin film of a two-dimensional laminate model subjected to a uniform temperature change have been investigated by using the viscoelastic boundary element method. The very large stress gradients are present at the interface corner and such stress singularity dominates a very small region relative to film thickness. Since the exceedingly large stresses at the interface corner cannot be borne by the viscoelastic thin film, edge cracks or delamination can occur in the vicinity of free surface.

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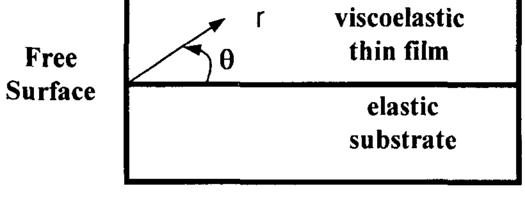

Fig.1 Region near interface corner between the elastic substrate and the viscoelastic thin film

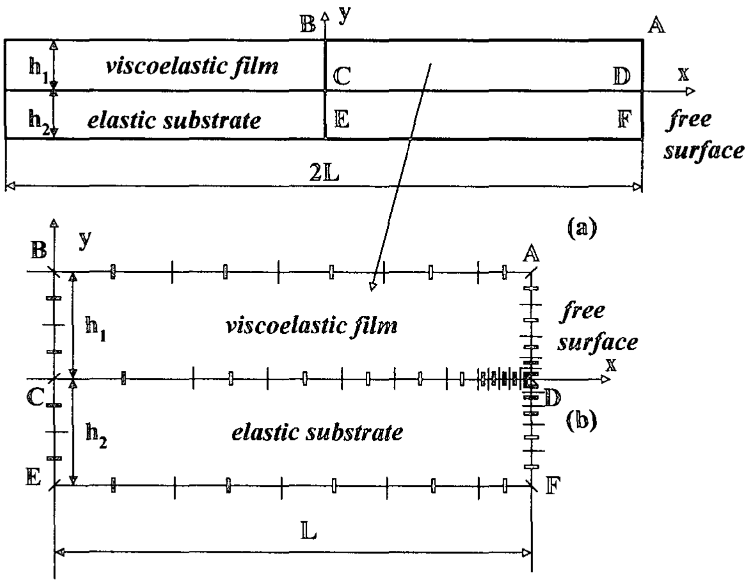

Fig.2 Boundary element model for determination of interface stresses.

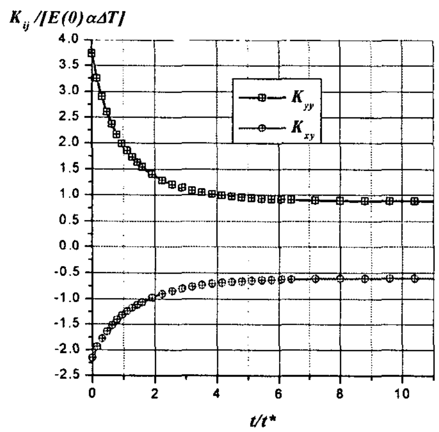

Fig.3. Variation of the stress singularity factors